Analysis, Settings and Simulations of Relay SISO Autotuners

Roman Prokop, Jiří Korbel, and Radek Matušů

***Abstract*—** A relay based procedure for estimating and controller autotuning is addressed. Autotuning principles usually combine a relay feedback test with control synthesis. This paper presents some results of the first part of this scheme, i.e. relay plant identification for continuous-time plants. The estimation of the controlled system parameters plays the key role in the quality of control. There are many types of relays used in feedback relay schemes. The contribution deals with four ones of them, namely unbiased and biased relays without or with hysteresis.

Many industrial plants can be satisfactory estimated by a first or second order linear stable system with a time delay term. The main relay parameters are the asymmetry, hysteresis and amplitudes. The aim of this paper is to study and analyze the influence of these parameters for the quality of estimation of the gain, time constant and time delay. As a result, some recommendations for settings of relay features can be given.

Then, control synthesis follows in algebraic philosophy. This approach brings a tuning positive real parameter which highly influences the control behavior. All simulations were performed in Matlab and Simulink program environment. A program system for automatic estimation, design and simulation was developed.

Keywords—Autotuning, Relay experiment, Limit cycle oscillations, Biased and unbiased relay, Hysteresis, Describing function.

I. Introduction

The Åström and Hägglund relay feedback test [1] started in 1984 an important tool for automatic controller tuning because it identifies two main parameters for the Ziegler-Nichols method [3]. Previously, relay was mainly used as an amplifier or as a relay back control. The Åström-Hägglund test is based on the observation, when the output lags behind the input by -π radians, the closed loop oscillates with a constant period. Then, the ultimate gain and frequency are identified by a simple symmetrical relay feedback experiment proposed in [1]. From the critical values the controller setting was applied by the Ziegler-Nichols rule which is simple but it suffers from several drawbacks.

From that time, many studies have been reported to extend and improve both, the relay feedback experiment as well as tuning and control design principles; see e.g. [2] - [4], [10], [18]-[20]. Many of them need an estimation of transfer function parameters and the original approach provides no explicit parameters of the identified transfer function. During the period of almost three decades, the direct estimation of transfer function parameters instead of critical values began to appear. The extension in relay utilization was performed in e.g. [8] - [12], [27] by an asymmetry and hysteresis of a relay. Nowadays, almost all commercial industrial PID controllers provide the feature of autotuning.

This paper brings a study how the asymmetry and hysteresis influence the quality and accuracy of identification process. Also the length of the experiment and the relay amplitude can influence the quality of the estimation.

Probably Luyben in [5] was the first who used the approximate describing function (DF) method to estimate the process transfer function from limit cycle measurements.

The main scheme for the relay estimation and/or identification is depicted in Fig. 1.

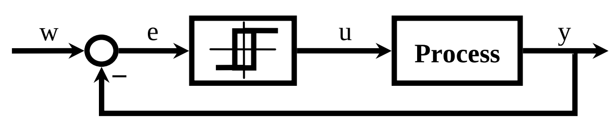

Fig. 1 relay based identification

The goal of the original test was to indicate the critical point in the Nyquist curve of the open loop. However, there are other relays used in identification experiments, e.g. the biased (asymmetrical) relay, two positions symmetrical and asymmetrical (biased) relay without and with hysteresis characteristic are depicted in Fig. 2. A biased (asymmetrical) one characteristic is obtained by a simple vertical moving by an asymmetry shift. Also, the relay without hysteresis is obtained by putting $\varepsilon = 0$.

Many research works have been done to improve and refine the effect of fundamental harmonic by using different shapes and structures of the relay element, see [6], [7], [24] - [26]. A limit cycle oscillation for a stable system with positive steady state gain with a biased relay is shown in Fig. 3.

