Mathematical and Computational Analysis of Moebius Strip

HOOI MIN YEE and SAMSUDIN

***Abstract*—** This research highlights the problem of mathematical modelling of the Moebius strip. Shape-finding of tensioned membrane surface bordered by Moebius strip is investigated. Moebius strip has the mathematical property of being non-orientable and with only one side and only one boundary component. In this research, the possibility of adopting the form of Moebius strip as surface shape for tensioned membrane structure has been scrutinised. The combination of shape and internal forces for the purpose of stiffness and strength is an important feature of tensioned membrane surface. For this purpose, shape-finding needs to be carried out. Nonlinear analysis method is used for computational shape-finding analysis in this research. Development and pattern of prestress in the resulting tensioned membrane surface is also studied. Shape-finding has been found to converge for Moebius strip with midcircle radius R over half-width W, R/W= 0.7, 1.2, 1.7, 2.2 and 2.7. The way of mathematical modelling presented in this paper forms the basis for computer designer to consider the Moebius strip, R/W= 0.7, 1.2, 1.7, 2.2 and 2.7 applied in tensioned membrane structure. Such in-sight will lead to improvement of rural basic infrastructure, economic gains, sustainability of built environment and green technology initiative.

***Keywords*—N**on-orientable, nonlinear analysis method, shape-finding and tensioned membrane.

I. Introduction

Tensioned Membrane Structures (TMS) include a wide variety of systems that are distinguished by their reliance upon tensile only members to support load. TMS have been employed throughout recorded history as in rope bridges and tents. However, large permanent tension structures were generally a 19th century development in bridges and a 20th century development in buildings. Tensioned membrane structure is the best alternative to cover area with low cost. One of the greatest benefit is their translucent. Woven fabric coated with a polymeric resin allows a light transmission value of around 10%. This provides a very comfortable level of illumination compared to the full brightness of outside the structure.

One of the most exciting shapes that captured the focus of the mathematics is Moebius strip surface. Moebius strip surface is a surface with only one side, one boundary component and can be made by twisting a strip by 360° and joining both ends of the strip. It was discovered by two German mathematicians name August Ferdinand Moebius and Johann Benedict Listing in 1858.

Moebius strip has the mathematical property that is non-orientable surface. [1] have mentioned that Moebius strip surface has great potential be as an architecture form. Reference [2] have carried out the possibility implement Moebius strip surface in a shell structure. References [3]-[4] have proposed nonlinear analysis method for shape-finding of tensioned membrane structures in the form of Moebius strip. Applicability of the computational strategies proposed has been verified by shape-finding carried out models of tensioned membrane structures in the one of the form of Moebius strip (with $R/W > 1.3699$) have shown good agreement with mathematically defined surfaces. Shape-finding of Moebius strip TMS models by assuming an initial assumed shape with the opening at the center is not able to yield converged result for value $R/W \leq 1.3699$. The shape obtained after shape-finding are found to be different from the form of typical Moebius strip surface with opening at the center. The opening at the center of Moebius strip is found to be non-existent. Shape of Moebius strip (without opening) has been verified through soap film model with $R/W = 1$. Shape-finding has been found to converge for Moebius strip TMS model (with $R/W \leq 1.3699$) with initial assumed shape specified to follow the topology without opening which has been observed in experiment.

References [5]-[8] have studied the problem of mathematical modeling in the industry. In this research, shape-finding using nonlinear analysis method and soap film model of Moebius strip TMS with R/W = 0.7, 1.2, 1.7, 2.2 and 2.7 has been carried out. Shape-finding is the step to determine the initial equilibrium shape that satisfies the prescribed prestress system and boundary condition.

II. Development of Moebius Strip

For this research, the software [9] has been used for the purpose of model generation. Aspect of modeling of surface of Moebius strip and form as well prestress pattern of the resulting TMS through shape-finding using nonlinear analysis method are studied based on [3]. Moebius strip as shown in Fig.1 can be represented parametrically by the following set of equations [10]:

H. M. Yee is with the Faculty of Civil Engineering, Universiti Teknologi MARA, 13500 Permatang Pauh, Pulau Pinang, Malaysia (e-mail: yhooimin@yahoo.com, * Corresponding Author).

