Numerical Approximation of Static Characteristics of McKibben Pneumatic Artificial Muscle

Mária Tóthová, Ján Piteľ, Alexander Hošovský and József Sárosi

***Abstract*—**Pneumatic artificial muscles belong to progressive non-conventional actuators with several appealing characteristics as extremely high ratio of force to weight and high stiffness. But they exhibit highly non-linear behavior especially due to inherent properties of their materials and compressibility of air. Therefore, it is difficult to control them without adequate modeling and simulation. The paper deals with the numerical approximation of the non-linear static characteristics of McKibben pneumatic artificial muscle. The muscle force dependence on the muscle contraction and pressure in the muscle was approximated using available software tools as Microsoft Excel and Matlab Curve Fitting Toolbox and four obtained functions are expressed and shown for comparison in the paper.

***Keywords*—**pneumatic artificial muscle, static characteristic, numerical approximation

I. Introduction

PNEUMATIC artificial muscles (PAMs) belong to the group of non-conventional actuators which have great potential in mechatronic [1] and biomechanical [2], [3] applications due to an advantageous power to weight ratio, high stiffness, structural simplicity, natural compliance and self-dampening. When compressed air is supplied inside PAM its elastic tube has tendency to expand in the radial direction, whereas a contraction occurs in the axial direction and thus the tensile force is generated. This force is characterized by highly non-linear responses [4], [5]. The principle of PAM is described in more details for example in [6], [7], [8].

PAMs can be classified into the groups, namely [9]: braided muscles (McKibben muscle, sleeved bladder muscle), netted muscles (Yarlott muscle, ROMAC, Kukolj muscle), embedded muscles (Morin muscle, Baldwin muscle, under-pressure muscle, Kleinwachter torsion device, Paynter knitted muscle, Paynter hyperboloid muscle), pleated muscles (PPAM muscle) and special muscles (rotary muscle, 3-DOF muscle, single-action elastic tube and combination). Their some examples can be seen in Fig. 1 [10], [11] and the most used type of muscle in practice is McKibben muscle.

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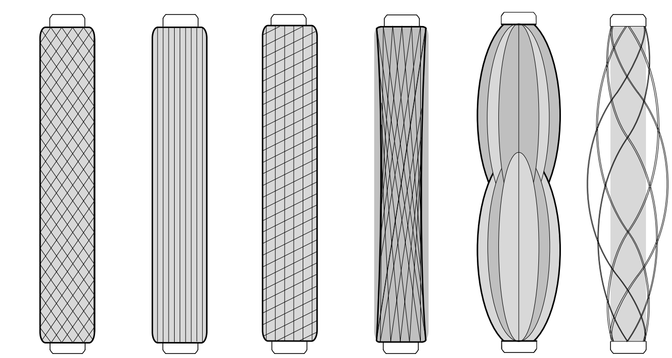

Fig. 1 Types of PAMs: a) McKibben muscle, b) pleated muscle, c) Yarlott netted muscle, d) Painter hyperboloid muscle, e) ROMAC muscle, f) Kukolj muscle

II. Static Characteristics and Geometric Parameters of PAM

The static characteristic of McKibben PAM represents the relation (a function) between the muscle force F and muscle contraction κ (under a constant muscle pressure p) regardless the time factor. PAMs manufactured by FESTO are now the most popular and commercially available and therefore static characteristics of fluidic muscles of this company were used for approximation. These characteristics of PAM type FESTO MAS-20-200N in a working range between 0 % and 25 % of the muscle contraction (which is recommended by FESTO catalogue [12]) are shown in Fig. 2.

PAM consists of the inner elastic tube inserted into the non-extensible fibers which define an expansion in the context of increasing pressure. Typical materials used for the tube are latex and silicone rubber. Fibers are made typically from nylon. The tube is connected to the terminals at both ends of muscle through which the mechanical energy is transferred to the load. These terminals are most often made from metallic materials. The commercially available PAM MAS-20-200N by FESTO differs slightly from the general McKibben type muscle. The fibers of the fluidic muscle are knit into the tube, offering easy assembly compared to conventional designs of muscle [13].

