Evolutionary Control of Chaotic Burgers Map by means of Chaos Enhanced Differential Evolution

Roman Senkerik, Ivan Zelinka, Michal Pluhacek and Zuzana Kominkova Oplatkova

***Abstract*—** In this paper, evolutionary technique Differential Evolution (DE) is used for the evolutionary tuning of controller parameters for the stabilization of chaotic Burgers map system. The novality of the approach is that the identical selected discrete dissipative chaotic system is used also as the chaotic pseudo random number generator to drive the mutation and crossover process in the DE. The optimization was performed for two types of case studies and developed cost functions.

***Keywords*—** Differential Evolution, Optimization, Chaos control, Evolutionary algorithms, Burgers map.

I. Introduction

THESE days the methods based on soft computing such as neural networks, evolutionary algorithms (EA's), fuzzy logic, and genetic programming are known as powerful tool for almost any difficult and complex optimization problem.

The interest about the interconnection between evolutionary techniques and control of chaotic systems is spread daily. First steps were done in [1], [2], [3] where the control law was based on Pyragas method: Extended delay feedback control – ETDAS [4], [5], [6]. These papers were concerned to tune several parameters inside the control technique for chaotic system. The big advantage of the Pyragas method for evolutionary computation is the amount of accessible control parameters, which can be easily tuned by means of EA's.

This paper is aimed at investigating the chaos driven Differential Evolution (DE). Although a number of DE variants have been recently developed, the focus of this paper is the embedding of chaotic systems in the form of chaos pseudo random number generator (CPRNG) for DE and its application to optimization of chaos control.

Firstly, the problem design is proposed. The next sections are focused on the description of used cost functions, evolutionary algorithm DE and the concept of chaos driven DE. Results and conclusion follow afterwards.

R.Senkerik, Ivan Zelinka, M. Pluhacek and Z. Kominkova Oplatkova are with the Department of Informatics and Artificial Intelligence, Tomas Bata University in Zlin, Nad Stranemi 4511, 760 05 Zlin, Czech Republic, (phone: +420576035189; fax: +420576035279; e-mail: senkerik@fai.utb.cz).

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II. Motivation

This paper extends the research of evolutionary chaos control optimization by means of DE algorithm [3].

In this paper the DE/rand/1/bin strategy driven by Burgers chaotic map (system) was utilized to solve the issue of evolutionary optimization of chaos control for the very same chaotic system. Thus the idea was to utilize the hidden chaotic dynamics in pseudo random sequences given by chaotic Burgers map system to help Differential evolution algorithm in searching for the best controller settings for the very same chaotic system.

Recent research in chaos driven heuristics has been fueled with the predisposition that unlike stochastic approaches, a chaotic approach is able to bypass local optima stagnation. This one clause is of deep importance to evolutionary algorithms. A chaotic approach generally uses the chaotic map in the place of a pseudo random number generator [7]. This causes the heuristic to map unique regions, since the chaotic map iterates to new regions. The task is then to select a very good chaotic map as the pseudo random number generator.

The initial concept of embedding chaotic dynamics into the evolutionary algorithms is given in [8]. Later, the initial study [9] was focused on the simple embedding of chaotic systems in the form of chaos pseudo random number generator (CPRNG) for differential evolution (DE) [10] and SOMA [11] in the task of optimal PID tuning

Several papers have been recently focused on the connection of heuristic and chaotic dynamics either in the form of hybridizing of DE with chaotic searching algorithm [12] or in the form of chaotic mutation factor and dynamically changing weighting and crossover factor in self-adaptive chaos differential evolution (SACDE) [13]. Also the PSO (Particle Swarm Optimization) algorithm with elements of chaos was introduced as CPSO [14] or CPSO combined with chaotic local search [15].

The focus of our research is the embedding of chaotic systems in the form of chaos pseudo random number generator for evolutionary algorithms.