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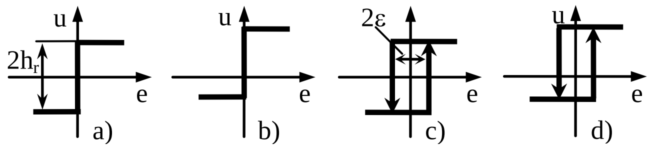

Fig. 2 types of relay

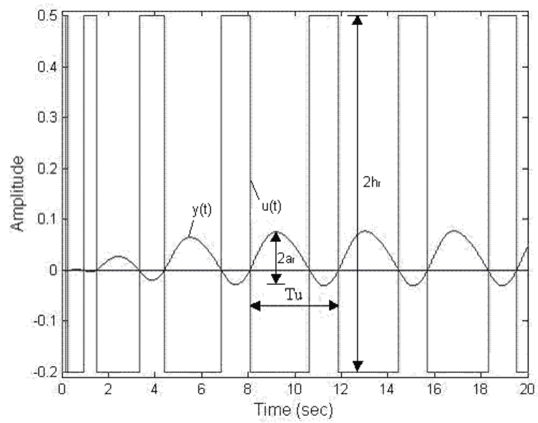

Fig. 3 biased relay oscillation of stable processes

Many stable industrial processes can be approximated by a first or second order linear system with a time delay term. The model for first order (stable) systems plus dead time (FOPDT) is supposed in the form:

$$G(s) = \frac{K}{Ts+1} \cdot e^{-\Theta s} \tag{1}$$

The second order model plus dead time (SOPDT) is assumed in the form:

$$G(s) = \frac{K}{(Ts+1)^2} \cdot e^{-\Theta s} \tag{2}$$

II. RELAY FEEDBACK ESTIMATION

A. Critical Values Estimation

Critical values estimation is based on a simple symmetric (unbiased) relay experiment which output is depicted in Fig. 4.

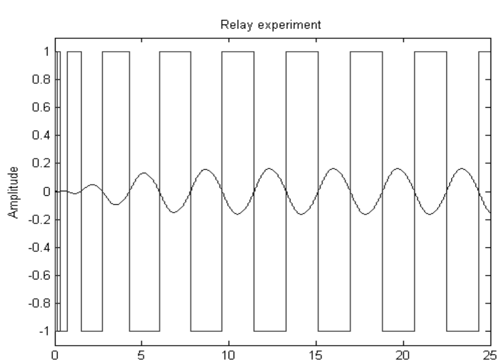

Fig. 4 unbiased relay oscillation of stable processes

The critical gain is then given by the relation (see e.g. [1])

$$r_u = \frac{4}{\pi} \cdot \frac{h_r}{\sqrt{a_r^{\,2} - \varepsilon^2}} \Big|_{\varepsilon=0} = \frac{4}{\pi} \cdot \frac{h_r}{a_r} \tag{3}$$

and the ultimate period T_u can be read according to Fig. 3.

B. Unbiased relay experiment

The relay feedback experiment according to Fig. 1 yields stable harmonic oscillations, i.e. it causes rise of the stable limit cycles (Fig. 4). The describing function method ([5], [11], [14], [29]) is a tool for verification the limit cycle rise. The describing function of the relay *N(a)* is considered as a complex gain which depends on the harmonic oscillation amplitude a and angular frequency ω in the relay input *e(t)*

$$e(t) = a \sin \omega t \tag{4}$$

The condition for the limit cycle follows from the critical point of non-linear closed-loop system in Fig. 1 which gives

$$N(a) G_P(s) + 1 = 0 \tag{5}$$

where $G_P(s) = A_P(\omega)\, e^{j\varphi(\omega)}$ is the plant transfer function, $A_P(\omega)$ and $\varphi(\omega)$ are called the (transfer function) magnitude and phase, respectively.

For the symmetric relay without or with hysteresis $\varepsilon > 0$, the describing function and the critical characteristic have the form (see e.g. [8], [12], [29])

$$A_N(a)\, e^{j\varphi_N(a)} = -\frac{1}{N(a)} \quad for \quad 0 \le \varepsilon \le a \tag{6}$$

otherwise $N(a) = \infty$. Values $A(a)$ and $\varphi_N(a)$ represent the critical magnitude and critical characteristic phase, respectively

$$A_N(a) = \frac{\pi a_r}{4 h_r}$$
$$\varphi_N(a) = arctg \frac{\varepsilon}{\sqrt{a^2 - \varepsilon^2}} - \pi \tag{7}$$

The frequency transfer function $G(j\omega) = A_P(\omega) e^{j\varphi_P(\omega)}$ for the first order system (1) gives

$$A_P(a) = \frac{K}{\sqrt{T^2\omega^2 + 1}}$$
$$\varphi_P(a) = -arctg\, \omega T - \omega \Theta \tag{8}$$

