Effective Hybrid Adaptive Temperature Control inside Plug-flow Chemical Reactor

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***Abstract*—** The paper deals with two methods of hybrid adaptive control of the nonlinear system represented by the plug-flow tubular chemical reactor. The mathematical model of this type of technological processes is described by the set of partial differential equations which were solved numerically by the finite difference method and Runge-Kutta's method. The adaptivity of the controller is satisfied by the recursive identification of the external linear model as a linear representation of the originally high nonlinear controlled system. The first method uses well known Pole-placement method and the second is based on the more sophisticated LQ approach. The advantage of these method is that both have tuning parameter which can affect control results. Although the controlled system has highly nonlinear behavior, used adaptive controller has good results.

***Keywords*—**Adaptive Control, Pole-placement Method, Recursive Identification, LQ Approach, Plug-flow Tubular Chemical Reactor.

I. Introduction

THE controlling of chemical reactors is always challenging because of the complexity of the system, hazardous and cost savings. The modeling of such processes usually ends with the complicated set of ordinary or even partial differential equations depending on the type of system [1].

The tubular plug-flow reactor belongs to the ring of systems with continuously distributed parameters, mathematical model of which uses partial differential equations (PDE) unfortunately in the nonlinear form [2].

The mathematical solution of the set of PDE uses Finite differences method which discretize the equation in the axial variable which means that the set of PDE is transformed into the set of ordinary differential equations ODE that can be then solved for example by Runge-Kutta's method [3] which is easily programmable or even build-in function in mathematical software. Other numerical methods are also discussed in [4].

Once we have done the simulation of the steady-state and dynamic behavior, we can continue with the choice of the optimal control strategy. There are several let's say "modern" control methods which were tested on this or similar types of systems – the robust control, the predictive control or the adaptive control.

The adaptive control [5] has various improvements and applications. The approach applied in this work uses results from the dynamic analysis for the choice of the External Linear Model (ELM) parameters of which are estimated recursively during the control which satisfies adaptivity of the controller [6].

Control synthesis uses a polynomial approach [7] which satisfies basic control requirements like stability of the control loop, the reference signal tracking and the disturbance attenuation. Another big advantage of this method is that it provides not only the structure of the controller but also relations for computing of the controller's parameters. This method could uses also other methods like the Pole-placement method [7] and LQ approach. These two methods are discussed in this work.

Other, let us say, modern control methods are robust control [8] and predictive control [9]. Advantage of these methods can be found in better efficiency and versability.

All experiments in the work are done by simulations using mathematical software Matlab, version 7.0.1. These methods were tested and can be used also for the controlling of real systems, similarly as in [10].

II. Model of Tubular Chemical Reactor

The system under the consideration is a tubular chemical reactor [11] as typical nonlinear equipment used in industry. The reaction inside is a simple exothermic reaction in the liquid phase and the reactant is cooled by the cooling liquid inside the jacket of the reactor. The scheme of the reactor could be found in Fig. 1.

The convection of the liquids in the pipes and the cooling jacket is expected to be plug-flow. That is why are these types of reactors called Plug-Flow Reactors (PFR). The mathematical model uses material and heat balances inside the reactor.

The PFR displayed in Fig. offers theoretically two types of cooling from the direction point of view – co-current and counter-current cooling. It was proofed for example in [12], that the counter-current cooling, where the direction of the cooling flow is opposite to the direction of the reactant has better cooling efficiency. This type of cooling is considered in this work mainly because of this efficiency.

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The mathematical description of such model is very complex and there must be introduced simplifications which reduce the complexity of the system: we expect, that all densities, heat capacities and heat transfer coefficients are expected to be constant. Also, we neglect heat losses and conduction along the metal wall of pipes. On the other hand, the heat transfer through the wall is consequential for the dynamic study. As the space variable is also important in the mathematical description, the mathematical model with all mentioned simplification is described by the set of five partial differential equations (PDE)

$$\begin{aligned}
&\frac{\partial c_A}{\partial t}+v_r\cdot\frac{\partial c_A}{\partial z}=-k_1\cdot c_A\\
&\frac{\partial c_B}{\partial t}+v_r\cdot\frac{\partial c_B}{\partial z}=k_1\cdot c_A-k_2\cdot c_B\\
&\frac{\partial T_r}{\partial t}+v_r\cdot\frac{\partial T_r}{\partial z}=\frac{h_r}{\rho_r\cdot c_{pr}}-\frac{4\cdot U_1}{d_1\cdot\rho_r\cdot c_{pr}}\cdot\left(T_r-T_w\right)\\
&\frac{\partial T_w}{\partial t}=\frac{4}{\left(d_2^2-d_1^2\right)\cdot\rho_w\cdot c_{pw}}\cdot\left[d_1\cdot U_1\cdot\left(T_r-T_w\right)+d_2\cdot U_2\cdot\left(T_c-T_w\right)\right]\\
&\frac{\partial T_c}{\partial t}-v_c\cdot\frac{\partial T_c}{\partial z}=\frac{4\cdot n_1\cdot d_2\cdot U_2}{\left(d_3^2-n_1\cdot d_2^2\right)\cdot\rho_c\cdot c_{pc}}\left(T_w-T_c\right)
\end{aligned}\tag{1}$$

where T denotes temperature, d are diameters of the pipes – d_1 is inner diameter of the pipe, d_2 is outer diameter of the pipe and d_3 denotes diameter of the jacket. Then, ρ are used for densities, c_p for specific heat capacities, U denotes heat transfer coefficients, n_1 is used for number of individual pipes and L is length of the reactor.

