Approximate solution of the Navier-Stokes equation and Magnus effect in the fluid-film bearings

L. Savin, A. Kornaev, E. Kornaeva

Abstract**—**The Magnus effect, which occurs with the oncoming flow around the rotating bodies, is known for a long time. The main purpose of the present paper was to identify the conditions of Magnus effect occurrence in a thin lubricant film of the fluid-film bearing and to obtain an approximate analytical dependency for its quantity evaluation. The mathematical model of the viscous incompressible fluid flow is based on the Navier-Stokes equation and the condition of incompressibility considering the non-stationarity of the process, inertia, viscous resistance and mass forces effect. As a result of evaluation of significance of the mathematical model equations terms by means of similarity theory and dimension analysis the conditions were determined as a dimensionless criteria, when inertia forces from the Magnus effect are significantly bigger than the mass and viscosity forces. Given the fulfillment of these conditions an analytical form was obtained to determine the hydrodynamic reaction of the lubricant.

Keywords**—**Continuum mechanics, hydrodynamic lubrication theory, Magnus effect, fluid-film bearing, similarity criterion.

I. Introduction

The Magnus effect [1] occurs with the combination of the rotational flow and the oncoming flow of the media around the body, which results in the drop of the pressure and in the resulting force perpendicular to the direction of the oncoming flow.

According to the Joukovsky theory on the lifting force [2] the Magnus effect in the quantitative terms is as follows:

$$\vec{F} = -\rho\vec{\Gamma} \times \vec{V}_{\infty}\,, \tag{1}$$

where ρ - density of the media, $\vec{\Gamma}$ - circulation of the rotational velocity of the cylinder, $\vec{V}_{\infty}$ - velocity of the oncoming flow.

The module of the reaction (1) in the case of the flow over the cylinder of the unit length can be written in a more simple form suggested by Rayleigh [3]:

$$F = 2\pi\omega r^2 \rho V_{\infty}\,, \tag{2}$$

where ω - the angular rotational velocity of the cylinder, r - radius of the cylinder.

The Magnus effect in connected with the viscosity properties of the media, namely the stratified flow and adhesion properties, and quantitatively it is due to the inertia forces of the media motion, which is evidenced by the presence of the ρ in (1).

The object of the study is a circular flow of the viscous incompressible media in the fluid-film bearings. The importance of the correct calculation is hard to overestimate. It is only necessary to highlight that they are the key elements of the rotor systems in the fluid rocket engines.

The vibration of the rotating tip in the fluid-film bearing cause the media motion in the same way as it happens when the oncoming flow flows over the rotating cylinder. So there is a theoretical possibility of a significance of the influence of the Magnus effect on the hydrodynamic reaction of the lubricant.

It is possible to show that the Reynolds equation [4] which is most frequently used in hydrodynamic theory, does not consider the influence of the Magnus effect. So it is an urgent problem to indicate the conditions of occuring and to evaluate the significance of the Magnus effect in the fluid-film bearings, as it is the objection of this paper.

II. Conceptual Model

A problem under study considers a media flow in the gap between the tip of the rotor with the radius r which rotates and vibrates, and the bearing with the radius R of the rotor machine (fig. 1).

Some of the assumptions in the conceptual model are made subjectively, due to the fact of solving some particular class of problems of the hydrodynamics [5]. Among them are the following:

This work was supported in part by the Ministry of Education and Science of the Russian Federation under the projects No 9.101.2014/K and No 363 of a part of the state task for the "State University - ESPC" and under grant of the President of the Russian Federation No 14.Z56.14.6000-MK.

L. A. Savin is with the Faculty of New Technologies and Production Automation, State University – Education-Science-Production Complex, Oryol, 302020, Russia (e-mail: savin@ostu.ru).

A. V. Kornaev is with the Faculty of New Technologies and Production Automation, State University – Education-Science-Production Complex, Oryol, 302020, Russia (corresponding author to provide phone: +7953-478-1591; e-mail: rusakor@inbox.ru).

E. P. Kornaeva is with the Education-Science-research institute of information technologies, State University – Education-Science-Production Complex, Oryol, 302020, Russia (e-mail: lenoks_box@mail.ru).

1. The rotor rotates with a constant angular velocity ω and vibrates in the plain which is perpendicular to its axis.
2. The continuous media is incompressible and has constant mechanical and thermophysical properties.
3. On the surfaces of the tip and the bearing the no-slip condition is met.

Another part of the assumptions was made on the basis of the results of the experimental study on the bearing dynamics of the high-speed lightly loaded rotor systems [6].

4. The media motion occurs in the plain perpendicular to the bearing axis, the lubricant consumes the whole area between the tip and the bearing.
5. The axis of the tip vibrates close to the center of the bearing.
6. The transverse vibrations frequency in equal to the rotational frequency, and the trajectory of the vibration is close to a circle form.