M. A. Samsudin is with the Faculty of Civil Engineering, Universiti Teknologi MARA, 13500 Permatang Pauh, Pulau Pinang, Malaysia (e-mail: madzimsamsudin@gmail.com).

$$X = \left(R + S\cos\frac{\tau}{2}\right)\cos\tau$$

$$Y = \left(R + S\cos\frac{\tau}{2}\right)\sin\tau$$

$$Z = S\sin\left(\frac{\tau}{2}\right)$$

for $\{S\text{:-}W,W\}$ and $\{\ \tau : 0,2\pi\}$ (1)

The problem with this development model is single continuous surface involve only one curve along the band in a Cartesian plane. The problem is impossible to join the band border and to overcome this problem, [3] has proposed by applying discontinuous surface in modeling with small gap across the width of the strip. It was therefore decided to settle for a sufficiently close approximation of this shape, the solution adopted was to introduce a small gap (1˚) across the width of the strip as depicted in Fig. 1. In this way, the surface becomes discontinuous over the gap.

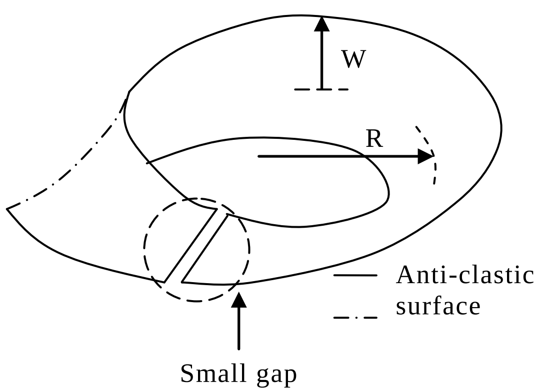

Fig.1 a Moebius strip with midcircle radius *R* and half-width *W*

III. Shape-Finding using Nonlinear Analysis Method

A nonlinear finite element analysis program by [3] for the analysis of tensioned membrane structures has been used in this study. The procedure adopted is based on the work by [3]. 3-node plane stress element has been used as element to model the surface of TMS. All x, y and z translation of nodes lying along the boundary edge of the Moebius strip have been restrained. Similarly, all nodes lying on either side of the gap introduced were also restrained from translating in all directions. The member pretension in warp and fill direction, is 2000N/m, respectively. The shear stress is zero.

Nonlinear FE analysis procedures for stress analysis of tensioned membrane structures have been used as basis for form-finding in this study based on [3]. Reference [3] has mentioned that a first shape for the start of form-finding procedure adopted in this study which is called initial assumed shape is needed. The proposed computational strategies by [3] involve two phases of analysis in one cycle. The first phase (denoted as Phase I) is an analysis which starts with an initial assumed shape in order to obtain an updated shape for initial assumed shape. This is then followed by the second phase of analysis (denoted as Phase II) aimed at checking the convergence of updated shape obtained at the end of Phase I by means of iterative calculation. The resultant shape at the end of iterative step is considered to be in the state of initial equilibrium under the prescribed warp and fill stresses and boundary condition if difference between the obtained and the prescribed tensioned fabric stresses relative to the prescribed stress is negligibly small.

Different combination parameters radius, *R* and haft-width, *W* for Moebius strip have been studied. The determination of *R* and *W* has been carried out as follow: W = 10 and R = 7, 12, 17, 22 and 27. Moebius strip R/W = 0.7, 1.2, 1.7, 2.2 and 2.7 has been studied. The opening of the opening at the center becomes larger when *R* has been increased. Two different variables R/W = 0.7 and R/W = 1.2 which is $R/W \leq 1.3699$ for Moebius strip have been chosen for the first discussion. Total number of nodes and elements used in the model are 343 and 576, respectively. Fig. 2 shows the initial assumed shape for Moebius strip R/W = 0.7. The surface models R/W = 0.7 has been found intersect with each other after shape-finding as shown in Fig. 3. The convergence of R/W = 0.7 not able to achieve due to intersection of surface.