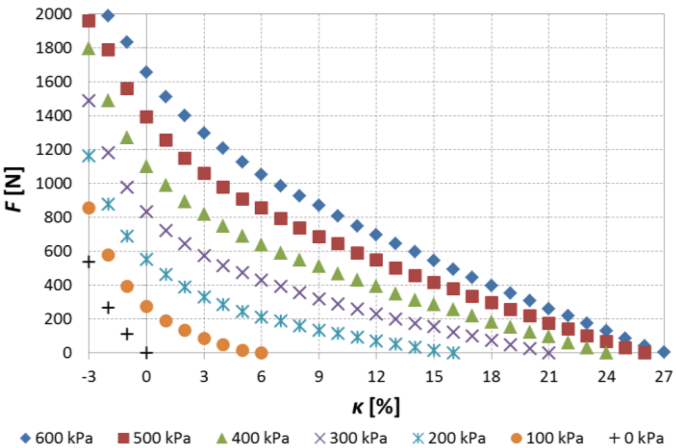

Fig. 2 Static characteristics of PAM type FESTO MAS-20-200N

Then the main geometric parameters of PAM in Fig. 3 are the initial muscle length l_0, the initial angle α_0, the initial radius in the middle of the muscle r_0, the actual muscle length l, the actual angle α, the actual radius in the middle of the muscle r, number N wrapped of single fibers and the half nylon thread length L.

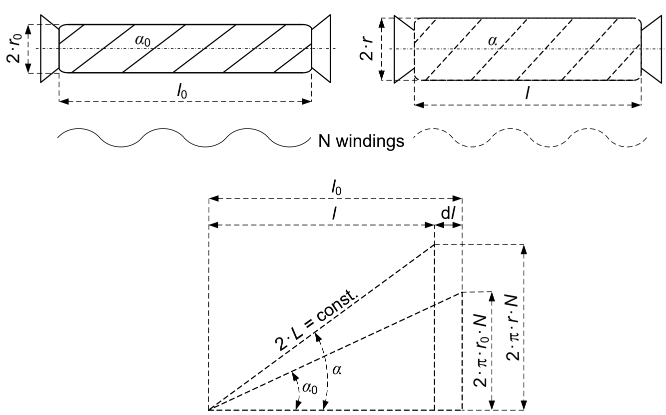

Fig. 3 Correlation between geometric parameters of PAM

III. Approximation of Static Characteristics of PAM

There are several basic approximation functions for description of static characteristics of PAM which are the subject of many works [13], [14], [15], [16], [17]. Following there are presented four possible ways of numerical approximation of static characteristics of PAM using the given measured data.

A. Approximation Using an Analytical Modeling of PAM

On the basis of physical laws (law of energy conservation, Bernoulli equation, etc. [18], [19], [20]) and the geometric parameters of PAM (Fig. 2), relation of the static characteristics was derived [17]:

$$F(p,\kappa) = \pi \cdot r_0^2 \cdot p \cdot \left(\frac{3}{\tan^2 \alpha_0} \cdot \frac{l^2}{l_0^2} - \frac{1}{\sin^2 \alpha_0} \right). \tag{1}$$

Equation (1) does not include impact of the muscle membrane on the pressure change; it was assumed that the maximal contraction will be the same for the different pressures. In order to balance the relation between pressure p and contraction κ, the member $\varepsilon(p)$ was added to (1). This new member was suitable for higher pressures p but there were always the differences between the experimentally obtained values and the theoretical model for smaller pressure p in relation to contraction κ. In order to obtain the approximation for small values of pressure, member $\mu(\kappa)$ was also added to equation (1). After substituting new members and using relation for the contraction $\kappa = (l_0-l)/l_0$, it can be obtained [21], [22]:

$$F(p,\kappa) = \mu(\kappa) \cdot \pi \cdot r_0^2 \cdot p \cdot \left(a \cdot (1 - \varepsilon(p) \cdot \kappa)^2 - b \right), \tag{2}$$

where values $a = 3/\tan^2\alpha_0$, $b = 1/\sin^2\alpha_0$ depend on the parameters of PAM (for our type of PAM: $a = 5.4$, $b = 2.8$, $r_0 = 10$ mm).