A third part of the assumptions will be formed based on the similarity theory and the dimensional analysis applied to the equations of the mathematical model of the media flow.

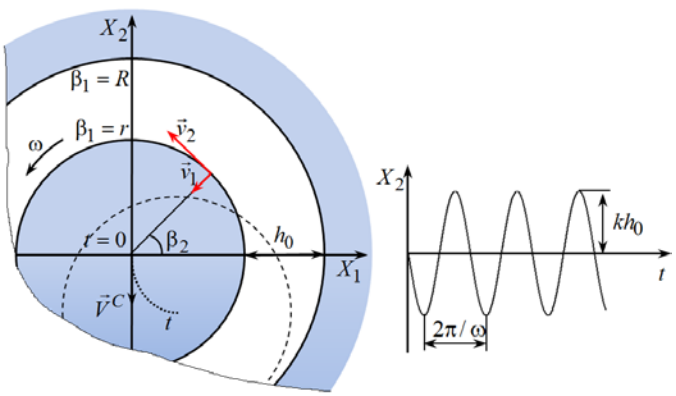

Fig. 1 Conformity model of the fluid-film bearing

III. The Mathematical Model

According to the assumptions of the conceptual model, and taking for the sake of simplicity that the trajectory of the vibrations of the tips is a circle of a known radius kh_0 ($0 \le k < 1$) with a center on a horizontal symmetry axis of the bearing and goes at initial point of time $t = 0$ through the center of the bearing (fig. 1), so the problem of the continuous media motion at this point of time is significantly simplified due to the symmetry of the flow area and the convenience of setting the boundary conditions. To obtain the approximate analytical dependencies for the lifting force calculation, the analysis of the significance of the mathematical model equation terms will be implemented on the surface of the tip at an initial point of time.

The continuous media motion is more convenient to study in polar coordinates [7]. Lamé parameters H_i in polar coordinates β_i , where β_1 - a radial coordinate, β_2 - a tangential coordinate, will take the following form:

$$\begin{aligned} H_1 &= 1, \\ H_2 &= \beta_1. \end{aligned} \tag{3}$$

The Navier-Stokes equation in the projection on the β_i axis considering (3) take the following form [5, 8, 9]:

$$\begin{aligned}
&\rho\left\{\frac{\partial v_1}{\partial t} + v_1\frac{\partial v_1}{\partial \beta_1} + \frac{v_2}{\beta_1}\frac{\partial v_1}{\partial \beta_2} - \frac{v_2^2}{\beta_1}\right\} = \rho f_1 - \\
&-\frac{\partial p_0}{\partial \beta_1} + \frac{\mu}{\beta_1^2}\left[\beta_1^2\frac{\partial^2 v_1}{\partial \beta_1^2} + \beta_1\frac{\partial v_1}{\partial \beta_1} - v_1 + \frac{\partial^2 v_1}{\partial \beta_2^2} - 2\frac{\partial v_2}{\partial \beta_2}\right], \\
&\rho\left\{\frac{\partial v_2}{\partial t} + v_1\frac{\partial v_2}{\partial \beta_1} + \frac{v_2}{\beta_1}\frac{\partial v_2}{\partial \beta_2} + \frac{v_1 v_2}{\beta_1}\right\} = \rho f_2 - \\
&-\frac{1}{\beta_1}\frac{\partial p_0}{\partial \beta_2} + \frac{\mu}{\beta_1^2}\left[\beta_1^2\frac{\partial^2 v_2}{\partial \beta_1^2} + \beta_1\frac{\partial v_2}{\partial \beta_1} - v_2 + \frac{\partial^2 v_2}{\partial \beta_2^2} + 2\frac{\partial v_1}{\partial \beta_2}\right],
\end{aligned} \tag{4}$$

where v_i - components of the velocity vector of the media flow in the polar coordinates β_i, t - time, f_i - specific mass force, p_0 - pressure, μ - dynamic viscosity coefficient (viscosity).

The left hand part of the equations (4) characterize the inertia forces, the first term of the right hand part characterizes the mass forces (gravity, electromagnetic interaction forces), the second term of the right hand part is a hydrostatic force, the third term is a viscosity forces (dissipative term).

The incompressibility condition takes the form [5, 8, 9]:

$$\frac{\partial v_1}{\partial \beta_1} + \frac{1}{\beta_1}\frac{\partial v_2}{\partial \beta_2} + \frac{v_1}{\beta_1} = 0. \tag{5}$$

The equations of the vibration of the center of the tip in the X_i coordinates with the given assumptions take the following form[1]:

$$\begin{aligned} X_1^C &= -kh_0\cos\omega t + kh_0, \\ X_2^C &= -kh_0\sin\omega t, \end{aligned} \tag{6}$$

where k - amplitude of the vibration coefficient ($0 \le k < 1$), h_0 - average gap ($h_0 = R - r$), ω - rotational velocity of the vibrations (coincides the angular velocity of the rotor).