Another set of Moebius strip R/W = 0.7 has been carried out by assuming the initial assumed shape similar to Fig. 3. Total number of nodes and elements used in the model are 154 and 270, respectively. Fig. 4 shows the convergence of the Moebius strip R/W = 0.7. The convergence curve in Fig. 5 shows the total warp and fills deviation < 0.01. Similar case obtained also for the Moebius strip R/W = 1.2.

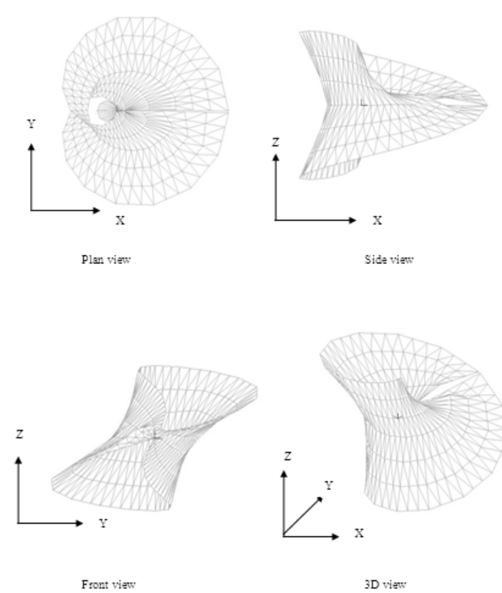

Fig. 2 initial assumed shape for Moebius strip (R/W = 0.7)

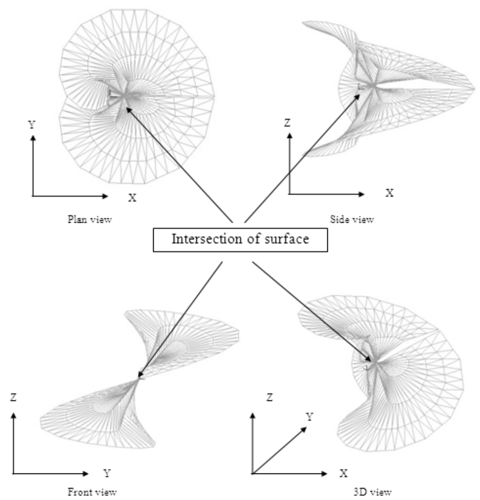

Fig. 3 models of TMS intersect with each other in the form of Moebius strip after shape-finding ($R/W = 0.7$)

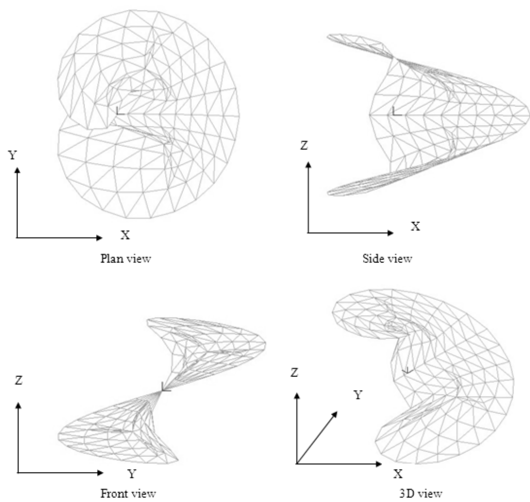

Fig. 4 TMS model in the form Moebius strip (without opening) after shape-finding ($R/W = 0.7$)