This research was later extended with the successful experiments with chaos driven DE [16], [17] with simple test functions in low dimensions and in the task of chemical reactor geometry optimization [18].

The concept of Chaos DE proved itself to be a powerful heuristic also in combinatorial problems domain [19], [20].

At the same time the chaos embedded PSO with inertia weigh strategy was closely investigated [21] and more experiments were performed with the concept of chaos driven DE in higher dimensions and with more complex benchmark functions [22].

The interconnection between PSO algorithm and pure CPRNGs was intensively studied within the PID controller optimization issue [23], followed by the introduction of a PSO strategy driven alternately by two chaotic systems [24] and novel chaotic Multiple Choice PSO strategy (Chaos MC-PSO) [25].

Recently, it was proven that the evolutionary algorithms do not require random processes at all and works well and even better with simple deterministic sequences [26], [27].

III. Selected Chaotic System

The chosen example of discrete dissipative chaotic system used both as a CPRNG and within the evolutionary optimization of chaos control problem was the two-dimensional Burgers map system.

The Burgers mapping is a discretization of a pair of coupled differential equations which were used by Burgers to illustrate the relevance of the concept of bifurcation to the study of hydrodynamics flows. The map equations are given in (1) with control parameters a = 0.75 and b = 1.75 as suggested in [28].

For these value, the system exhibits chaotic behavior. The example of this behavior is depicted in x-y plot (Fig. 1) and numerical simulation of direct system output (x or y) in the uncontrolled state (Fig. 2 and Fig. 3).

$$\begin{aligned} X_{n+1} &= aX_n - Y_n^2 \\ Y_{n+1} &= bY_n + X_n Y_n \end{aligned} \tag{1}$$

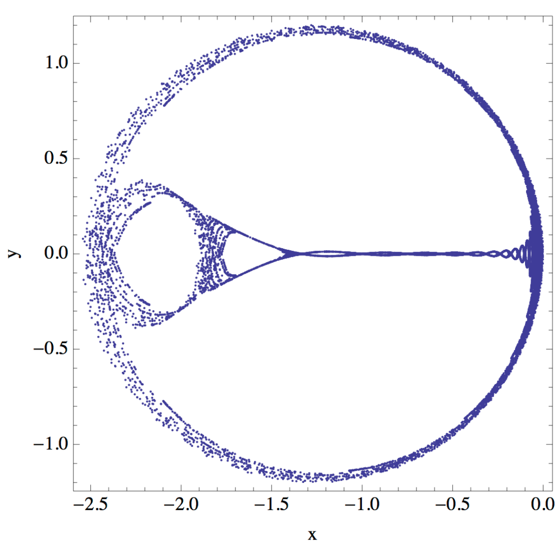

Fig. 1. x, y plot of the Burgers map

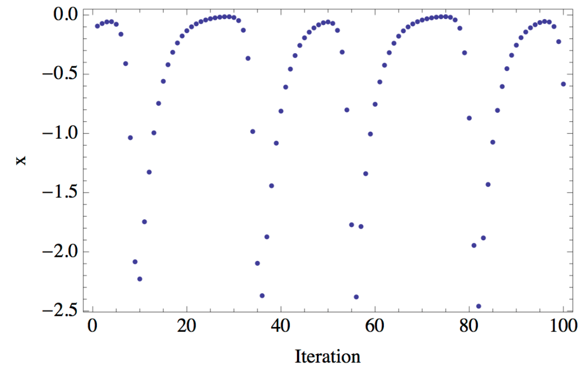

Fig. 2. Iterations of the uncontrolled Burgers map (variable x)