Comparing $A_N(a) = A_P(a)$ and $\varphi_N(a) = \varphi_P(a)$ in (7) and (8) gives two equations for the calculation of T and Θ. The final relations for the time constant and time delay terms for

FOPDT (1) are given by:

$$T = \frac{T_y}{2\pi} \cdot \sqrt{\frac{16 \cdot K^2 \cdot u_0^2}{\pi^2 \cdot a_y^2} - 1}$$
$$\Theta = \frac{T_y}{2\pi} \cdot \left[\pi - arctg\, \frac{2\pi T}{T_y} - arctg\, \frac{\varepsilon}{\sqrt{a_y^2 - \varepsilon^2}} \right] \tag{9}$$

where a_y and T_y are depicted in Fig. 3 and ε is hysteresis.

The second order system SOPDT (2) is estimated by relations

$$T = \frac{T_y}{2\pi} \cdot \sqrt{\frac{4 \cdot K \cdot u_0}{\pi \cdot a_y} - 1}$$
$$\Theta = \frac{T_y}{2\pi} \cdot \left[\pi - 2arctg\, \frac{2\pi T}{T_y} - arctg\, \frac{\varepsilon}{\sqrt{a_y^2 - \varepsilon^2}} \right] \tag{10}$$

Relations (9), (10) represent a suitable identification tool for computing time and time delay terms but a relay unbiased experiment is not able to estimate the gain of the controlled system.

C. Biased relay experiment

Asymmetrical relays with or without hysteresis bring further progress, see e.g. [2], [6], [7], [14], [27], [28]. After the relay feedback test, the estimation of process parameters can be performed. A typical data response of such relay experiment is depicted in Fig. 5. The relay asymmetry is required for the process gain estimation (11) while a symmetrical relay would cause the zero division in the appropriate formula. In this paper, an asymmetrical relay with hysteresis was used. This relay enables to estimate transfer function parameters as well as a time delay term. The proportional gain can be computed by the relation [11]:

$$K = \frac{\int_0^{iT_y} y(t)dt}{\int_0^{iT_y} u(t)dt}; \quad i = 1,2,3,\ldots \tag{11}$$

when the asymmetric relay is used for the relay feedback test, it is shown in Fig. 5, the output y converges to the stationary oscillation in one period. These oscillations are characterized by equations (see [8]):

$$A_u = (\mu_0 + \mu) \cdot K \cdot \left(1 - e^{-\frac{\Theta}{T}} \right) + \varepsilon \cdot e^{-\frac{\Theta}{T}} \tag{12}$$

$$A_d = (\mu_0 - \mu) \cdot K \cdot \left(1 - e^{-\frac{\Theta}{T}} \right) - \varepsilon \cdot e^{-\frac{\Theta}{T}} \tag{13}$$

$$T_{u1} = T \cdot \ln \frac{2 \cdot \mu \cdot K \cdot e^{\frac{\Theta}{T}} + \mu_0 \cdot K - \mu \cdot K + \varepsilon}{\mu \cdot K + \mu_0 \cdot K - \varepsilon} \tag{14}$$

$$T_{u2} = T \cdot \ln \frac{2 \cdot \mu \cdot K \cdot e^{\frac{\Theta}{T}} - \mu_0 \cdot K - \mu \cdot K + \varepsilon}{\mu \cdot K - \mu_0 \cdot K - \varepsilon} \tag{15}$$

The normalized dead time of the process ($L=\Theta/T$) is obtained from (12) or (13) in the form (see e.g. [8]):

$$L = \ln \frac{(\mu_0 + \mu) \cdot K - \varepsilon}{(\mu_0 + \mu) \cdot K - A_u} \tag{16}$$

or

$$L = \ln \frac{(\mu - \mu_0) \cdot K - \varepsilon}{(\mu - \mu_0) \cdot K + A_d} \tag{17}$$

Next, the time constant can be computed from (14) or (15) by solving these formulas [8]:

$$T = T_{u1} \cdot \left(\ln \frac{2 \cdot \mu \cdot K \cdot e^L + \mu_0 \cdot K - \mu \cdot K + \varepsilon}{\mu \cdot K + \mu_0 \cdot K - \varepsilon} \right)^{-1} \tag{18}$$

or

$$T = T_{u2} \cdot \left(\ln \frac{2 \cdot \mu \cdot K \cdot e^L - \mu_0 \cdot K - \mu \cdot K + \varepsilon}{\mu \cdot K - \mu_0 \cdot K - \varepsilon} \right)^{-1} \tag{19}$$

and a time delay term is $\Theta = T \cdot L$.