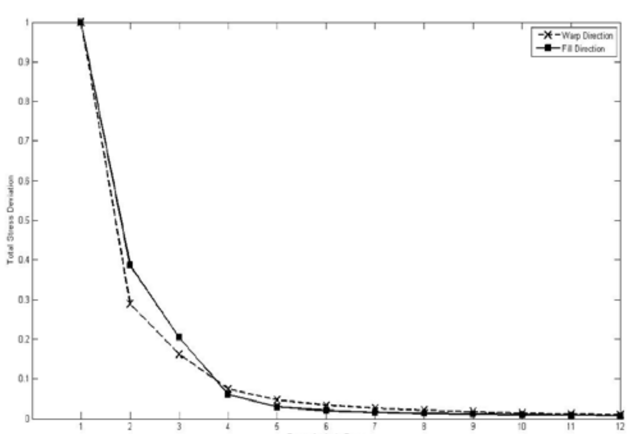

Fig. 5 variation of total stress deviation in warp and fill direction verses stress analysis stage for Moebius strip TMS Model ($R/W = 0.7$)

Moebius strip $R/W = 1.7$, 2.2 and 2.7 have been chosen for the following discussion. The surfaces of a Moebius strip also generated from (1). Total number of nodes and elements used in the model are 343 and 576, respectively. Fig. 6 shows the initial assumed shape for Moebius strip ($R/W = 1.7$). Fig. 7 shows the convergence of the Moebius strip $R/W = 1.7$. The convergence curve in Fig. 8 shows the total warp and fills deviation < 0.01. The similarity convergence from the mathematical shape can be clearly seen in Fig. 7. Similar case obtained also for the Moebius strip $R/W = 2.2$ and 2.7.

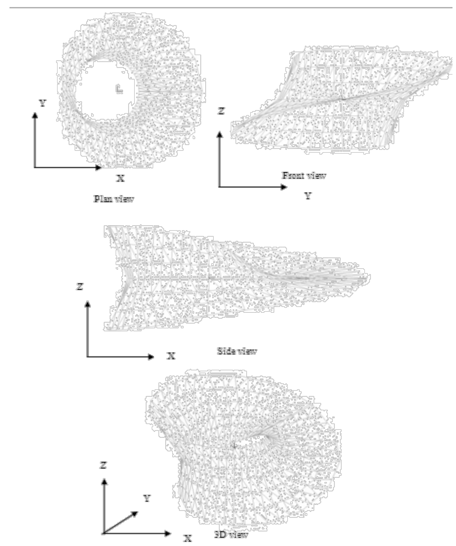

Fig. 6 initial assumed shape for Moebius strip ($R/W = 1.7$)

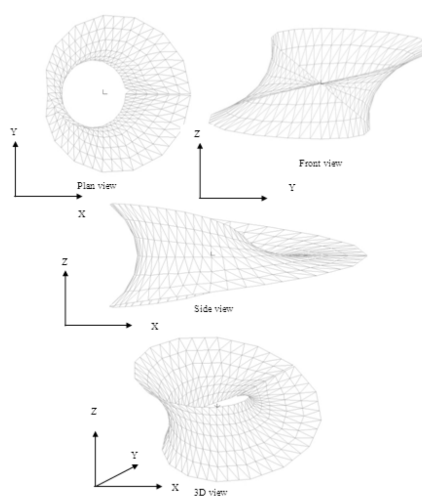

Fig. 7 TMS model in the form Moebius strip after shape-finding ($R/W = 1.7$)

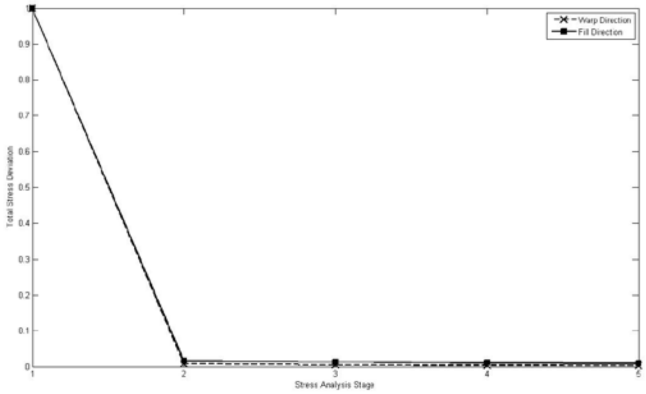

Fig. 8 variation of total stress deviation in warp and fill direction verses stress analysis stage for Moebius Strip TMS Model (*R/W* = 1.7)

The result obtained from soap film model and computational analysis are compared for the purposed of verification accuracy. Soap film model is minimal surface, so the comparison both methods are useful for determination of the capability of computational method to predict initial equilibrium shape in the form of minimal surface.