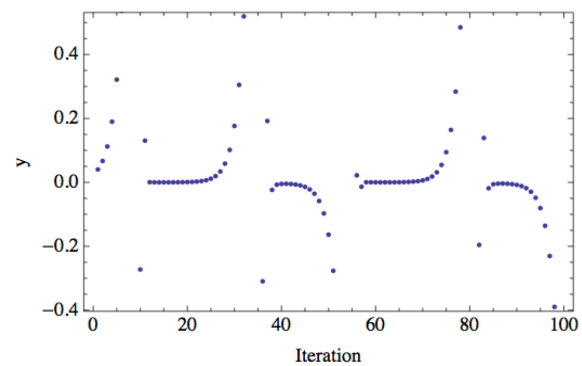

Fig. 3. Iterations of the uncontrolled Burgers map (variable y)

IV. Original ETDAS Chaos Control Method

This work is focused on the utilization of the chaos driven DE for tuning of parameters for ETDAS control method to stabilize desired Unstable Periodic Orbits (UPO). In the described research, desired UPO was p-1 (stable state). The original control method – ETDAS in the discrete form suitable for Burgers map has the form (2).

$$\begin{aligned} x_{n+1} &= aX_n - Y_n^2 + F_n \\ F_n &= K\left[(1-R)S_{n-m} - x_n\right] \\ S_n &= x_n + RS_{n-m} \end{aligned} \tag{2}$$

Where: K and R are adjustable constants, which have to be evolutionary tuned. F is the perturbation; S is given by a delay equation utilizing previous states of the system, m is the period of m-periodic orbit to be stabilized. The perturbation F_n in equations (2) may have arbitrarily large value, which can cause diverging of the system outside the output interval of Burgers map system {-1.2, 1.2}. Therefore, F_n should have a value between $<-F_{max}, F_{max}>$. The suitable F_{max} value was also obtained from evolutionary optimization process.

V. Cost Functions

This research utilizes and compares two cost function design.

The proposal of the first basic cost function (CF) is in general based on the simplest CF, which could be used problem-free only for the stabilization of p-1 orbit. The idea was to minimize the area created by the difference between the required state and the real system output on the whole simulation interval – τ_i. The simple CF is given in (3).

$$CF_{SIMPLE} = \sum_{t=0}^{\tau_i} \left| TS_t - AS_t \right| \tag{3}$$

Nevertheless this simple approach has one big disadvantage, which is the including of initial chaotic transient behavior of not stabilized system into the cost function value. As a result of this, the very tiny change of control method setting for extremely sensitive chaotic system causing very small change of CF value, can be suppressed by the above-mentioned including of initial chaotic transient behavior.

Another universal cost function had to be used for securing the stabilization of either p-1 orbit (stable state) or higher periodic orbit and having the possibility of adding penalization rules. It was synthesized from the simple CF and other terms were added.

This CF is in general based on searching for desired stabilized periodic orbit and thereafter calculation of the difference between desired and found actual periodic orbit on the short time interval - τ_s (approx. 20 - 50 iterations) from the point, where the first min. value of difference between desired and actual system output is found (i.e. floating window for minimization – see Fig. 4). The CF_{UNI} has the form (4).

$$CF_{UNI} = pen_1 + \sum_{t=\tau 1}^{\tau 2} \left| TS_t - AS_t \right| \tag{4}$$

Where:
TS - target state, *AS* - actual state
τ_1 - the first minimal value of difference between *TS* and *AS*
τ_2 – the end of optimization interval ($\tau_1 + \tau_s$)
$pen_1 = 0$ if $\tau_i - \tau_2 \geq \tau_s$;
$pen_1 = 10^*(\tau_i - \tau_2)$ if $\tau_i - \tau_2 < \tau_s$ (i.e. late stabilization)