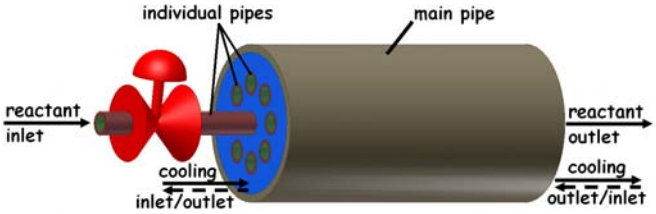

Fig. 1 Scheme of the plug-flow tubular chemical reactor

The variables v_r and v_c are fluid velocities computed from the volumetric flow rate q and constant f, e.g.

$$v_r=\frac{q_r}{f_r};\quad v_c=\frac{q_c}{f_c}\tag{2}$$

Where constants f_r and f_c are connected to the structure of reactor

$$f_r=n_1\cdot\frac{\pi\cdot d_1^2}{4};\quad f_c=\frac{\pi}{4}\left(d_3^2-n_1\cdot d_2^2\right)\tag{3}$$

The main nonlinearity of this system can be found in reaction velocities k_1 and k_2 which are nonlinear functions of the rectant's temperature T_r according to the *Arrhenius law*:

$$k_j=k_{0j}\cdot\exp\left(-\frac{E_j}{R\cdot T_r}\right),\text{for } j=1,2\tag{4}$$

with k_{0j} as a pre-exponential factors, E_j as a activation energies and R as a universal gas constant.

The last, unmentioned variable in (1) is a reaction heat h_r computed from

$$h_r=h_1\cdot k_1\cdot c_A+h_2\cdot k_2\cdot c_B\tag{5}$$

where h_j are reaction enthalpies.

Fixed parameters of the reactor [11] are shown in the following Table 1:

Table 1 Fixed parameters of the reactor

Parameter	*Notation and value*
Inner diameter of the tube	$d_1 = 0.02\ m$
Outer diameter of the tube	$d_2 = 0.024\ m$
Inner diameter of the reactor	$d_3 = 1\ m$
Number of pipes	$n_1 = 1200$
Length of the reactor	$L = 6\ m$
Volum. flow rate of the reactant	$q_r = 0.15\ m^3.s^{-1}$
Volum. flow rate of the cooling	$q_c = 0.275\ m^3.s^{-1}$
Density of the reactant	$\rho_r = 985\ kg.m^3$
Density of the metal wall	$\rho_w = 7800\ kg.m^3$
Density of the cooling	$\rho_c = 998\ kg.m^3$
Heat capacity of the reactant	$c_{pr} = 4.05\ kJ.kg^{-1}.K^{-1}$
Heat capacity of the metal wall	$c_{pw} = 0.71\ kJ.kg^{-1}.K^{-1}$
Heat capacity of the cooling	$c_{pc} = 4.18\ kJ.kg^{-1}.K^{-1}$
Heat transfer coefficient 1	$U_1 = 2.8\ kJ.m^{-2}.K^{-1}.s^{-1}$
Heat transfer coefficient 2	$U_2 = 2.56\ kJ.m^{-2}.K^{-1}.s^{-1}$
Pre-exponential factor 1	$k_{10} = 5.61\times10^{16}\ s^{-1}$
Pre-exponential factor 2	$k_{20} = 1.128\times10^{16}\ s^{-1}$
Activation energy 1/gas constant	$E_1/R = 13477\ K$
Activation energy 2/gas constant	$E_2/R = 15290\ K$
Reaction enthalpy 1	$h_1 = 5.8\times10^4\ kJ.kmol^{-1}$
Reaction enthalpy 2	$h_2 = 1.8\times10^4\ kJ.kmol^{-1}$
Input concentration of comp.A	$c_{A0}{}^s = 2.85\ kmol.m^{-3}$
Input temperature of the reactant	$T_{r0}{}^s = 323\ K$
Input temperature of the cooling	$T_{c0}{}^s = 293\ K$

Since the mathematical model of the system (1) is described by the set of nonlinear partial differential equations, we are talking about the *nonlinear distributed-parameters system*.

III. Steady-State and Dynamic Analyses

The static and dynamic analyses are usually the first steps after the modelling part. The goal of these studies is at first verify proposed mathematical model with measurements on the real system. Sometimes simplifications reduce the accuracy of the mathematical description and the use of the mathematical model is unacceptable. The second reason why we do these analyses is that we need to know the behavior of the system for finding of the optimal working point, limitations etc. The step responses in the dynamic analysis are also used for the choice of the External Linear Model in adaptive control described later in this work.

As there are theoretically more input and output variables, the change of the cooling volumetric flow rate, q_c, was chosen as a input variable for the reactant temperature, T_r, as an output variable. The volumetric flow rate as an input was chosen from the practical point of view – it is represented by the twist of the valve in this case. On the other hand, the

output temperature is better measured than the output concentration.

A. Steady-state Analysis

The static analysis explores the behavior of the system in steady-state, i.e. in the state when state variable does not change. Mathematically speaking, the derivatives with respect to time are equal to zero in the steady-state and the set of partial differential equations (1) is transformed to the set of ordinary differential equations with respect to space variable z.