Obviously, the components of the velocity of the tip center in the X_i coordinates take the form:

[1] The values in the Cartesian coordinates are denoted with the capital letters (X, V, F, etc.)

$$\begin{aligned} V_1^C &= kh_0\omega \sin \omega t, \\ V_2^C &= -kh_0\omega \cos \omega t. \end{aligned} \tag{7}$$

Then, at the point of time $t=0$ the cinematic boundary conditions take the form:

$$\begin{aligned} & v_1\big|_{\beta_1=r} = V_1^C\Big|_{t=0} cos\,\beta_2 + V_2^C\Big|_{t=0} sin\,\beta_2 = -kh_0\omega\, sin\,\beta_2, \\ & v_1\big|_{\beta_1=R} = 0, \\ & v_2\big|_{\beta_1=r} = \omega r - V_1^C\Big|_{t=0} sin\,\beta_2 + V_2^C\Big|_{t=0} cos\,\beta_2 = \\ & = \omega r - kh_0\omega\, cos\,\beta_2, \\ & v_2\big|_{\beta_1=R} = 0. \end{aligned} \tag{8}$$

It can be shown that on the surface of the tip $\beta_1 = r$ at a moment of time $t=0$ the incompressibility condition (5) with the cinematic boundary conditions (8) take the form:

$$\frac{\partial v_1}{\partial \beta_1} = 0\,. \tag{9}$$

Additionally, with the (7) and (8) on the surface of the tip $\beta_1 = r$ at a moment of time $t=0$ the form of the functions $\partial v_i/\partial\beta_2$ and $\partial v_i/\partial t$ can be simply determined:

$$\begin{aligned} \frac{\partial v_1}{\partial \beta_2} &= -kh_0\omega\, cos\,\beta_2, \\ \frac{\partial v_2}{\partial \beta_2} &= kh_0\omega\, sin\,\beta_2, \\ \frac{\partial v_1}{\partial t} &= kh_0\omega^2\, cos\,\beta_2, \\ \frac{\partial v_2}{\partial t} &= -kh_0\omega^2\, sin\,\beta_2. \end{aligned} \tag{10}$$

Same with (7), (8) on the surface of the tip $\beta_1 = r$ at a moment of time $t=0$ the partial derivatives of higher orders can be determined with respect to a tangential coordinate β_2 and time t.

Then a similarity theory and a dimensional analysis are applied to evaluate the significance of the terms in the Navier-Stokes equation (4) and the incompressibility condition (5) in order to simplify the mathematical model (table I). Due to the fact that to determine the lifting force it is necessary to solve the Navier-Stokes equation on the surface of the tip $\beta_1 = r$ at a moment of time $t=0$, the nondimensionalization of the equations of the mathematical model was implemented with the a priori knowledge of the range of the equation terms change on the surface of the tip (8)-(10).

I The components of the Navier-Stokes equations (4) and the continuity equation (5)

Dimensional form		Dimensionless form
Value	Range	
Radial coordinate	$r \le \beta_1 \le R$	$\tilde{\beta}_1 = \dfrac{\beta_1 - r}{h_0}$
Tangential coordinate	$0 \le \beta_2 \le 2\pi$	$\tilde{\beta}_2 = \dfrac{\beta_2}{2\pi}$
Time	$0 \le t \le \infty$	$\tilde{t} = \omega t$
Radial component of velocity	$-kh_0\omega \le v_1 \le$ $\le kh_0\omega$	$\tilde{v}_1 = \dfrac{v_1}{2kh_0\omega}$
Tangential component of velocity	$\omega r - kh_0\omega \le v$ $\le \omega r + kh_0\omega$	$\tilde{v}_2 = \dfrac{v_2}{v^*}$, $v^* = \sqrt{(\omega r)^2 + (2kh_0\omega)^2}$
Mass force	$-g \le f_i \le g$	$\tilde{f}_i = \dfrac{f_i}{g}$
Pressure	$p_0 \sim p_0^0$	$\tilde{p}_0 = \dfrac{p_0}{p_0^0}$