The following functions are used to achieve the best approximation of the muscle curve:

$$\varepsilon(p) = a_\varepsilon \cdot e^{-p} - b_\varepsilon, \tag{3}$$

$$\mu(\kappa) = a_\kappa \cdot e^{-\kappa \cdot c_\kappa} - b_\kappa. \tag{4}$$

Coefficients a_ε, b_ε, a_κ, b_κ, c_κ were determined using Matlab Curve Fitting Toolbox. The obtained values of these coefficients are shown in Table 1.

Table 1 The values of coefficients from (3) and (4)

Coefficient	Value
a_ε	0.4678000
b_ε	-0.0108600
a_κ	0.0008969
b_κ	-0.0025230
c_κ	0.2938000

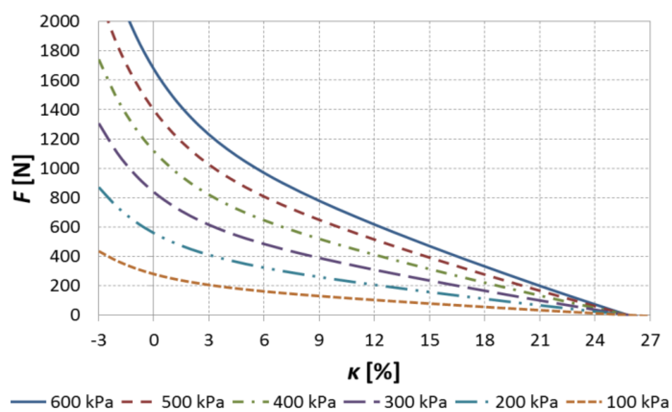

Fig. 4 Force-contraction relations approximated by (2) for various pressures in the muscle

Fig. 4 presents the static characteristics of Fluidic Muscle type MAS-20-200N (Fig. 2) obtained by approximation and they were created using (2) in MS Excel. To describe the nature and force of the relation between the calculated results and the specified by FESTO, comparison of results were used in Fig. 5.

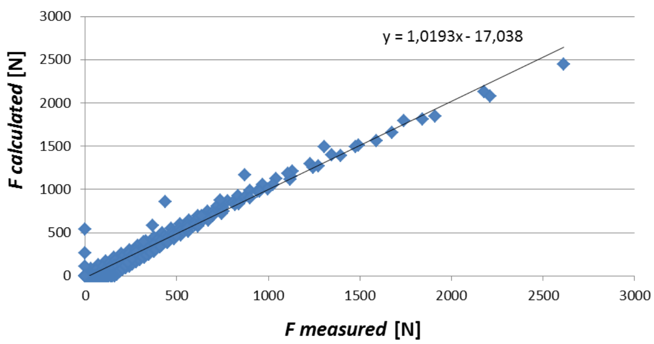

Fig. 5 *F* calculated in correlation with *F* measured for Fig. 4

The fit results are shown in Table 2 and approximated surface in Matlab is shown in Fig. 6.

Table 2 Fit results for the approximation using an analytical modeling of PAM

SSE	R-square	adj R-square	RMSE
2.784E+06	0.9713	0.9711	81.22

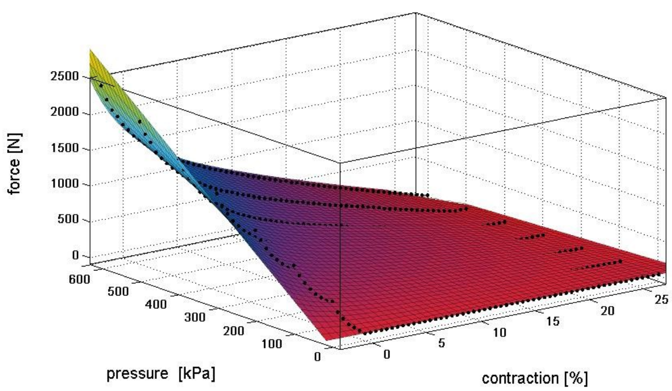

Fig. 6 Force-contraction-pressure relations approximated by (2)