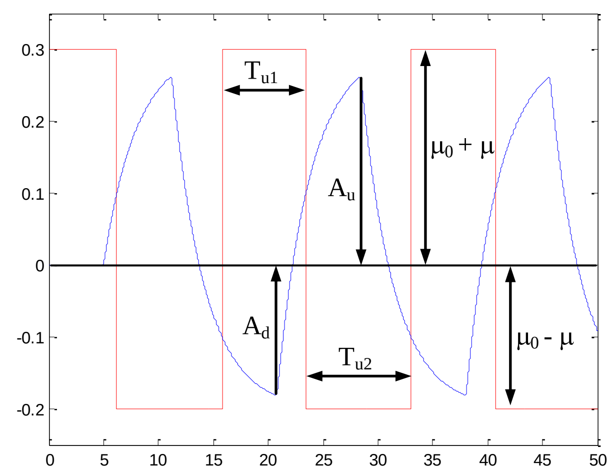

Fig. 5 biased relay oscillation of first order stable system

III. ALGEBRAIC CONTROL DESIGN

The control design is based on the fractional approach; see e.g. [9], [15], [17], [20], [31]. Any transfer function $G(s)$ of a (continuous-time) linear system is expressed as a ratio of two elements of R_{PS}. The set R_{PS} means the ring of (Hurwitz) stable and proper rational functions [31]. Traditional transfer functions as a ratio of two polynomials can be easily transformed into the fractional form simply by dividing, both the polynomial denominator and numerator by the same stable polynomial of the appropriate order.

Then all transfer functions can be expressed by the ratio:

$$G(s) = \frac{b(s)}{a(s)} = \frac{\dfrac{b(s)}{(s+m_0)^n}}{\dfrac{a(s)}{(s+m_0)^n}} = \frac{B(s)}{A(s)} \tag{20}$$

$$n = \max(\deg(a), \deg(b)), \quad m_0 > 0 \tag{21}$$

The feedback control loop can be in the traditional structure (one degree of freedom – 1DOF), see Fig. 6 or in the two degree of freedom (2DOF), see Fig. 7. In both cases, all feedback stabilizing controllers for the feedback systems are given by a general solution of the Diophantine equation:

$$AP + BQ = 1 \tag{22}$$

which can be expressed with Z free in R_{PS}:

$$\frac{Q}{P} = \frac{Q_0 - AZ}{P_0 + BZ} \tag{23}$$

In contrast of polynomial design, all controllers are proper and can be utilized.

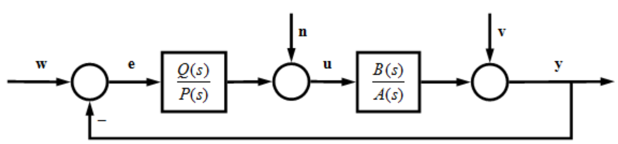

Fig. 6 one degree of freedom (1DOF) control loop

The Diophantine equation for the design of the feedforward controller depicted in Fig.7 is:

$$F_w S + BR = 1 \tag{24}$$

with parametric solution:

$$\frac{R}{P} = \frac{R_0 - F_w Z}{P_0 + BZ} \tag{25}$$

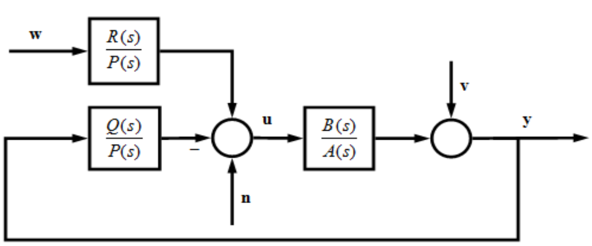

Fig. 7 two degree of freedom (2DOF) control loop

Asymptotic tracking is then ensured by the divisibility of the denominator P in (23) by the denominator of the reference $w = G_w / F_w$. The most frequent case is a stepwise reference with the denominator in the form:

$$F_w = \frac{s}{s+m_0}; \quad m_0 > 0 \tag{26}$$

The similar conclusion is valid also for the load disturbance (Fig. 6, Fig. 7) $n = G_n / F_n$. The load disturbance attenuation is then achieved by divisibility of P by F_n. More precisely, for tracking and attenuation in the closed loop the multiple of AP must be divisible by the least common multiple of denominators of all input signals. The divisibility in R_{PS} is defined through unstable zeros and it can be achieved by a suitable choice of rational function [15] for details.