The Navier-Stokes equations (4), written in a dimensionless form (table I) take the form:

$$\begin{aligned} & \left\{ \frac{k(h_0\omega)^2}{v^{*2}} \left(\frac{\partial \tilde{v}_1}{\partial \tilde{t}} + \tilde{v}_1 \frac{\partial \tilde{v}_1}{\partial \tilde{\beta}_1} \right) + \frac{k(h_0\omega)}{(\tilde{\beta}_1+\gamma)v^*} \tilde{v}_2 \frac{\partial \tilde{v}_1}{\partial \tilde{\beta}_2} - \right. \\ & \left. - \frac{\tilde{v}_2^{\,2}}{(\tilde{\beta}_1+\gamma)} \right\} = \frac{\tilde{f}_1}{Fr} - Eu \frac{\partial \tilde{p}_0}{\partial \tilde{\beta}_1} + \frac{k(h_0\omega)}{Re(\tilde{\beta}_1+\gamma)^2 v^*} \times \\ & \times \left[(\tilde{\beta}_1+\gamma)^2 \frac{\partial^2 \tilde{v}_1}{\partial \tilde{\beta}_1^2} + (\tilde{\beta}_1+\gamma) \frac{\partial \tilde{v}_1}{\partial \tilde{\beta}_1} - \tilde{v}_1 + \frac{\partial^2 \tilde{v}_1}{\partial \tilde{\beta}_2^2} - 2 \frac{\partial \tilde{v}_2}{\partial \tilde{\beta}_2} \right], \\ & \frac{k(h_0\omega)}{v^*} \left\{ \frac{(h_0\omega)}{v^*} \frac{\partial \tilde{v}_2}{\partial \tilde{t}} + \tilde{v}_1 \frac{\partial \tilde{v}_2}{\partial \tilde{\beta}_1} + \frac{\tilde{v}_2}{(\tilde{\beta}_1+\gamma)} \frac{\partial \tilde{v}_2}{\partial \tilde{\beta}_2} + \frac{\tilde{v}_1\tilde{v}_2}{(\tilde{\beta}_1+\gamma)} \right\} = \\ & = \frac{\tilde{f}_2}{Fr} - \frac{Eu}{\pi(\tilde{\beta}_1+\gamma)} \frac{\partial \tilde{p}_0}{\partial \tilde{\beta}_2} + \frac{1}{Re(\tilde{\beta}_1+\gamma)^2} \left[(\tilde{\beta}_1+\gamma)^2 \frac{\partial^2 \tilde{v}_2}{\partial \tilde{\beta}_1^2} + \right. \\ & \left. + (\tilde{\beta}_1+\gamma) \frac{\partial \tilde{v}_2}{\partial \tilde{\beta}_1} - \tilde{v}_2 + \frac{k(h_0\omega)}{v^*} \left(\frac{\partial^2 \tilde{v}_2}{\partial \tilde{\beta}_2^2} + 2 \frac{\partial \tilde{v}_1}{\partial \tilde{\beta}_2} \right) \right], \end{aligned} \tag{11}$$

where k - dimensionless coefficient of the vibrations amplitude ($k \le 1$), $v^* = \sqrt{(\omega r)^2 + (2kh_0\omega)^2}$ - characteristic velocity, $\gamma = r/h_0$ - geometry parameter, $Fr = v^{*2}/(gh_0)$ - Froude number, $Eu = p_0^0/(v^{*2}\rho)$ - Euler number, p_0^0 - the characteristic pressure, $\mathrm{Re} = v^*\rho h_0/\mu$ - Reynolds number.

It is easy to demonstrate that the described method of nondimensionalization applied to the incompressibility condition (5) proves its terms orders are equal. In many cases the substitution of the specific values of the dimensionless criteria in the dimensionless equations of the mathematical model allows to eliminate the non-significant terms of the model and simplify the model [10, 11]. Below these examples will be shown.

The unknown hydrodynamic reaction of the moving media $\vec{F}$ on the surface of the tip $\beta_1 = r$ at a moment of time $t = 0$ is determined by means of integrating the pressure distribution p_0, which was found by solving the simplified Navier-Stokes equations (4) with accuracy up to the additive constant together with the incompressibility condition (9) and taking the cinematic boundary conditions (8) into account:

$$\begin{aligned} F_1 &= -r\int_0^{2\pi} p_0\big|_{\beta_1=r} \cos\beta_2 d\beta_2, \\ F_2 &= -r\int_0^{2\pi} p_0\big|_{\beta_1=r} \sin\beta_2 d\beta_2. \end{aligned} \tag{12}$$

IV. HYDRODYNAMIC FORCES IN THE FLUID-FILM BEARINGS

On the basis of the developed mathematical model with the initial and boundary conditions it is necessary to determine, under which circumstances the influence of the Magnus effect on the resulting hydrodynamic force is significant. For this, the problem of determination of the horizontal component F_1 of the hydrodynamic reaction $\vec{F}$, which acts on the rotating cylinder of a unit length, is considered. The reaction is determined in the Cartesian coordinates X_i (fig. 2) by the solution (12), nonzero value of this reaction evidences the presence of the Magnus effect (Fig. 2).