B. Approximation Deducted from the Maximum Force of PAM

The muscle force as a function $F(p,\kappa)$ of the muscle contraction for different pressures in the muscle can be deducted from the maximum muscle force F_{max}. Note that if the muscle contraction is constant, the muscle force depends almost linearly on the pressure. However, the proportionality factor decreases with increasing contraction. Then, the result of the muscle force is defined as follows [23]:

$$F(p,\kappa) = F_{\max}(\kappa) - (p_{\max} - p) \cdot \left(\frac{a_0 - a_1 \kappa}{a_2} \right), \tag{5}$$

where p_{max} is the maximum and p is the actual pressure in muscle. Coeficients a_0 [N], a_1 [N·m^{-1}], a_2 [Pa] were found using Matlab Curve Fitting Toolbox. The maximum force F_{max} as a function of the muscle contraction is introduced by a fourth-order polynomial function for the response at the maximum pressure p_{max} = 600 kPa [23]:

$$F_{\max}(\kappa) = b_0 + b_1\kappa + b_2\kappa^2 + b_3\kappa^3 + b_4\kappa^4, \tag{6}$$

where coeficients b_0, b_1, b_2, b_3, b_4 were also found using Matlab Curve Fitting Toolbox. All coefficients from (5) and (6) are shown in Table 3.

Table 3 The values of coefficients from (5) and (6)

Coefficient	Value
a_0	461.700000
a_1	17.050000
a_2	168.000000
b_0	1700.000000
b_1	-166.700000
b_2	12.680000
b_3	-0.591600
b_4	0.009848

The result of approximation of the static characteristics of Fluidic Muscle type MAS-20-200N (Fig. 2) is shown in Fig. 7. They were created using (5), (6) in MS Excel.

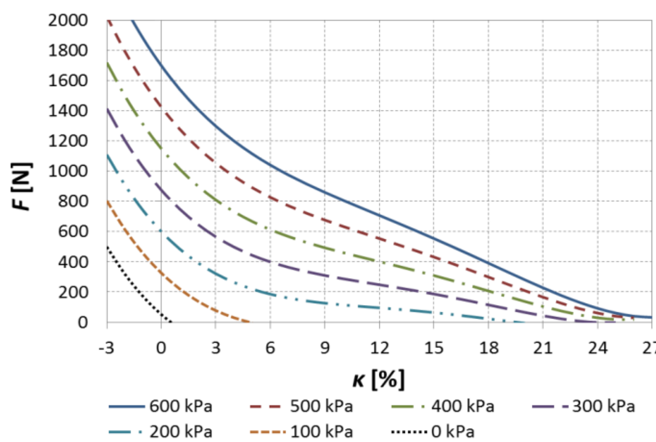

Fig. 7 Force-contraction relations approximated by (5) and (6) for various pressures in the muscle

Fig. 8 describes relation between the calculated result and the specified by FESTO.

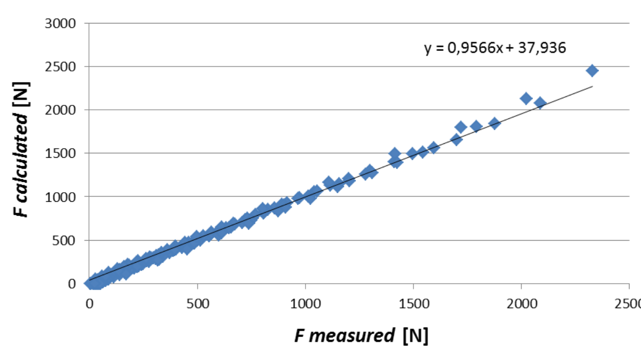

Fig. 8 *F* calculated in correlation with *F* measured for Fig. 7

Table 4 shows the fit results and approximated surface in Matlab is shown in Fig. 9.

Table 4 Fit results for the approximation deducted from the maximum force of PAM

SSE	R-square	adj R-square	RMSE
9.374E+07	0.9974	0.9974	473

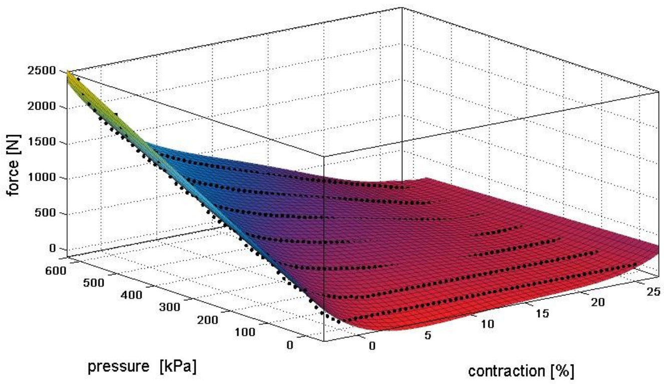

Fig. 9 Force-contraction-pressure relations approximated by (5) and (6)