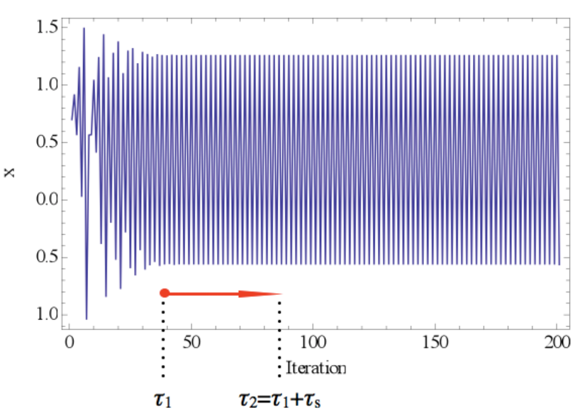

Fig. 4. Floating window for optimization

VI. Differential Evolution

DE is a population-based optimization method that works on real-number-coded individuals [10]. For each individual $\vec{x}_{i,G}$ in the current generation G, DE generates a new trial individual $\vec{x}'_{i,G}$ by adding the weighted difference between two randomly selected individuals $\vec{x}_{r1,G}$ and $\vec{x}_{r2,G}$ to a randomly selected third individual $\vec{x}_{r3,G}$. The resulting individual $\vec{x}'_{i,G}$ is crossed-over with the original individual $\vec{x}_{i,G}$. The fitness of the resulting individual, referred to as a perturbed vector $\vec{u}_{i,G+1}$, is then compared with the fitness of $\vec{x}_{i,G}$. If the fitness of $\vec{u}_{i,G+1}$ is greater than the fitness of $\vec{x}_{i,G}$, then $\vec{x}_{i,G}$ is replaced with $\vec{u}_{i,G+1}$; otherwise, $\vec{x}_{i,G}$ remains in the population as $\vec{x}_{i,G+1}$. DE is quite robust, fast, and effective, with global optimization ability. It does not require the objective function to be differentiable, and it works well even with noisy and time-dependent objective functions. Description of used DERand1Bin strategy is presented in (5). Please refer to [10], [29], for the description of all other strategies.

$$u_{i,G+1} = x_{r1,G} + F \cdot \left(x_{r2,G} - x_{r3,G} \right) \tag{5}$$

VII. Chaos Driven DE

The main principle of this concept is the embedding of chaotic systems in the form of chaos pseudo random number generator (CPRNG) for DE. In this research, direct output iterations of the chaotic map were used for the generation of real numbers in the process of crossover based on the user defined CR value and for the generation of the integer values used for selection of individuals. This concept is described in details in [30]. The initial concept of embedding chaotic dynamics into the evolutionary algorithms is given in [31].

VIII. Experimental Results

Within the research a total number of 50 simulations with chaos driven DE by means of Burgers map system were carried out for each CF design. All simulations were successful and have given new optimal settings for ETDAS control method securing the fast stabilization of the chaotic system at required behaviour (p-1 orbit).

Following Tables 2 and 4 contain the simple statistical overview of optimization/simulation results. Tables 3 and 5 contain the best founded individual solutions of parameters set up for ETDAS control method, corresponding final CF value, also the Istab. Value representing the number of iterations required for stabilization on desired UPO and further the average error between desired output value and real system output from the last 20 iterations.

Graphical simulation outputs of the best individual solutions for both case studies are depicted in Fig. 5 and Fig. 7, whereas the Fig. 6 and Fig 8 shows the simulation output of all 50 runs of CHAOS DE, thus confirm the robustness of this approach.

For the illustrative purposes, all graphical simulations outputs are depicted only for the variable x of the chaotic Burgers map system.

Settings of EA parameters for both processes were based on performed numerous experiments with chaotic systems (Table 1). Based on the mathematical analysis, the real p-1 UPO for unperturbed Burgers map system has following value: $x_S = 0.7499$.

The ranges of all estimated parameters were these: $-2 \le K \le 2$, $0 \le F_{max} \le 0.9$ and $0 \le R \le 0.99$,

TABLE I. CHAOS DE SETTINGS

DE Parameter	Value
PopSize	25
F	0.8
CR	0.8
Generations	250
Max. CF Evaluations (CFE)	6250

A. Case study 1 – Simple cost function

From the results presented in the Tables 2 and 3, it follows that the CF-simple is very convenient for evolutionary process, which means that repeated runs of EA are giving identical optimal results (i.e. very close to the possible global extreme). This is graphically confirmed in the Figure 6 when all 50 simulations are basically merged into the one line.