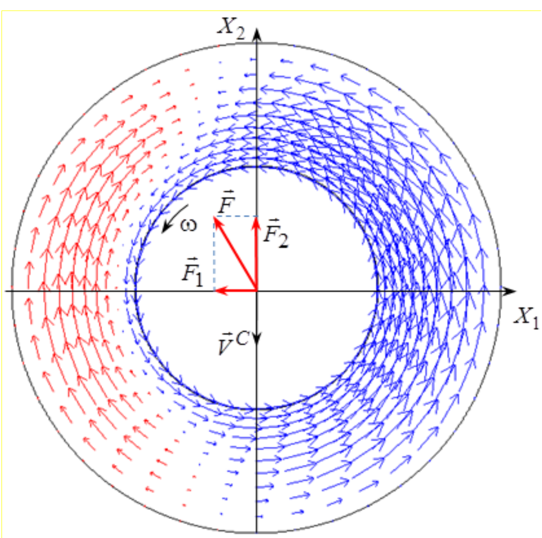

Fig. 2 Vector field of the media flow in a bearing with a rotating tip, the center of which vibrates

In the Fig. 2 a scheme of a joint action of the viscosity and inertia forces is shown in a resulting hydrodynamic force, and the flow velocity distribution in a bearing with a rotating and vibrating tip. The velocity distribution calculation is implemented using special software [12]. The flow in Fig. 2 is stretched along the radial coordinate to make it more demonstrative.

Let us consider a particular case, where the mathematical model parameters have the values of such orders which are close to the conditions of the high-speed rotor systems fluid-film bearings operation. For such systems $\gamma \sim 10^2$ or less, so in the characteristic velocity equation $v^* = \sqrt{(\omega r)^2 + (2kh_0\omega)^2}$ the peripheral speed is no less than 10^2 times more than the vibration velocity, since $r = \gamma h_0$ and $k < 1$. Then the characteristic velocity is approximately equal to the peripheral velocity on the surface of the tip $v^* \approx (\omega r)$. The order of the terms in (11) for the case in question is shown in the table II.

There is a rule, in accordance to which the terms of inertia and dissipation can be neglected as their order differs by three or more orders from the order of the older terms. In this case one exception is made and in the second Navier-Stokes equation the inertia terms, which differ by two orders from the older term, are omitted.

II The Navier-Stokes equations (11) terms orders of magnitude

Inertial				Mass	Pressure gradient	Dissipative				
$\frac{k}{\gamma}$	$\frac{k^2}{\gamma}$	$\frac{k}{\gamma}$	1	$\frac{\gamma}{Fr}$	γEu	$\frac{k}{Re}$	$\frac{k}{\gamma Re}$	$\frac{k}{\gamma^2 Re}$	$\frac{k}{\gamma^2 Re}$	$\frac{k}{\gamma^2 Re}$
$\frac{k}{\gamma}$	k	$\frac{k}{\gamma}$	$\frac{k}{\gamma}$	$\frac{\gamma}{Fr}$	Eu	$\frac{\gamma}{Re}$	$\frac{1}{Re}$	$\frac{1}{\gamma Re}$	$\frac{k}{\gamma^2 Re}$	$\frac{k}{\gamma^2 Re}$

The simplified Navier-Stokes equations after a number of simplest transformations can be written in a following form:

$$\begin{aligned} \frac{\partial p_0}{\partial \beta_1} &= \rho f_1 + \rho\frac{v_2^2}{\beta_1} + \frac{\mu}{\beta_1^2}\left[\beta_1^2\frac{\partial^2 v_1}{\partial \beta_1^2} + \beta_1\frac{\partial v_1}{\partial \beta_1}\right], \\ \frac{\partial p_0}{\partial \beta_2} &= \rho f_2 - \rho\beta_1 v_1\frac{\partial v_2}{\partial \beta_1} + \mu\left[\beta_1\frac{\partial^2 v_2}{\partial \beta_1^2} + \frac{\partial v_2}{\partial \beta_1}\right]. \end{aligned} \tag{13}$$

To compare and evaluate the significance of the terms between two equations (13) it is necessary to equate the left-hand parts, which can be achieved by differentiating the first equation (13) with respect to the β_2 coordinate, and the second equation – to the β_1 coordinate. After differentiation and nondimensionalization (13) take the form:

$$\frac{Eu}{\pi}\frac{\partial^2 \tilde{p}_0}{\partial\tilde{\beta}_1\partial\tilde{\beta}_2}=\frac{1}{Fr}\frac{\partial\tilde{f}_1}{\partial\tilde{\beta}_2}+\frac{2k\tilde{v}_2}{\gamma\left(\tilde{\beta}_1+\gamma\right)}\frac{\partial\tilde{v}_2}{\partial\tilde{\beta}_2}+$$
$$+\frac{k}{\gamma\, Re\left(\tilde{\beta}_1+\gamma\right)}\left[\left(\tilde{\beta}_1+\gamma\right)\frac{\partial^3\tilde{v}_1}{\partial\tilde{\beta}_1^2\partial\tilde{\beta}_2}+\frac{\partial^2\tilde{v}_1}{\partial\tilde{\beta}_1\partial\tilde{\beta}_2}\right],$$
$$\frac{Eu}{\pi}\frac{\partial^2 \tilde{p}_0}{\partial\tilde{\beta}_1\partial\tilde{\beta}_2}=-\left(\frac{k}{\gamma}\frac{\partial}{\partial\tilde{\beta}_1}\left(\tilde{\beta}_1\tilde{v}_1\right)\frac{\partial\tilde{v}_2}{\partial\tilde{\beta}_1}+\frac{k\left(\tilde{\beta}_1+\gamma\right)}{\gamma}\tilde{v}_1\frac{\partial^2\tilde{v}_2}{\partial\tilde{\beta}_1^2}\right)+ \quad (14)$$
$$+\frac{1}{Re}\left[2\frac{\partial^2\tilde{v}_2}{\partial\tilde{\beta}_1^2}+\left(\tilde{\beta}_1+\gamma\right)\frac{\partial^3\tilde{v}_2}{\partial\tilde{\beta}_1^3}\right]+\frac{1}{Fr}\frac{\partial\tilde{f}_2}{\partial\tilde{\beta}_1}.$$

It can be seen that the order of the inertia and the dissipative terms of the right-hand part of the first equation (14) is smaller by four or more than the orders of the according terms of the second equation (14). If the mass forces effect is insignificant: $Fr >> max\left(\gamma^2/k, \gamma\, Re/k\right)$, the Navier-Stokes equations can be additionally simplified, and the equations of the mathematical model with the incompressibility condition (5) take the following form:

$$\frac{\partial p_0}{\partial\beta_1}=0,$$
$$\frac{\partial p_0}{\partial\beta_2}=-\rho\beta_1 v_1\frac{\partial v_2}{\partial\beta_1}+\mu\frac{\partial}{\partial\beta_1}\left[\beta_1\frac{\partial v_2}{\partial\beta_1}\right], \quad (15)$$
$$\frac{\partial}{\partial\beta_1}\left(\beta_1 v_1\right)+\frac{\partial v_2}{\partial\beta_2}=0.$$

It has to be noted, that even with the simplifications the problem (15) is hard to solve analytically, so below only specific cases will be discussed.

First specific case: the inertia terms can be neglected $Re << \gamma/k$ (with $k \neq 0$), or $k=0$. Then the solution of (15) considering the cinematic border conditions (8) and the condition of β_2 periodicity of the functions, the pressure distribution on the surface of the tip $\beta_1 = r$ can be calculated with accuracy up to the constant p_0^0:

$$p_0 = p_0^0 + a\mu V^C \sin\beta_2, \quad (16)$$

where

$$a=\frac{-r\, ln\left(\frac{r}{R}\right)+ln\left(\frac{r^r}{R^R}\right)+h_0\left(1+ln(R)\right)}{\frac{1}{2}\left(r^2-R^2\right)ln\left(\frac{r}{R}\right)+h_0\left[ln\left(\frac{r^r}{R^R}\right)+ln\left(\frac{R^{h_0}r^r}{R^R}\right)+h_0\right]}\ \text{-}$$

geometry parameter, $V^C = kh_0\omega$ - amplitude of the tip vibration velocity(7).

Then the components of the hydrodynamic reaction will make:

$$F_1 = 0,$$
$$F_2 = -\pi a\mu r V^C. \quad (17)$$

The obtained solution (17) agrees well with the solution of the Reynolds equation for the two-dimensional flow case [13, 14].

Second specific case: the viscosity terms can be neglected $Re >> \gamma/k$ ($k \neq 0$). Then, the solution (15) after the second equation is differentiated with respect to β_1 and the minimal of two terms of the expanded differential of the multiplication result is omitted, the pressure distribution on the surface of the tip $\beta_1 = r$ can be calculated with accuracy up to the constant p_0^0:

$$p_0 = p_0^0 - \gamma\rho\omega r V^C \cos\beta_2. \quad (18)$$

Then the components of the hydrodynamic reaction will make:

$$F_1 = -\pi\gamma\rho\omega r^2 V^C,$$
$$F_2 = 0. \quad (19)$$

The obtained equation with precision up to $2/\gamma$ matches the initial calculation equation (2) obtained when solving the problem of the flow over the rotating cylinder by the oncoming flow. When the vibrations are not present $k=0$, the mathematical model equations are reduced to the form for the case of the fluid flow between the rotating coaxial cylinders.