C. Approximation Using an Exponential Function

The muscle force F dependence on the muscle contraction κ at constant pressure p in the muscle can be approximated with a good precision using an exponential function as follows [24]:

$$F(\kappa) = a_1 \cdot e^{(a_2 \cdot \kappa + a_3)} + a_4 \kappa + a_5 , \qquad (7)$$

where a_1, a_2, a_3, a_4, a_5 are unknown coefficients.

In order the equation (7) would have universal application for different pressures in the muscle an algorithm differing in pressure was necessary to create. Therefore a new approximation algorithm with several different unknown parameters was introduced [15]:

$$F(\kappa, p) = (a_1 \cdot p + a_2) \cdot e^{(a_3 \cdot \kappa + a_4)} + (a_5 \cdot p + a_6) \cdot \kappa + \\ + a_7 \cdot p + a_8 . \qquad (8)$$

With reduced number of parameters the force also can be calculated and therefore approximation algorithm with six different unknown parameters was expressed for comparison [15]:

$$F(\kappa, p) = (a_1 \cdot p + a_2) \cdot e^{a_3 \cdot \kappa} + a_4 \cdot \kappa \cdot p + a_5 \cdot p + a_6 , \qquad (9)$$

where κ is muscle contraction, p is pressure in the muscle and a_1, a_2, a_3, a_4, a_5, a_6 are unknown coefficients which values are also found using Matlab Curve Fitting Toolbox and they are presented in Table 5.

Table 5 The values of coefficients from (9)

Coefficients	Values	Coefficients	Values
a_1	0.11210	a_4	-0.08619
a_2	263.70000	a_5	2.62400
a_3	-0.35150	a_6	-245.60000

The static characteristics of Fluidic Muscle type MAS-20-200N (Fig. 2) approximated according to (9) and created in Microsoft Excel are shown in Fig. 10. Relation between the calculated results and the specified by FESTO describes Fig. 11.

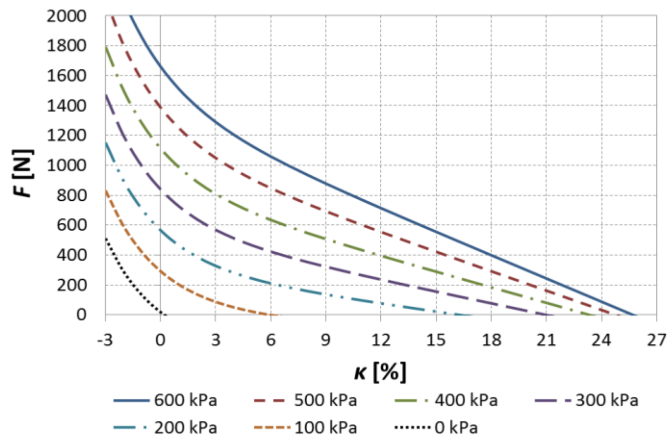

Fig. 10 Force-contraction relation approximated by (9) for various pressures in the muscle

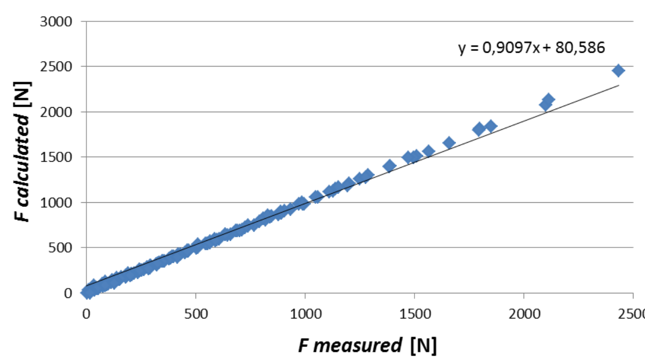

Fig. 11 F calculated in correlation with F measured for Fig. 10

Table 6 shows the fit results and approximated surface in Matlab is shown in Fig. 12.