On the other hand the disadvantage of including of initial chaotic transient behavior of not stabilized system into the cost function value and resulting very tiny change of control method setting for extremely sensitive chaotic system is causing suppression of stabilization speed and numerical precision.

TABLE II. CF-SIMPLE VALUES STATISTIC

Statistical data	CF Value
Min	2.16199
Max	2.16199
Average	2.16199
Median	2.16199
Std.Dev.	$5.58 \cdot 10^{-11}$
Avg. Full Stab. (Iteration)	45

TABLE III. CHARACTERISTICS OF THE BEST SOLUTION

Parameter	Value
K	1.22847
F_{max}	0.9
R	0.574997
CF Value	2.16199
Istab. Value	45
Avg. error per iteration	$5.86 \cdot 10^{-5}$

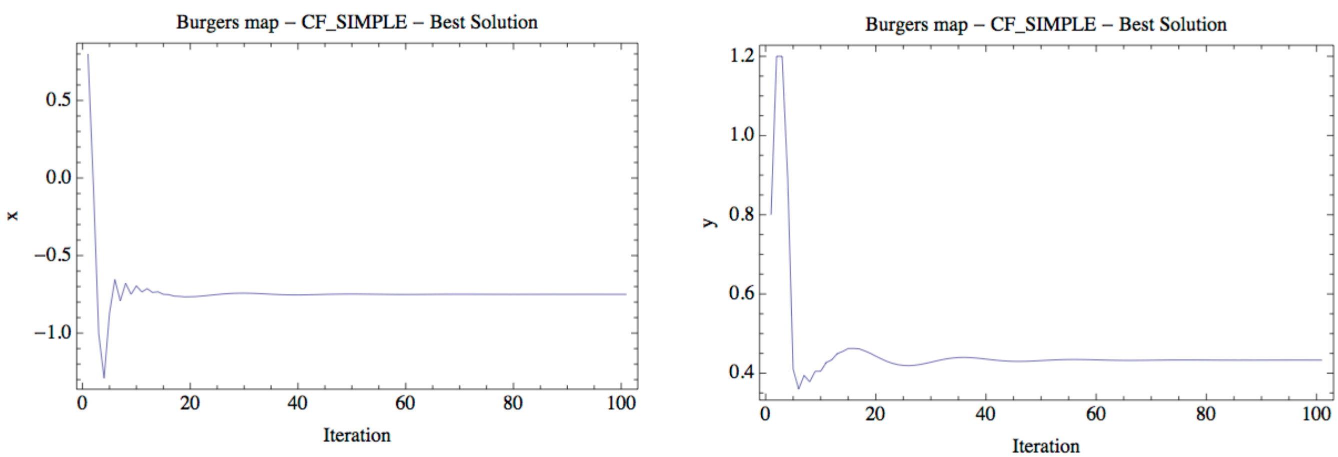

Fig. 5. Simulation of the best individual solution –Burgers map system CHAOS DE - CF Simple