The resulting control law is governed by the equation:

$$P(s)u(s) = Q(s)(w(s) - y(s)) \tag{27}$$

in the case of the 1DOF structure and

$$P(s)u(s) = R(s)w(s) - Q(s)y(s) \tag{28}$$

in the case of the 2DOF controller. The algebraic approach mentioned in this section generates a PI controller for the 1DOF structure and first order system (1) with $\Theta = 0$ and a PID (realistic one) for the second order system (2). Parameters of controllers are nonlinear functions of the tuning parameter $m_0 > 0$, e.g. in the simplest case of (1) the resulting PI controller is in the form:

$$C(s) = \frac{Q}{P} = \frac{q_1 s + q_0}{s} \tag{29}$$

where parameters q_1 a q_0 are given by:

$$q_1 = \frac{2Tm_0 - 1}{K} \qquad q_0 = \frac{Tm_0^{\,2}}{K} \tag{30}$$

More details can be found in [16], [17], [20]. The tuning parameter can be chosen e.g. for aperiodic behavior, see [20].

The control design for systems (1), (2) with a nonzero delay term should be treated in a different ways, see e.g. [17]-[20]. There are several possibilities, the first one is neglecting of this term, the response are acceptable only for normalized small time delays $L=\Theta/(T+\Theta)$. Another approach utilizes the Pade approximation, the simplest case is given by

$$e^{-\Theta s} \approx \frac{1 - \dfrac{\Theta s}{2}}{1 + \dfrac{\Theta s}{2}} \tag{31}$$

Naturally, this approximation increases the degrees of polynomials of the numerator as well as denominator of the nominal controlled transfer function. Poles of transfer function remain in the left half plane, while zeros are moved to the right half plane which can destabilize feedback loops. More details can be found in [22]. A different way consists in using of Smith predictor structure of the control loop, see [23]. A very modern way in the algebraic philosophy utilizes the ring of meromorphic function approach with quasipolynomials, see e.g. [19], [21].

IV. SIMULATION PROGRAM

A Matlab program system was developed for estimation of transfer function, controller design and simulation of feedback loop applications of auto-tuning principles. This program enables a choice for the relay estimation of the controlled system of arbitrary order. The estimated model is of a first or second order transfer function with time delay. The user can choose three cases for the time delay term. In the first case the time term is neglected, in the second one the term is approximated by the Pade expansion and the third case utilizes the Smith predictor control structure. The program is developed with the support of the Polynomial Toolbox. The Main menu of the program system can be seen in Fig. 8.

In the first phase of the program routine, the controlled transfer function is defined and parameters for the relay experiment can be adjusted. Then, the experiment is performed and it can be repeated with modified parameters if necessary. After the experiment, an estimated transfer function in the form of (1) or (2) is performed automatically and controller parameters are generated after pushing of the appropriate button. Parameters for experimental adjustment are defined in the upper part of the window.

The second phase begins with the “Design controller parameters” button and the actual control design is performed. According to above mentioned methodology and identified parameters, the controller is derived and displayed. The control scheme depends on the choice for the 1DOF or 2DOF structure and on the choice of the treatment with the time delay term.

During the third phase, after pushing the “Start simulation” button, the simulation routine is performed and required outputs are displayed. The simulation horizon can be prescribed as well as tuning parameter *m*, other simulation parameters can be specified in the Simulink environment. In all simulation a change of the step reference is performed in the second third of the simulation horizon and a step change in the load is injected in the last third. A typical control loop of the case with the Smith predictor in Simulink is depicted in Fig. 9.

Also the step responses can be displayed and the comparison of the controlled and estimated systems can be depicted. Another versions of the similar program systems were developed and they are referred in e.g. [17], [18].