The Finite differences method is employed here for solving of this problem. Derivatives with respect to space variable are replaced by the first back difference

$$\left.\frac{dx}{dz}\right|_{z=z_i} \approx \frac{x(i)-x(i-1)}{h_z}, \text{for } i=1,2,\ldots n \quad (6)$$

with x as a general variable and step size $h_z = L/N_z$. As the system has counter-current cooling, the temperature of the cooling T_c is described in the opposite coordinates and the last fifth equation in (1) uses the first forward difference

$$\left.\frac{dx}{dz}\right|_{z=z_j} \approx \frac{x(j+1)-x(j)}{h_z}, \text{for } j=n,\ n\text{-}1,\ldots 0 \quad (7)$$

The steady-state analysis is then solution of the cycle of discrete equations for different values of the input variable, in this case volumetric flow rate of the cooling q_c.

The static analysis was done for various values of the cooling volumetric flow rate q_c = <0.1; 0.35> $m^3.s^{-1}$ and values of the steady-state reactant temperature, T_r^s, through the length of the reactor (axial variable z = <0; 8> m) are shown in Fig. 2.

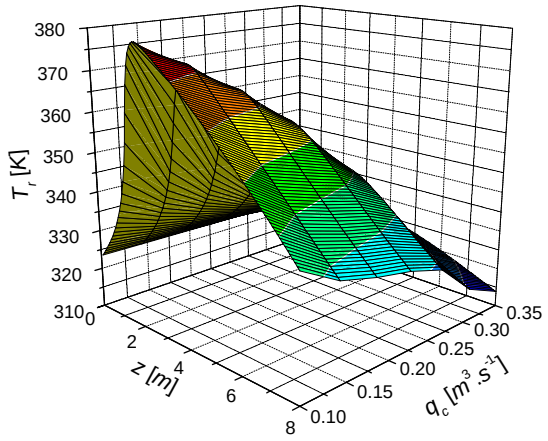

Fig. 2 Steady-state characteristic of the reactant's temperature, T_r, for different volumetric flow rate of the coolant q_c through the length of the reactor

Results of the steady-state analysis clearly show the nonlinearity of the system. The optimal working point is defined for the volumetric flow rate of the reactant q_r^s = 0.150 $m^3.s^{-1}$ and the volumetric flow rate of the coolant q_c^s = 0.275 $m^3.s^{-1}$ and this working point was used later in the dynamic analysis and also in the adaptive control.

B. Dynamic Analysis

The dynamic analysis observes the behavior of the output variable, reactant temperature at the end of the reactor $T_r(L)$, after the step change of the input variable, in this case step change of the volumetric flow rate of the coolant, Δq_c. The input, $u(t)$, and the output, $y(t)$ variables for both dynamic and control purposes are then

$$u(t)=\frac{q_c(t)-q_c^s}{q_c^s}\cdot 100[\%]; \quad y(t)=T_r(t,L)-T_r^s(L)[K] \quad (8)$$

where q_c^s is volumetric flow rate at the working point and $T_r^s(L)$ is the steady-state value of the output variable in the working point which is also initial value for the dynamic study. This means, that the graphs starts from zero.

From the mathematical point of view, the dynamic analysis is the numerical solution of the set of partial differential equations (1). The numerical solution of PDE is not simple and the combination of the Finite differences method described above which transforms the set of PDE to the set of ordinary differential equations (ODE) was used here. The set of ODE is then solved numerically with the use of Runge-Kutta's methods.

There were done several step changes and results are shown in the following Fig. 3.

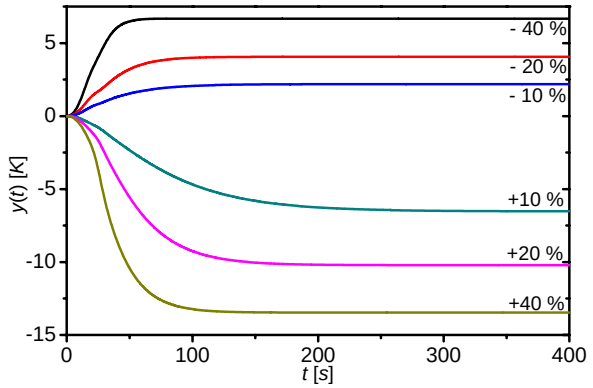

Fig. 3 Dynamic characteristic for various step changes of the input variable

It is clear, that the positive change of the input variable results in decreasing value of the output reactant temperature and conversely, the negative change of q_c produces positive change of the output temperature. All courses of the output variables could be described by second order transfer function which will be used later in the adaptive control.

IV. Adaptive Control

Once we have information about the system's behavior in the steady-state and dynamics, we can move on to the controller design. There are several control methods which can be used for such nonlinear process like predictive control, robust control etc. The adaptive approach was used in this work because authors have good experiences with the usage of this control method for similar types of technological

processes like heat exchangers, continuous stirred-tank reactors (CSTR), water tanks etc. An advantage of this method can be also find in the big theoretical background, modifications and applications.

The term "Adaptivity" comes from the nature, where animals and plants *adopt* their behavior depending on the living environment and conditions. Similarly, the adaptive controller could *adopt* (e.g. change) its parameters or structure according to the actual state of the system and control requirements.

There are, of course, various adaptive control strategies. As it is already mentioned, the adaptive approach here is based on on-line recursive identification of the External Linear Model (ELM) which represents original, nonlinear, process. Parameters of the controller depends on parameters of the ELM and changes in every identification step according to the identified parameters of the ELM.