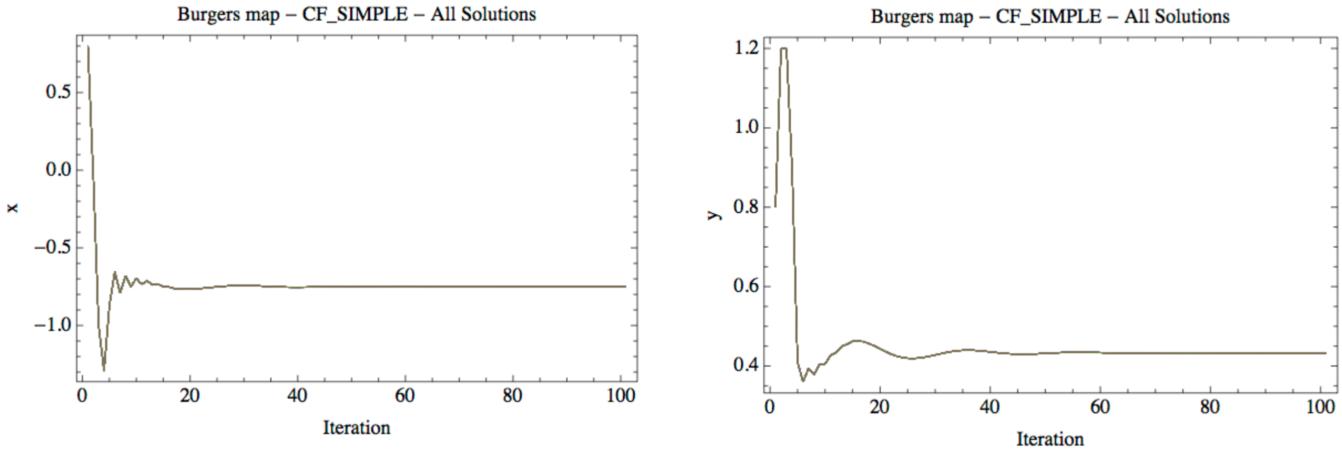

Fig. 6. Simulation of the all 50 solutions – Burgers map system CHAOS DE - CF Simple

B. *Case study 2 – Universal cost function*

TABLE IV. CF-UNIVERSAL VALUES STATISTIC

Statistical data	CF Value
Min	$1.05 \cdot 10^{-6}$
Max	0.0103
Average	$6.67 \cdot 10^{-4}$
Median	$5.32 \cdot 10^{-7}$
Std.Dev.	$1.89 \cdot 10^{-3}$
Avg. Stab. (Iteration)	35

Results obtained in this case study lend weight to the argument, that the technique of pure searching for periodic orbits is advantageous for faster and more precise stabilization of chaotic system.

TABLE V. CHARACTERISTICS OF THE BEST SOLUTION

Parameter	Value
K	0.732498
F_{max}	0.48495
R	0.811742
CF Value	$1.05 \cdot 10^{-6}$
Istab. Value	25
Avg. error per iteration	$1.21 \cdot 10^{-8}$

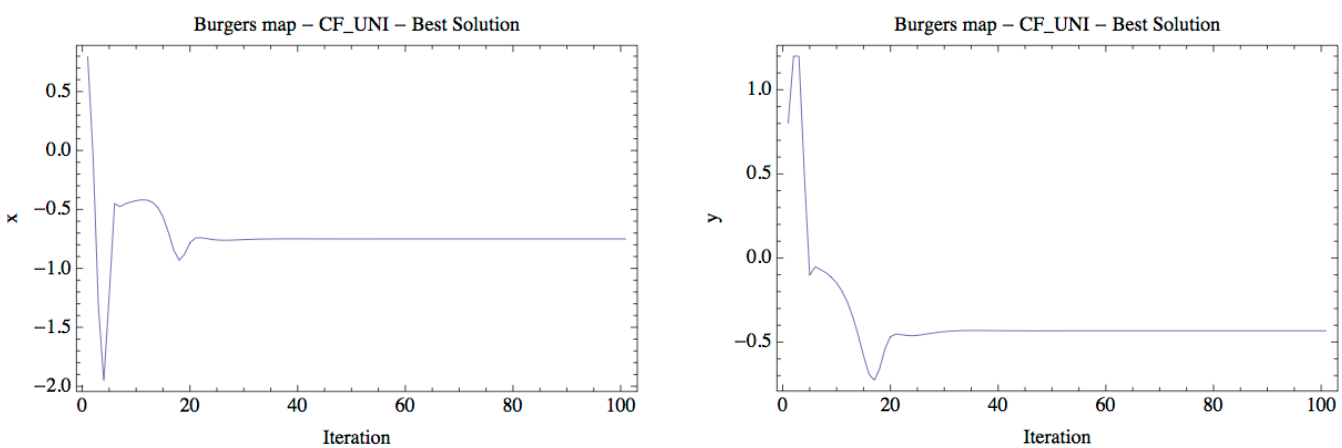

Fig. 7. Simulation of the best individual solution – Burgers map system CHAOS DE - CF Universal