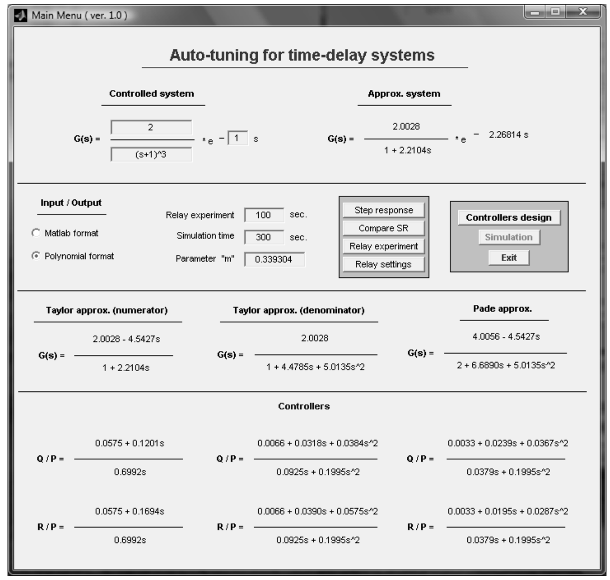

Fig. 8 main menu of program system

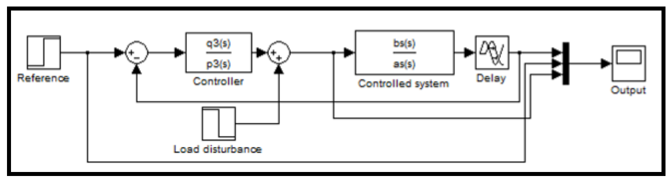

Fig. 9 simulation program in Simulink

V. ANALYSIS AND RELAY SETTING OF ESTIMATION

In this contribution, the main emphasis was laid on the accuracy of estimated parameters. The aim is to conclude how to set relay parameters and to give some recommendations.

In this approach, the identification relations have to estimate both, time constant as well as a system gain. The time parameters are estimated by a symmetrical relay, while the gain is estimated by a biased relay experiment. Then a contradictory question is concluded: How to utilize a biased relay experiment for estimation of all identified parameters in (1) and (2). The main aim of the research work was to investigate how a biased relay can be used with satisfactory accuracy and how to set up the relay experiment.

The first test transfer function for the first order system is given:

$$G(s) = \frac{3}{4s+1} \cdot e^{-6s} \tag{32}$$

Many relay feedback experiments were performed by the simulation program and the following sensitivity was investigated. The accuracy of estimated parameters depends on main parameters of the relay, namely:

- asymmetry
- hysteresis
- relay amplitude

Table I shows the influence of the asymmetry of the relay on the accuracy of estimation. All entries of Table 1 are in differences between the true and estimated values in %. The upper value of the relay output was *0.30*.

Asymmetry [%]	10	20	30	40	50	60
Δ K [%]	1.7	1.3	1.3	0.7	0.7	0.7
Δ T [%]	3.5	2.8	2.1	1.2	1.2	1.0
Δ Θ [%]	1.2	1.0	0.8	0.5	0.7	0.5

Table I estimation accuracy based on relay asymmetry.

Table II summarizes the sensitivity of the relay hysteresis for transfer function (32). All entries are for comparison in numerical values. The upper relay output was *1.2* lower value *-1.08*.

Hysteresis ε	0	0.2	0.4	0.6
Gain K	2.88	2.95	2.95	2.92
Time constant T	3.70	3.88	3.78	3.78
Time delay Θ	6.12	6.05	6.07	6.12

Table II estimation accuracy based on relay hysteresis

In a similar way, according to Table 2 also a set of experiments were for various values of the lower relay output *-0.96, 0.84, 0.72, 0.60*, respectively. The following observations and recommendations can be drawn from the obtained analysis:

- bigger values of asymmetry up to 40% caused better accuracy of all parameters
- better accuracy was achieved for smaller values of hysteresis $\varepsilon = 0.1;0.2$
- values of relay outputs have no relevant influence on the estimation accuracy

VI. EXAMPLES AND SIMULATIONS

The recommended values for a relay experiment were used for the estimation of the higher order system:

$$G(s) = \frac{5}{(s+1)^8} \cdot e^{-3s} \tag{33}$$

The relay parameters with $\varepsilon = 0.1;$ asymmetry *40%* with upper and lower relay outputs *0.30* and *-0.18* were used. The resulting first order estimation takes the form:

$$\tilde{G}(s) = \frac{4.97}{3.58s+1} \cdot e^{-8.19s} \tag{34}$$

Comparison of both step responses of systems (33) and (34) is depicted in Fig. 10. Other results of estimation and autotuning control can be found in [18] - [20].