A. External Linear Model

The choice of the ELM comes from the dynamic analysis presented above. The output responses have shown, that the change of the output temperature as the output $y(t)$ to the input variable $u(t)$ in Fig. 3 could be described by the continuous-time (CT) model

$$G(s)=\frac{b(s)}{a(s)}=\frac{b_1 s+b_0}{s^2+a_1 s+a_0} \tag{9}$$

On-line identification of the CT model is complicated. The discrete-time (DT) models are used more often. These models do not describe the system in the very accurate way – it depends on the choice od the sampling period T_v.

Compromise could be found in the use of delta-models as a special type of the DT models where values of the input and output variables are related to the sampling period and it was proofed, that parameters of the delta-model approaches to the parameters of the CT model.

The delta–model introduces a new complex variable γ [13]

$$\gamma=\frac{z-1}{T_v} \tag{10}$$

The ELM (9) could be then rewritten to the form of the differential equation

$$\begin{aligned} y_\delta(k)=&\, b_0^\delta u_\delta(k-1)+b_1^\delta u_\delta(k-2)-\dots \\ &\dots-a_0^\delta y_\delta(k-1)-a_1^\delta y_\delta(k-2) \end{aligned} \tag{11}$$

where $b_0{}^\delta$, $b_1{}^\delta$, $a_0{}^\delta$, $a_1{}^\delta$ are delta-parameters similar to those in (9) for small sampling period [14].

Delta values of input and output variables in Equation (11) can be computed as

$$\begin{gathered} y_\delta(k)=\frac{y(k)-2y(k-1)+y(k-2)}{T_v^2} \\ y_\delta(k-1)=\frac{y(k-1)-y(k-2)}{T_v} \quad u_\delta(k-1)=\frac{u(k-1)-u(k-2)}{T_v} \\ y_\delta(k-2)=y(k-2) \qquad u_\delta(k-2)=u(k-2) \end{gathered} \tag{12}$$

The regression vector $\boldsymbol{\varphi}_\delta$ and the vector of parameters $\boldsymbol{\theta}_\delta$ are

$$\begin{gathered} \boldsymbol{\varphi}_\delta(k-1)=\left[-y_\delta(k-1),-y_\delta(k-2),u_\delta(k-1),u_\delta(k-2)\right]^T \\ \boldsymbol{\theta}_\delta(k)=\left[a_1^\delta,a_0^\delta,b_1^\delta,b_0^\delta\right]^T \end{gathered} \tag{13}$$

and the differential equation (11) has then vector form

$$y_\delta(k)=\boldsymbol{\theta}_\delta^T(k)\cdot\boldsymbol{\varphi}_\delta(k-1)+e(k) \tag{14}$$

where $e(k)$ is a general random immeasurable component and the task of the identification is to estimate the vector of parameters $\boldsymbol{\theta}_\delta$ from known data vector $\boldsymbol{\varphi}_\delta$.

B. Recursive Identification

It was already mentioned, that adaptivity in this approach is based on the on-line parameter identification of the ELM. The recursive identification mathematically means the estimation of the vector of parameters $\boldsymbol{\theta}_\delta$ from the differential equation (14). The method used here is a simple Recursive Least-Squares (RLS) method [15] which can be easily programmed and also extended by the additional "forgetting" techniques. Generally, the RLS method used for estimation of the vector of parameters $\hat{\theta}_\delta^T(k)$ could be described by the set of equations:

$$\begin{gathered} \varepsilon(k)=y(k)-\boldsymbol{\varphi}^T(k)\cdot\hat{\boldsymbol{\theta}}(k-1) \\ \gamma(k)=\left[1+\boldsymbol{\varphi}^T(k)\cdot\mathbf{P}(k-1)\cdot\boldsymbol{\varphi}(k)\right]^{-1} \\ \boldsymbol{L}(k)=\gamma(k)\cdot\mathbf{P}(k-1)\cdot\boldsymbol{\varphi}(k) \\ \mathbf{P}(k)=\frac{1}{\lambda_1(k-1)}\left[\mathbf{P}(k-1)-\frac{\mathbf{P}(k-1)\cdot\boldsymbol{\varphi}(k)\cdot\boldsymbol{\varphi}^T(k)\cdot\mathbf{P}(k-1)}{\frac{\lambda_1(k-1)}{\lambda_2(k-1)}+\boldsymbol{\varphi}^T(k)\cdot\mathbf{P}(k-1)\cdot\boldsymbol{\varphi}(k)}\right] \\ \hat{\boldsymbol{\theta}}(k)=\hat{\boldsymbol{\theta}}(k-1)+\boldsymbol{L}(k)\varepsilon(k) \end{gathered} \tag{15}$$

where ε denotes a prediction error, $\mathbf{P}$ is a covariance matrix and λ_1 and λ_2 are forgetting factors. For example constant exponential forgetting [15] uses $\lambda_2 = 1$ and

$$\lambda_1(k)=1-K\cdot\gamma(k)\cdot\varepsilon^2(k) \tag{16}$$

where K is a very small value (e.g. $K = 0.001$).

C. Control Synthesis

It was already mentioned, that parameters of the ELM are used in the computation of the controller. The polynomial synthesis is employed here because it provides not only the structure of the controller but also relations for computing of the controller's parameters. Negligible advantage could be found also in the fulfillment of the basic control requirements and easily programmability.

The simplest one degree-of-freedom (1DOF) divides the control loop into two parts – the transfer function $G(s)$ representing controlled plant (i.e. the ELM of the system) and the transfer function of the controller $Q(s)$ – see Fig. 4.