In the Fig. 3-5 the results of the calculation of the resulting hydrodynamic force based on the models (17), (19) are presented for the bearing with a 50 mm diameter and an average gap 500 μm with kerosene, water, and liquid oxygen as lubricants. The rotation frequency as the main discrete variable was taken in interval $10^0 < n < 10^5$ rpm with fixed-increment. The coefficient of the vibrations amplitude $k = 0.3$ was taken as medium high [6]. For the convenience reasons, the results are presented in logarithmic coordinates, and it can be seen when the inertia and viscosity forces can be neglected and where they should be considered jointly.

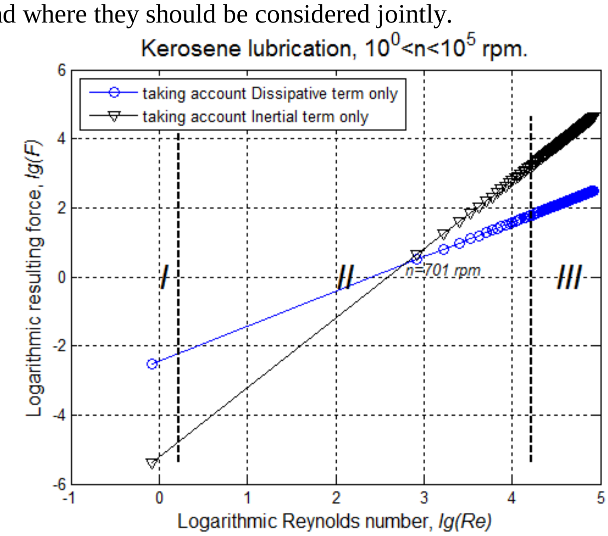

Fig. 3 The resulting hydrodynamic force (17), (19) in the fluid film bearing with kerosene lubrication and the zones of terms domination: I – viscosity term domination, II – both viscosity and inertia terms, III – inertia term domination

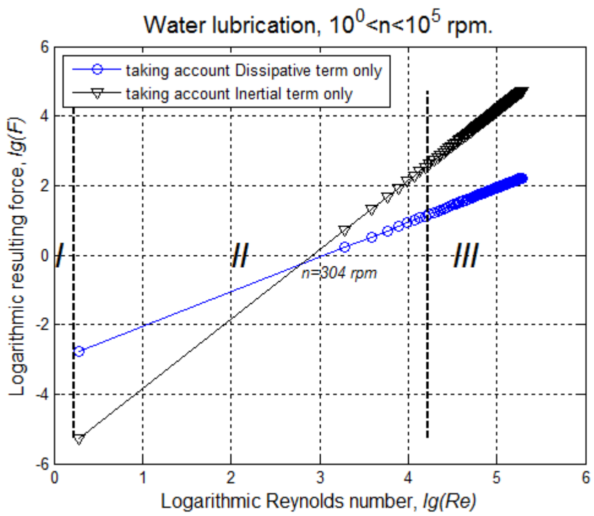

Fig. 4 The resulting hydrodynamic force (17), (19) in the fluid film bearing with water lubrication and the zones of terms domination: I – viscosity term domination, II – both viscosity and inertia terms, III – inertia term domination

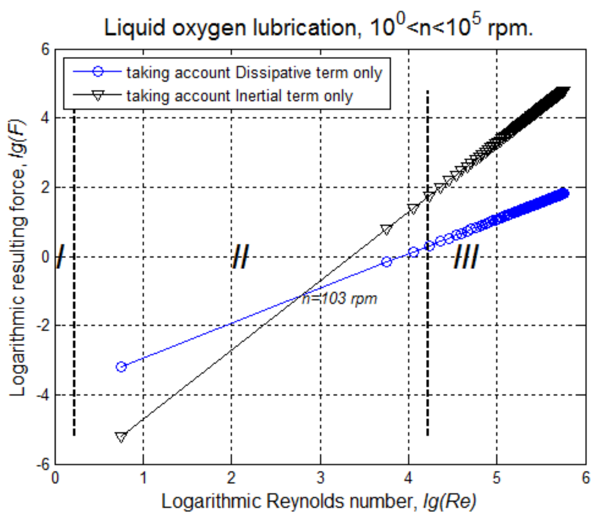

Fig. 5 The resulting hydrodynamic force (17), (19) in the fluid film bearing with liquid oxygen lubrication and the zones of terms domination: I – viscosity term domination, II – both viscosity and inertia terms, III – inertia term domination

V. Conclusion

In the fundamental papers on the hydrodynamic lubrication theory [2, 3] the problems of stationary or quasi-stationary media flow are considered. And presently the majority of the articles in the field of the hydrodynamic lubrication theory and the rotor system dynamics are based on the Reynolds equation solution [15-17]. The reason for the present research is the fact that the media flow in the fluid-film bearing with a vibrating tip is close to the oncoming flow over the cylinder, so the Magnus effect from the inertial forces, not considered in the Reynold equation, can be significant.