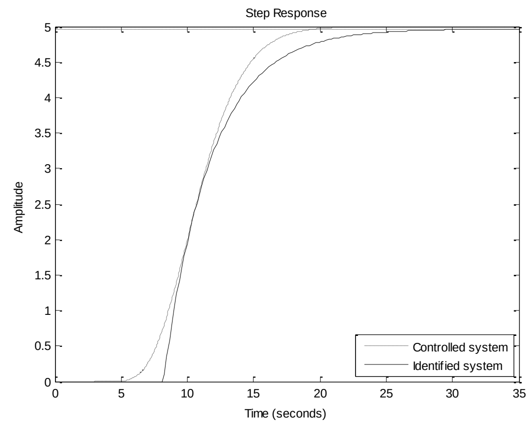

Fig. 10 step responses of systems (33) and (34)

The quantity of the normalized time delay $L=\Theta/(T+\Theta)=0.70$ for (33) indicates the difficulty of controlling. The second order approximation is in the form

$$\tilde{\tilde{G}}(s) = \frac{4.97}{(2.19s+1)^2} \cdot e^{-6.76s} \tag{35}$$

A simpler class of controllers was derived in R_{PS} representation (22), (23) with neglecting of time delay and the first order controller takes the form (for an appropriate choice of the tuning parameter, see [20])

$$G_{R1}(s) = \frac{0.01s+0.02}{s} \quad m_0 = 0.15 \tag{36}$$

The second order R_{PS} controller gives

$$G_{R2}(s) = \frac{0.23s^2+0.16s+0.03}{3.46s^2+s} \quad m_0 = 0.30 \tag{37}$$

Control responses of both feedback systems are shown in Fig. 11 and Fig. 12 outlines stability margins of both controllers with original control plant (33).

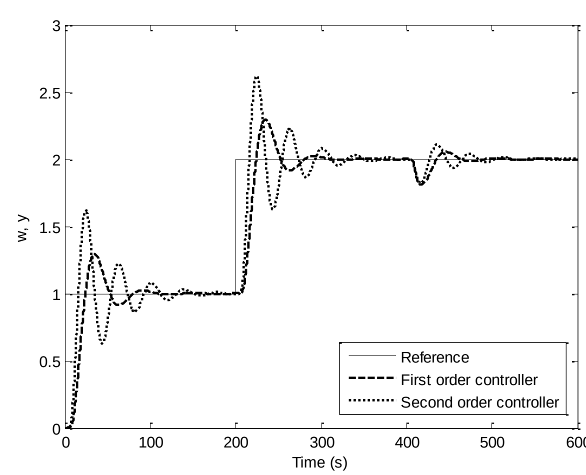

Fig. 11 R_{PS} control responses of (33) with (36), (37)

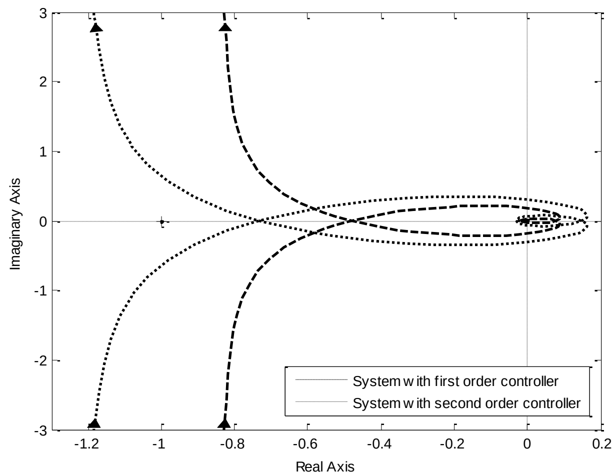

Fig. 12 R_{PS} Nyquist plots of (33) and (36), (37)

Further improvement of the control behavior can be achieved by introducing Pade approximation of the time delay term, see e.g. [17], [22]. The simplest approximation is given

$$e^{-\Theta s} = \frac{e^{-\frac{\Theta}{2}s}}{e^{\frac{\Theta}{2}s}} \approx \frac{2-\Theta}{2+\Theta} \tag{38}$$