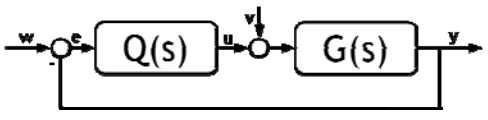

Fig. 4 One degree-of-freedom (1DOF) control configuration

The signal w in Fig. 4 represents reference signal (i.e. wanted value), u is control input, y controlled output, v denotes random error and e is control error – $e = w - y$.

The transfer function of the controlled plant $G(s)$ is known from the recursive identification and the transfer function of the controller is generally

$$\tilde{Q}(s) = \frac{q(s)}{s \cdot \tilde{p}(s)} \tag{17}$$

The parameters of the polynomials $\tilde{p}(s)$ and $q(s)$ are computed from the Diophantine equation

$$a(s) \cdot s \cdot \tilde{p}(s) + b(s) \cdot q(s) = d(s) \tag{18}$$

by the *Method of uncertain coefficients* which compares parameters of individual s-powers in (18). Polynomials $a(s)$ and $b(s)$ are known from the recursive identification and the polynomial $d(s)$ on the right side of the (18) is stable optional polynomial the choice of which affects mainly the quality of the control. Two methods of choosing of this polynomial are discussed and tested in the next chapters – *Pole-placement method* and *LQ approach*.

D. Pole-placement Method

The simplest way is to choose the polynomial $d(s)$ by the Pole-placement method which divides the polynomial generally to

$$d(s) = \prod_{i=1}^{\deg d(s)} (s + \alpha) \tag{19}$$

with the stability condition $\alpha > 0$.

Degrees of polynomials $\tilde{p}(s)$ and $q(s)$ from (17) and the polynomial $d(s)$ in (18) are for this second order transfer function with relative order one (9)

$$\begin{aligned} &\deg \tilde{p}(s) = \deg a(s) - 1 = 1 \\ &\deg q(s) = \deg a(s) = 2 \\ &\deg d(s) = \deg a(s) + \deg \tilde{p}(s) + 1 = 4 \end{aligned} \tag{20}$$

which means that the transfer function of the controller is

$$Q(s) = \frac{q_2 s^2 + q_1 s + q_0}{s \cdot (s + p_0)}, \tag{21}$$

and the polynomial $d(s)$ has four roots. Disadvantage of this method is that there is no rule how to choose there roots. We can have one quadruple root, two double roots, one ordinary and one triple root or four different roots. Our previous experiments have shown that it is good to connect the choice of the polynomial $d(s)$ with the controlled system, for example with the use of spectral factorization of the polynomial $a(s)$ in the numerator of the transfer function $G(s)$.

Let us introduce new polynomial $n(s)$ computed from the spectral factorization of the polynomial $a(s)$, i.e.

$$n^*(s) \cdot n(s) = a^*(s) \cdot a(s) \tag{22}$$

It is clear, that this polynomial has the same degree as the polynomial $a(s)$ and as it is a part of the polynomial $d(s)$, we can rewrite this polynomial to the form

$$d(s) = n(s) \cdot (s + \alpha)^2 \tag{23}$$

which means that we have reduced the uncertainty to one double root.

The controller designed with this method has one tuning parameter – α which could affect the quality of control.

E. LQ Approach

The second, let's say a bit sophisticated, method is for designing of the polynomial $d(s)$ is the use Linear-Quadratic (LQ) approach which is based on the minimization of the cost function

$$J_{LQ} = \int_0^\infty \left\{ \mu_{LQ} \cdot e^2(t) + \varphi_{LQ} \cdot \dot{u}^2(t) \right\} dt \tag{24}$$

in the complex domain. Parameters $\varphi_{LQ} > 0$ and $\mu_{LQ} \geq 0$ are weighting coefficients, $e(t)$ is the control error and $\dot{u}(t)$ denotes the difference of the input variable.

If we use again the spectral factorization of the polynomial $a(s)$, similarly as in previous case, the polynomial $d(s)$ is then divided into

$$d(s) = n(s) \cdot g(s) \tag{25}$$

where the polynomial is solution of the minimization of (24), mathematically solution of the spectral factorization

$$\begin{aligned} &\left(a(s) \cdot f(s)\right)^* \cdot \varphi_{LQ} \cdot a(s) \cdot f(s) + b^*(s) \cdot \mu_{LQ} \cdot b(s) = \ldots \\ &\qquad \ldots = g^*(s) \cdot g(s) \end{aligned} \tag{26}$$

Degrees of the controller's polynomials $\tilde{p}(s)$ and $q(s)$ and the polynomial $d(s)$ on the right side of Diophantine equation are for the second order ELM (9)

$$\begin{aligned} &\deg \tilde{p}(s) \geq \deg a(s) - 1 = 2 \\ &\deg q(s) = \deg a(s) + \deg f(s) - 1 = 2 \\ &\deg d(s) = 2 \deg a + 1 = 5 \end{aligned} \tag{27}$$

and the transfer function of the controller is

$$Q(s) = \frac{q_2 s^2 + q_1 s + q_0}{s \cdot \left(s^2 + a_1 s + p_0\right)} \tag{28}$$

The LQ adaptive controller has two tuning parameters, weighting factors φ_{LQ} and μ_{LQ} but our experiments have shown that is good to fix one parameter and change only the second one [16].