Fig. 9 shows the soap film model for Möbius strip (*R/W* = 0.7). Fig. 10 shows picture captured soap film model for Möbius Strip (*R/W* = 0.7) and the shape of computational model obtained is superimposed for the purpose of comparison. Fig. 11 shows the soap film model for Möbius strip (*R/W* = 1.7). Fig. 12 shows picture captured soap film model for Möbius strip (*R/W* = 1.7) and the shape of computational model obtained is superimposed for the purpose of comparison. Figs. 10 and 12 show the comparison of computational and soap film model results for Möbius strip (*R/W* = 0.7 and 1.7). For Möbius strip surface with *R/W* = 0.7, 1.2, 1.7, 2.2 and 2.7, computational and experimental results obtained show good agreement.

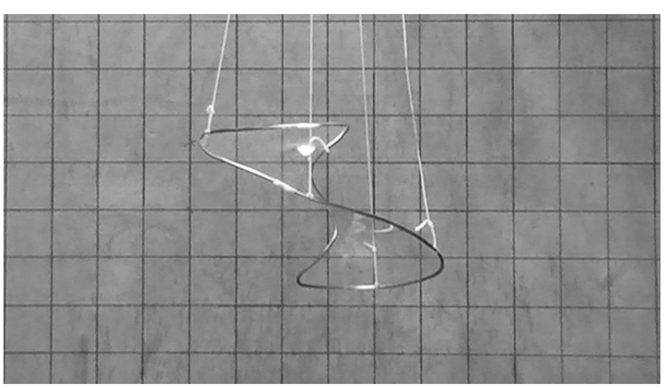

Fig. 9 Soap Film Model for Möbius Strip (*R/W* = 0.7)

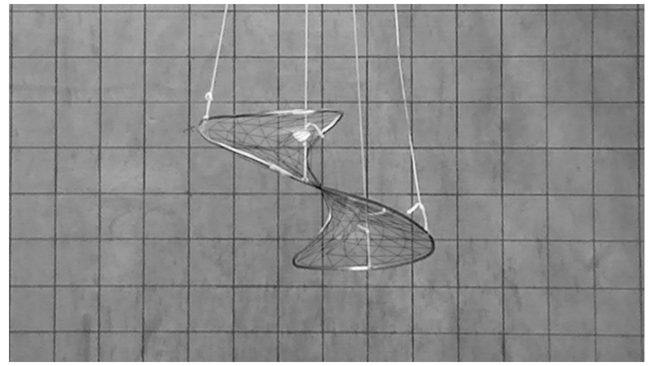

Fig. 10 Computational and Soap Film Model for Möbius Strip (*R/W* = 0.7)

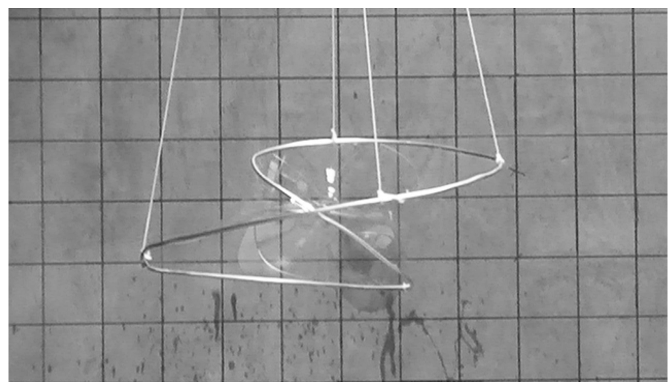

Fig. 11 Soap Film Model for Möbius Strip (*R/W* = 1.7)