Table 6 Fit results for the approximation using an exponential function

SSE	R-square	adj R-square	RMSE
1.842E+07	0.9996	0.9996	209.2

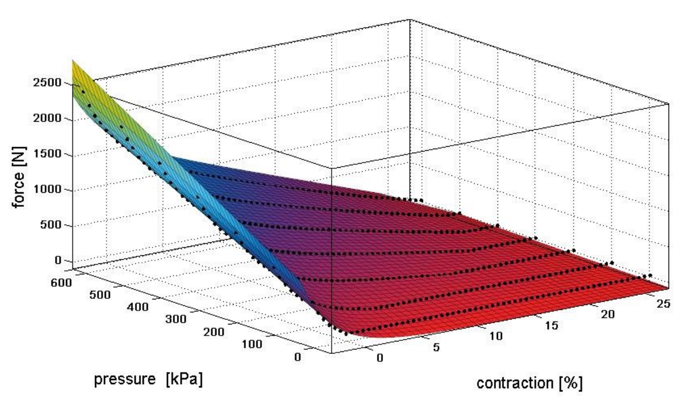

Fig. 12 Force-contraction-pressure relations approximated by (9)

D. Approximation Using a Polynomial Function

The fourth method applied for approximation of static characteristics of PAM (Fig. 2) was a polynomial approximation. In order to approximate these static characteristics with good accuracy, a fifth-order polynomial function of two variables was used. This polynomial function contains twenty-one coefficients and its form is as follows [25]:

$$\begin{aligned}F(\kappa, p) &= a_{00} + a_{10}\cdot\kappa + a_{01}\cdot p + a_{20}\cdot\kappa^2 + a_{11}\cdot\kappa\cdot p + \\ &+ a_{02}\cdot p^2 + a_{30}\cdot\kappa^3 + a_{21}\cdot\kappa^2\cdot p + a_{12}\cdot\kappa\cdot p^2 + \\ &+ a_{03}\cdot p^3 + a_{40}\cdot\kappa^4 + a_{31}\cdot\kappa^3\cdot p + a_{22}\cdot\kappa^2\cdot p^2 + \\ &+ a_{13}\cdot\kappa\cdot p^3 + a_{04}\cdot p^4 + a_{50}\cdot\kappa^5 + a_{41}\cdot\kappa^4\cdot p + \\ &+ a_{32}\cdot\kappa^3\cdot p^2 + a_{23}\cdot\kappa^2\cdot p^3 + a_{14}\cdot\kappa\cdot p^4 + a_{05}\cdot p^5.\end{aligned} \tag{10}$$

The values of all coefficients in (10) were determined also using Matlab Curve Fitting Toolbox and they are shown in Table 7.

Table 7 The values of coefficients from (10)

Coefficient	Value	Coefficient	Value
a_{00}	-14.570000	a_{31}	4.21E-04
a_{10}	-86.050000	a_{22}	-1.13E-05
a_{01}	3.148000	a_{13}	-2.85E-07
a_{20}	16.040000	a_{04}	-2.39E-08
a_{11}	-0.262400	a_{50}	-0.0004392
a_{02}	-0.002405	a_{41}	-7.03E-07
a_{30}	-1.112000	a_{32}	-5.54E-07
a_{21}	-0.003664	a_{23}	2.76E-08
a_{12}	0.0004941	a_{14}	-3.27E-10
a_{03}	1.08E-05	a_{05}	1.85E-11
a_{40}	0.035290		

The static characteristics generated by Fluidic Muscle type MAS-20-200N (Fig. 2) and approximated using (10) were created again in MS Excel and they are shown in Fig. 13. Relation between the calculated results and the specified by FESTO describes Fig. 14.