V. Simulation Results

Both techniques were tested by the simulation on the mathematical model (1). The control output is the change of the input volumetric flow rate of the coolant in % and the controlled output is the change of the output temperature, similarly as it is in (8):

$$u(t) = \frac{q_c(t) - q_c^s}{q_c^s} \cdot 100[\%]; \quad y(t) = T_r(t, L) - T_r^s(L)[K] \tag{29}$$

Due to better comparability of these methods are also simulation parameters the same. The sampling period was

T_v = 1.5 *s*, the simulation time was 8 000 *s* and there were done four different step changes to the positive and negative value during this time.

The first control simulation was done for the Pole-placement method and various values of the parameter α. = 0.07; 0.01 and 0.02.

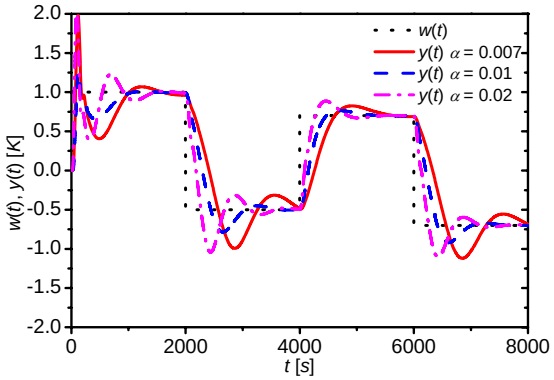

Fig. 5 The course of the reference signal, $w(t)$, and the output variable, $y(t)$, for various values of α, Pole-placement method

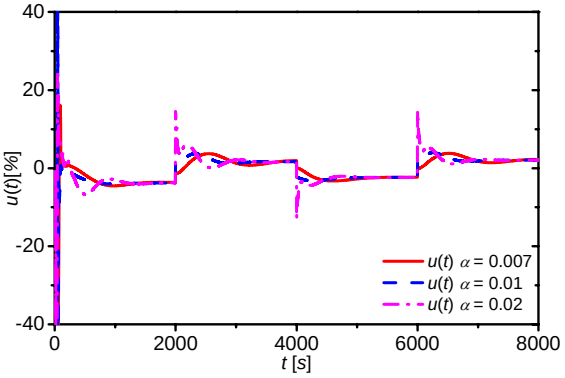

Fig. 6 The course of the input variable, $u(t)$, for various values of α, Pole-placement method

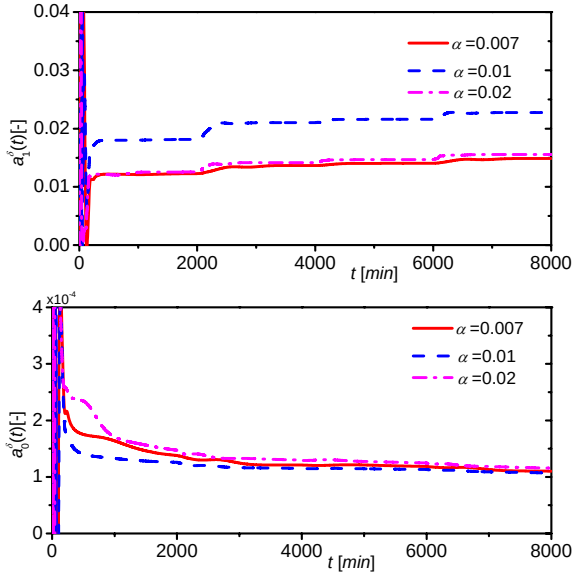

Fig. 7 The course of identified parameters $a_1^{\delta}(t)$ and $a_0^{\delta}(t)$ for various values of α, Pole-placement method

Obtained simulation results in Fig. 5 a 6 have shown that the increasing value of α results in quicker output response but overshoots of the output variable $y(t)$. The course of the control (input) variable $u(t)$ is smoother for lower values of α.

As it was already written, adaptive approach here is based on the recursive identification of the ELM (9). The recursive least squares method with exponential forgetting was used for online identification of parameters $a_1^{\delta}(t)$, $a_0^{\delta}(t)$, $b_1^{\delta}(t)$ and $b_0^{\delta}(t)$ and results are shown in Fig. 7 and 8.

You can see that there is only problem with identification at the very beginning of the control where controller does not have any information about the system and starting value of the vector of parameters is generally $\boldsymbol{\theta}_{\delta}(0) = [0.1, 0.1, 0.1, 0.1]^T$. The controller needs some time for "adaptation" but the estimation is much smoother after initial 50 *min* for all identified parameters.

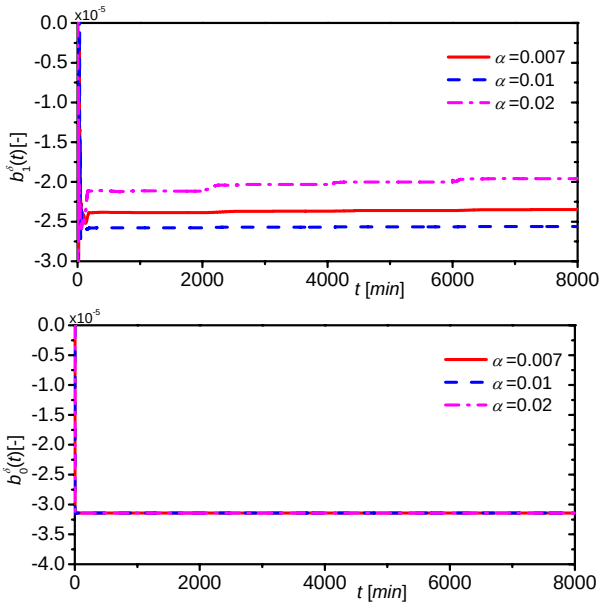

Fig. 8 The course of identified parameters $b_1^{\delta}(t)$ and $b_0^{\delta}(t)$ for various values of α, Pole-placement method

The second analysis was done for LQ approach and different values of weighting parameter ϕ_{LQ} = 0.005; 0.01 and 0.02 and the results are shown in Fig. 9 and 10.