During the research the analysis of the dimensionless equations of the mathematical model of the non-stationary isothermal flow of the viscous incompressible media in the fluid-film bearing considering the mass forces, inertial forces and dissipation was made. The conditions were determined, when the Magnus effect from the inertia forces influences most significantly on the hydrodynamic reaction of the lubricant film, namely the requirement $Re >> \gamma/k$ has to be fulfilled. So, the Magnus effect can have a significant influence on the bearings and contactless seals dynamics of the high-speed rotor machines lubricated with the low-viscous media, e.g. liquefied gases [18, 19]. For such conditions, based on the approximate equations of the mathematical model, an analytical equation was obtained for the further calculation of the hydrodynamic force (19). The obtained formula matches the known Rayleigh formula up to a constant.

Acknowledgment

The authors would like to thank to organizing committee of the 19th International Conference on Circuits, Systems, Communicatioins and Computers (CSCC 2015) for the possibility for us to publish this article.

References

[1] G. Magnus “Űber die Abweichung der Geschosse”, *Poggendorfs Annalen*, 88, I, 1853.
[2] N.E. Joukovsky, *Collected works. In seven volumes*. Moscow and Saint Petersburg, Gostehizdat Publishing, 1950 (In Russian).
[3] L. Prandtl. “Magnus effect und Windkraftschiff”, *Die Naturwissenschaften*, vol 6, 93, 1925.
[4] O. Reynolds, “On the Theory of Lubrication”, *Ibid.*, 177 (1886), pp. 157–234.
[5] L.M. Milne-Thomson. *Theoretical hydrodynamics*. Fours edition, London: Macmilan and Co LTD, 1960.
[6] R. Polyakov, L. Savin, D. Shutin, “Reliability improvement of rotor supports by combining rolling-element bearings and fluid-film bearings”, *Applied Mechanics and Materials*, vol. 630, pp. 188 – 198, 2014.
[7] G. A. Korn, T. M. Korn, *Mathematical Handbook for Scientists and Engineers.* Dover Publications, 2000.
[8] N.E. Kochin, I.A. Kibel, and N.V. Roze, *Theoretical hydromechanics*. Translated from the fifth Russian ed. by D. Boyanovitch. Edited by J.R.M. Radok. New York: Interscience Publishers, 1965.
[9] B. V. Koutcheryaev, *Mechanics of Continua: Theoretical Basics of Processing Metalls under the pressure.* Moscow, MISiS Publishing, 2006 (In Russian).
[10] D.R. Oliver, “The influence of fluid inertia, viscosity and extra stress on the load bearing capacity of a squeeze film of oil”. Applied Scientific Research, vol 35, 1979, pp. 217-135.
[11] L.M. Caligiuri, “The variability of gravitational constant G in constitutive medium-based models of space: the vacuum-polarizable approach and the fluid dynamic model of gravity as pressure force”. International Journal of Mechanics, vol. 9, 2015, pp. 90-104.
[12] L. Savin, A. Kornaev and E. Kornaeva, “Variational principle in the hydrodynamic lubrication theory”. *International Journal of Mathematical Models and Methods in Applied Sciences*, vol. 9, 2015, pp. 114-119.
[13] H. Yukio, *Hydrodynamic Lubrication*. Tokyo: Springer, 2006, 250p.
[14] A. Z. Szeri and A. Al-Sharif, “Flow Between Finite, Steady Rotating Eccentric Cylinders”. *Theoretical and Computational Fluid Dynamics*, vol. 7, 1995, pp. 1–28.
[15] C. Shu, L. Wang, Y. T. Chew, N. Zhao, “Numerical study of eccentric Couette-Taylor flows and effect of eccentricity on flow patterns”. *Theoretical and Computational Fluid Dynamics*, vol. 18, 2004, pp. 43–59.
[16] A. Z. Szeri, A. Al-Sharif, “Flow Between Finite, Steady Rotating Eccentric Cylinders”. *Theoretical and Computational Fluid Dynamics*, vol. 7, 1995, pp. 1–28.

[17] J. H. Vohr, “An Experimental Study of Taylor Vortices and Turbulence in Flow Between Eccentric Rotating Cylinders” in *Proc. of ASLE – ASME Conf.*, Chicago, 1967, pp. 299–315.
[18] A.V. Ivanov, V.A. Korobchenko and A.V. Shostak, *Design of the seals of pumps wet end and turbo–pump assemblies in liquid rocket engine*, Voronezh State University, 2005 (in Russian).
[19] S. Danaila and M. L. Niculescu, "Proper Orthogonal Decomposition Analysis for Unsteady Rotor-Stator Interaction in a Low Pressure Centrifugal Compressor", *WSEAS TRANSACTIONS on FLUID MECHANICS*, vol.5, 2010, pp. 226-235.