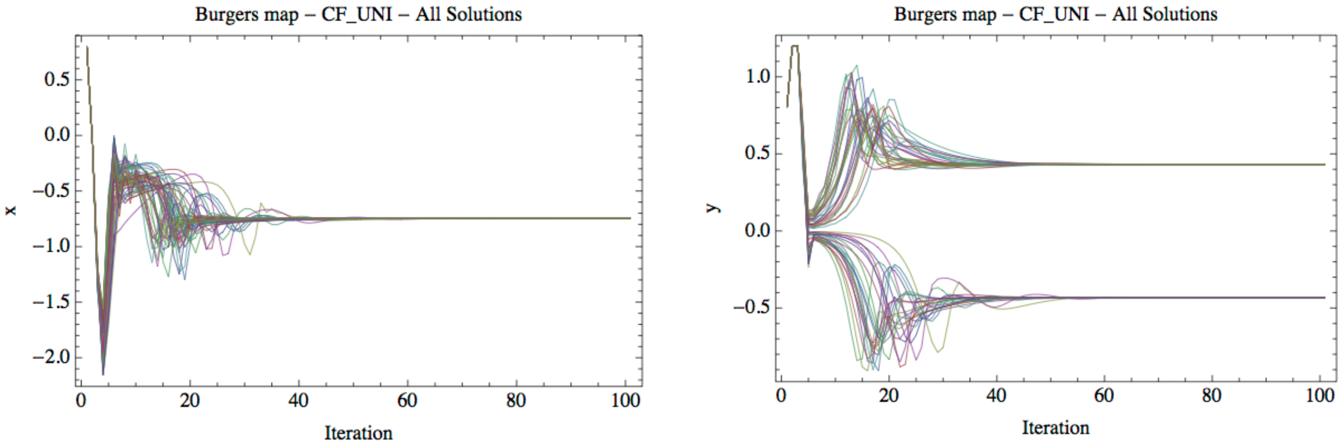

Fig. 8. Simulation of the all 50 solutions – Burgers map system CHAOS DE - CF Universal

IX. Conclusion

Based on obtained results, it may be claimed, that the presented Chaos DE driven by selected discrete dissipative chaotic system has given satisfactory results in the chaos control optimization issue.

The results show that embedding of the chaotic dynamics in the form of chaotic pseudo random number generator into the differential evolution algorithm may help to improve the performance and robustness of the DE. Thus to obtain optimal solutions securing the very fast and precise stabilization for both convenient CF surface in case of the CF-simple and very chaotic and nonlinear CF surface in case of the CF-universal.

When comparing the both CF designs, the CF-simple is very convenient for evolutionary process (i.e. repeated runs are giving identical optimal results), but it has many limitations.

The second universal CF design brings the possibility of using it problem free for any desired behavior of arbitrary chaotic systems, but at the cost of the highly chaotic CF surface. Nevertheless the embedding of the chaotic dynamics into the evolutionary algorithms helped to deal with such an issue.

The primary aim of this work was not to develop any new pseudo random number generator, which should normally pass many statistical tests, but to show that through embedding the hidden chaotic dynamics into the evolutionary process in the form of chaotic pseudo random number generators may help to obtain better results and avoid problems connected with evolutionary computation such as premature convergence and stagnation in local extremes.

Future plans include testing of different chaotic systems, either manually or evolutionary tuning of chaotic maps parameters, comparisons with different heuristics and obtaining a large number of results to perform statistical tests.

The future research will include the development of better cost functions, testing of different AP data sets, and performing of numerous simulations to obtain more results and produce better statistics, thus to confirm the robustness of this approach.

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