Although there are similar values of the weighting parameter ϕ_{LQ} as α in previous case, the meaning of this parameter is different. In this case, increasing value of parameter ϕ_{LQ} results in slower, more oscillating output response but smoother course of the input variable which could be sometimes good from the practical point view.

The use of LQ approach produces generally more oscillating output responses but both control techniques could be used for controlling of such strongly nonlinear processes.

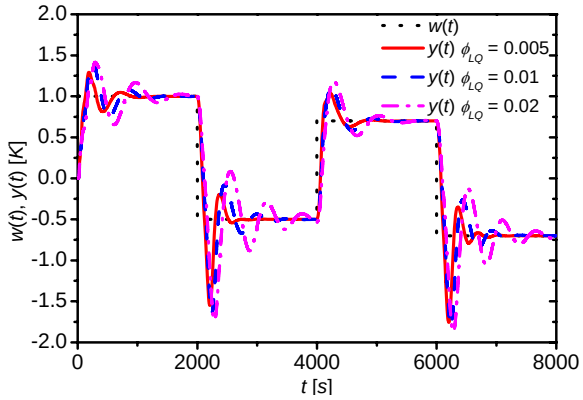

Fig. 9 The course of the reference signal, $w(t)$, and the output variable, $y(t)$, for various values of ϕ_{LQ}, LQ approach

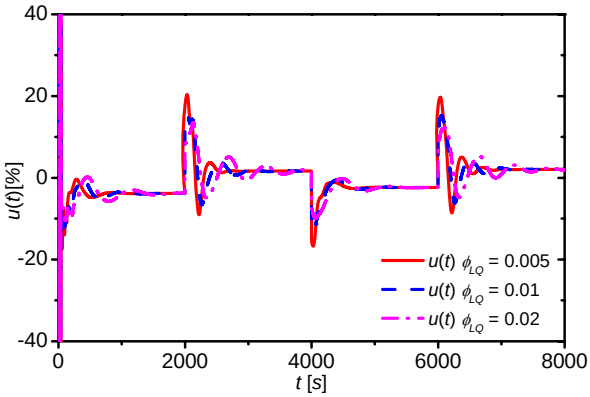

Fig. 10 The course of the input variable, $u(t)$, for various values of ϕ_{LQ}, LQ approach

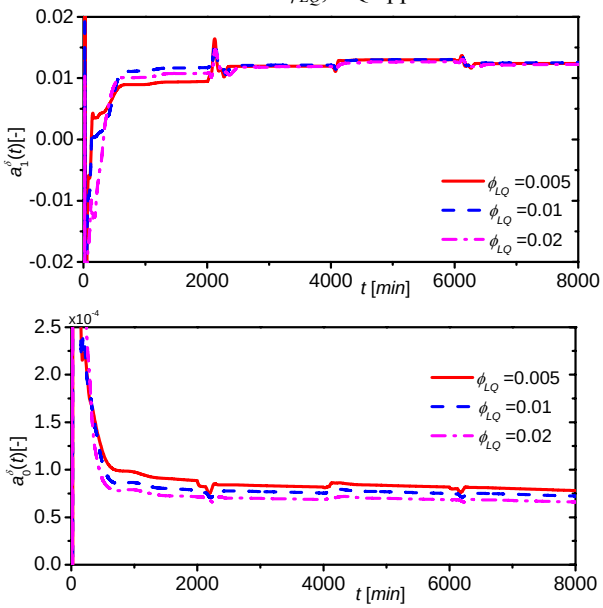

Fig. 11 The course of identified parameters $a_1^{\delta}(t)$ and $a_0^{\delta}(t)$ for various values of ϕ_{LQ}, LQ approach

Fig. 11 and Fig. 12 representing results of online identification show very similar results to those mentioned above for previous control approach. We can say here that initial "adaptation" is much quicker than for previous case.

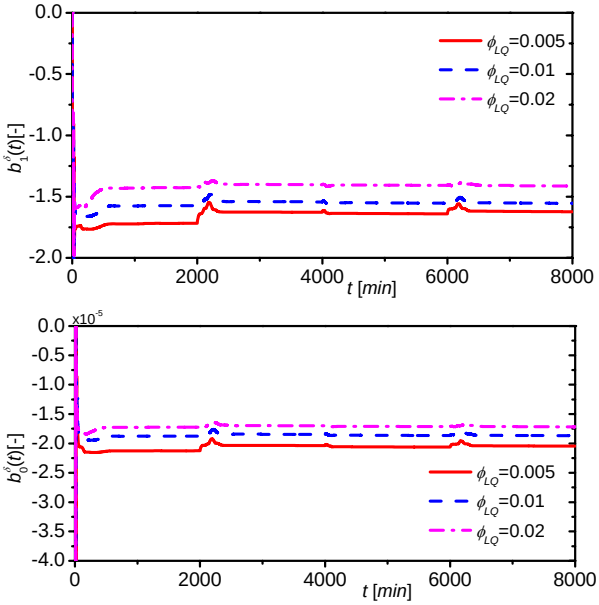

Fig. 12 The course of identified parameters $b_1^{\delta}(t)$ and $b_0^{\delta}(t)$ for various values of ϕ_{LQ}, LQ approach

All results of identification presented in Fig. 7, Fig. 8, Fig. 11 and Fig. 12 show usability of this recursive least-squares method. Moreover, we can see, that identified parameters do not change dramatically after some, already mentioned, initial "adaptation" time. Here rise the question: Is online recursive identification important here, where parameters does not change? Of course, we can use controllers with fixed parameters but what if the control conditions changes? What if there occur unexpected disturbances. In these cases is the use of online identification very good option. Those controllers react to these changes quickly and provide more optimal results.