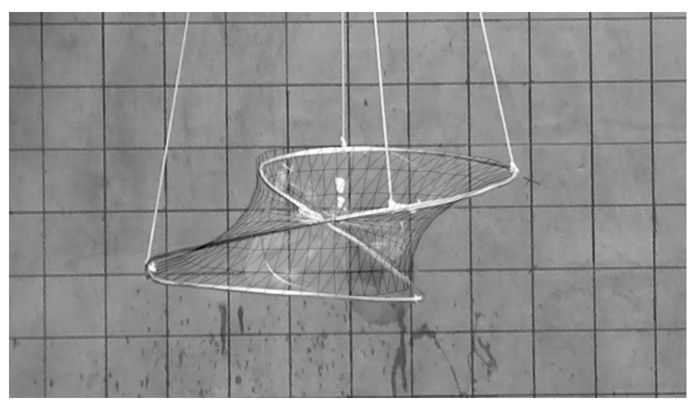

Fig. 12 Computational and Soap Film Model for Möbius Strip (*R/W* = 1.7)

IV. Conclusion

Shape-finding on TMS with surface in the form of Moebius strip *R/W*= 0.7, 1.2, 1.7, 2.2 and 2.7 has been carried out successfully. It provides an alternative choice for designer to consider the Moebius strip, *R/W*= 0.7, 1.2, 1.7, 2.2 and 2.7 applied in TMS or shell and spatial structures. Its can be used to improve rural basic infrastructure. TMS can save materials and help to achieve economy in construction. TMS can reduce the environment impact by giving natural diffuse light with reduced heat load.

REFERENCES

[1] V. Petresin and L. Robert, “The double Möbius strip studied”, *Nexus Network Journal*, vol. 4, pp. 4, 2002.

[2] K.K. Choong and M.S. Kuan, “Structural behaviour of shell surface in the form of Möbius strip Published Conference Proceedings style),” in *Proc. 6th Sixth International Conference on Computation of Shell and Spatial Structures*, New York, 2008, pp. 1–4.

[3] H.M. Yee, “A computational strategy for form-finding of tensioned fabric structure using nonlinear analysis method,” Ph.D. dissertation, School of Civil Engineering, Universiti Sains Malaysia, Pulau Pinang, Malaysia, 2011.

[4] H.M. Yee, K.K. Choong and J.Y. Kim, “Form-finding analysis of tensioned fabric structures using nonlinear analysis method,” *Advanced Materials Research*, vol. 243-249, pp. 1429–1434, May 2011.

[5] P. Suchánek, R.Bucki, F.Marecki and E.Litavcová, “Information modelling of the manufacturing centre with the use of the Heuristic Algorithms,” *International Journal of Mathematics and Computers in Simulation*, vol. 8, pp. 60-66, 2014.

[6] K. Kyas, M. Stanek, J. Navratil, M. Manas, D. Manas, V. Senkerik, A. Skrobak, “Rubber product properties influenced by runners trajectory,” *International Journal of Mathematics and Computers in Simulation*, vol. 7, pp. 1-8, 2013.

[7] B. Chramcov, “The optimization of production system using simulation optimization tools in witness,” *International Journal of Mathematics and Computers in Simulation*, vol. 7, pp. 95-105, 2013.

[8] S. Hubalovsky, M. Musilek, “Modeling, simulation and visualization of real processes in LOGO programming language as a method of development of algorithm thinking and programming skills,” *International Journal of Mathematics and Computers in Simulation*, vol. 7, pp.144-152, 2013.

[9] *ADINA*, System 8.1. R&D Inc., 2003.

[10] Weisstein and W. Eric. (2014, March 5). Networks [Online]. Available: http://mathworld.wolfram.com/ MöbiusStrip.html