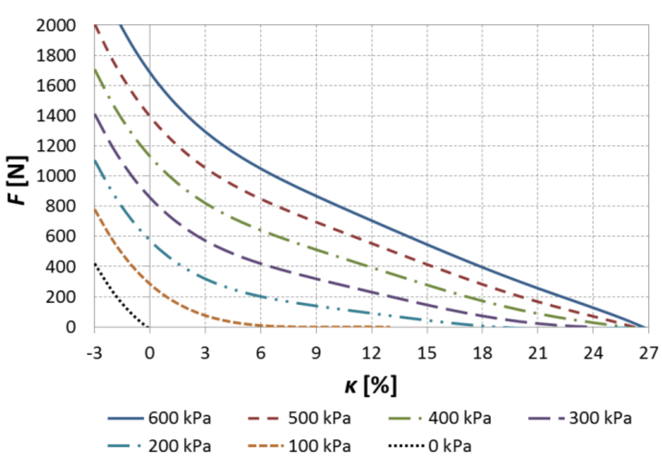

Fig. 13 Force-contraction relations approximated by (10) for various pressures in the muscle

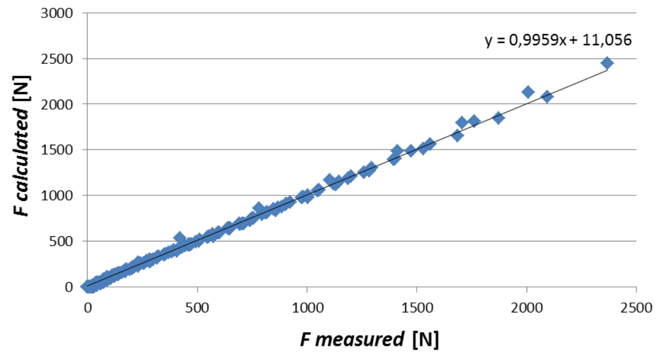

Fig. 14 F calculated in correlation with F measured for Fig. 13

Table 8 shows the fit results and approximated surface is shown in Fig. 15.

Table 8 Fit results for the approximation using the fifth-order polynomial

SSE	R-square	adj R-square	RMSE
1.099E+07	0.9997	0.9997	164.5

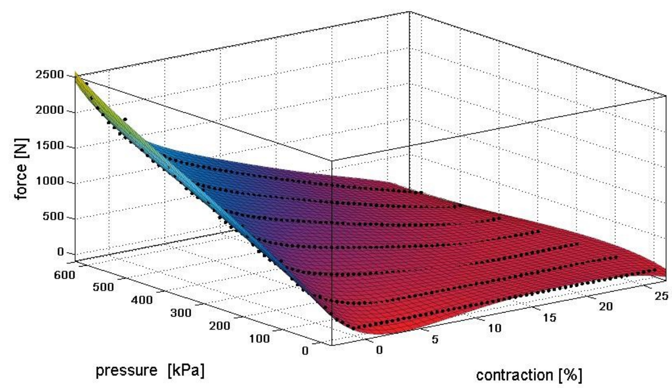

Fig. 15 Force-contraction-pressure relations approximated by the fifth-order polynomial

IV. Results of Comparison and Conclusion

The muscle force dependence on the muscle contraction and pressure in the muscles was approximated for modified version of McKibben PAM produced by FESTO Company. Four different approximation functions were tested for possible approximation of the given static characteristics of Fluidic Muscle type MAS-20-200N (Fig. 2). Analyses were carried out in Matlab Curve Fitting Toolbox environment and obtained results of approximation were shown using MS Excel and Matlab.

Different approaches using an analytical modeling (presented in section A), results of experimental modeling (presented in section B), an exponential function (presented in section C) and only numerical approximation (presented in section D) were used for comparison. Firstly, the results of approximation using an analytical modelling of PAM (Fig. 4) were compared and the achieved coefficient of determination was $R^2 = 0.9713$. Secondly, the results of approximation deducted from the maximum force of PAM were compared and the achieved coefficient of determination was $R^2 = 0.9974$.

Thirdly, the results of approximation using an exponential function were compared and the achieved coefficient of determination was $R^2 = 0.9996$. The best approximation results were reached using a fifth-order polynomial function of two variables with twenty-one coefficients (the achieved coefficient of determination was $R^2 = 0.9997$). That is why the function (10) with the values of coefficients in Table 7 was used in dynamic modeling of one DoF (Degree of Freedom) PAM-based pneumatic antagonistic actuator using modified Hill's muscle model [25].

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