This approximation is then used in estimated transfer functions of the first and second order (34), (35). The first order system is then

$$G_1(s) = \frac{-20.38s + 4.97}{14.678s^2 + 7.68s + 1} \tag{39}$$

According to control synthesis equation (22), the resulting controller for asymptotic tracking takes the form

$$G_{R1}(s) = \frac{0.33s^2 + 0.12s + 0.01}{7.01s^2 + s} \quad m_0 = 0.15 \tag{40}$$

and it is also parameterized by a tuning parameter

The second order approximation (35) with (38) yields to the following system of the third order

$$G_2(s) = \frac{-16.78s + 4.965}{16.28s^3 + 19.65s^2 + 7.77s + 1} \tag{41}$$

The resulting controller is no more of the PID structure but it has the third order transfer function

$$G_{R2}(s) = \frac{0.17s^3 + 0.22s^2 + 0.09s + 0.01}{5.27s^3 + 3.12s^2 + s} \quad m_0 = 0.30 \tag{42}$$

The control responses are depicted in Fig. 13. The improvement of the control behavior is apparent. However, the controller structure is more complex and approximation (38) brings unstable zeros to transfer functions (39), (41).

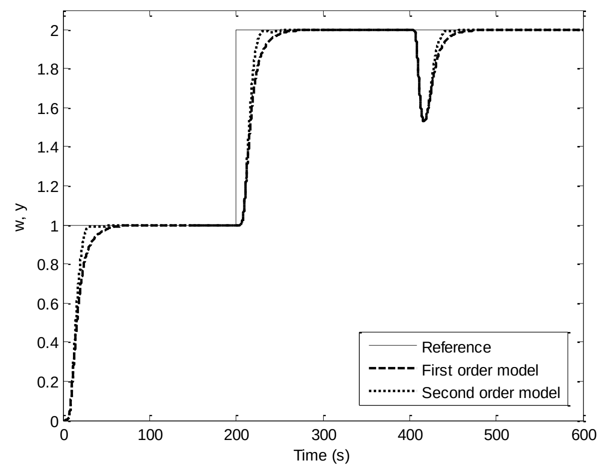

Fig. 13 control responses with Pade approximation (38)

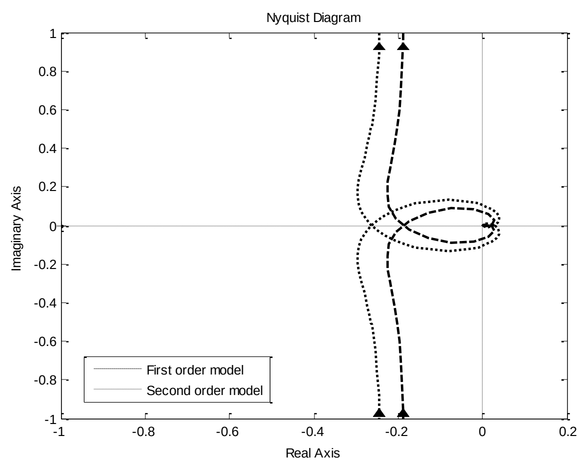

Fig. 14 Nyquist plots of (33) and (40), (42)

Fig. 14 shows the Nyquist plots of the connection of the controlled system (33) and controllers (40), (42). It is evident that the Pade approximation brings evident improvement in the stability margin which is a measure of the robustness of designed controllers.

Another principle how to tackle with delay systems can be found in meromorphic function approach. There is no approximation in delay terms, however, the price is more complex controller structure, see e.g. [18], [21].

VII. Conclusion

The paper presents some results of research whose aim is to develop and analyze main features for improving of single input- output autotuners. The first interest is devoted to relay parameters for a feedback relay experiment. The proper and accurate parameter estimation plays a key role for a control design, especially in autotuning utilization. Various relay improvements and utilization for control design can be found also in [13], [14], [20], [30]. The goal of the paper is to investigate how the estimation is sensitive on the relay settings. Main relay characteristics as asymmetry, hysteresis and amplitude can be recommended for the correct adjustment for the relay experiment. Further, some results of the algebraic approach are summarized. The tuning scalar parameter is proposed and this parameter can influence control behavior, robustness and aperiodicity of the control response. The illustrative example for a delay system compares neglecting and Pade approximation of the delay term in controller design and also a robust performance is outlined. A Matlab+Simulink program was developed for automatic estimation, control design and simulation of single input-output continuous-time systems.

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