Obtained results were discussed only from the visual view until now but it is good to have any mathematical description of results for comparison. We can use for example simple quadratic criterions S_u and S_y which quantitatively describes the courses of the output variable, $y(t)$, or its difference from the reference signal, $w(t)$,respectively and the changes of the input variable, $u(t)$:

$$S_u = \sum_{i=2}^{N} \left(u(i) - u(i-1)\right)^2 [-]; \quad \text{for } N = \frac{T_f}{T_v} \qquad (30)$$
$$S_y = \sum_{i=2}^{N} \left(w(i) - y(i)\right)^2 [K^2]$$

Obtained values of these quadratic criterions are shown in Tables 2, 3 and following figures 13-17.

Table 2 Computed values of quadratic criterions S_u and S_y in control with Pole-placement method

	S_u [-]	S_y [K^2]
$\alpha = 0.007$	24 759	1 105
$\alpha = 0.01$	79 963	649
$\alpha = 0.02$	16 233	519

Table 3 Computed values of quadratic criterions S_u and S_y in control with LQ method

	S_u [-]	S_y [K^2]
ϕ_{LQ} = 0.005	61 806	7.69
ϕ_{LQ} = 0.01	68 823	19.99
ϕ_{LQ} = 0.02	60 865	18.23

Presented values of criterions can help us with the choice of the optimal value of tuning parameter α or ϕ_{LQ}. For example, Table 2 and graphs in Figures 13 and 15 which represent control approach with Pole-placement method indicates, that from the input point of view are the best results for control with α = 0.02. This is represented by the lowest value of the criterion S_u that sums square of changes of the input variable. Also, the value of the second criterion S_y denoting the sum of control errors ($w - y$) is also the lowest for the last control strategy. We can say, that this setting has the best results and could be tested on the real process.

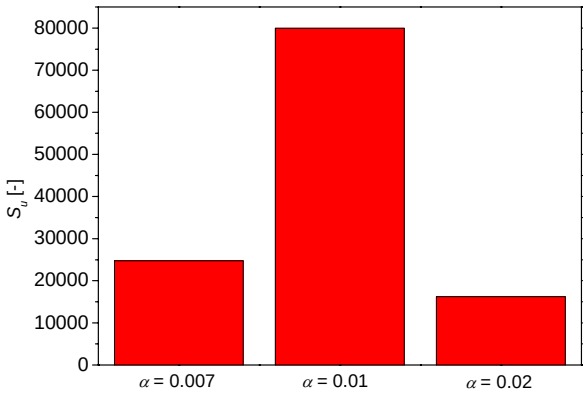

Fig. 13 Values of the quadratic criterion S_u for various values of α, Pole-placement method

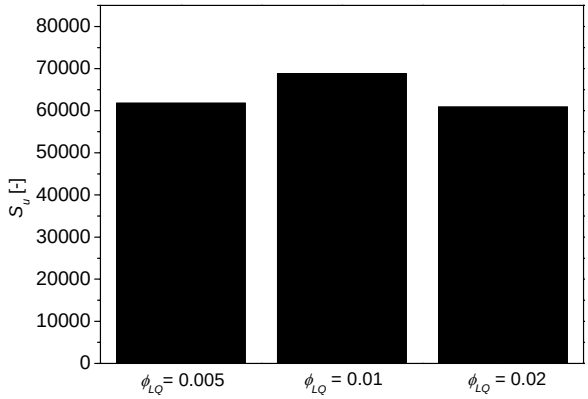

Fig. 14 Values of the quadratic criterion S_u for various values of ϕ_{LQ}, LQ approach

On the other hand, results for the next, LQ, strategy does not indicate so clear results. Values of input quadratic criterion S_u are very similar and the criterion S_y is the lowest for the first value of ϕ_{LQ} = 0.005. As the value of the criterion S_u is also one of the lowest for this setting, we can say that control with ϕ_{LQ} = 0.005 has the best results.

If we compare also these criterions for both control strategies, Pole-placement method produces better values of the criterion S_u then LQ method but this method has, on the other hand, better results of the criterion S_y.

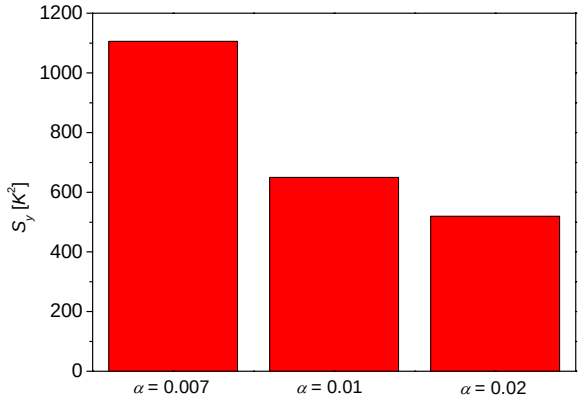

Fig. 15 Values of the quadratic criterion S_y for various values of α, Pole-placement method

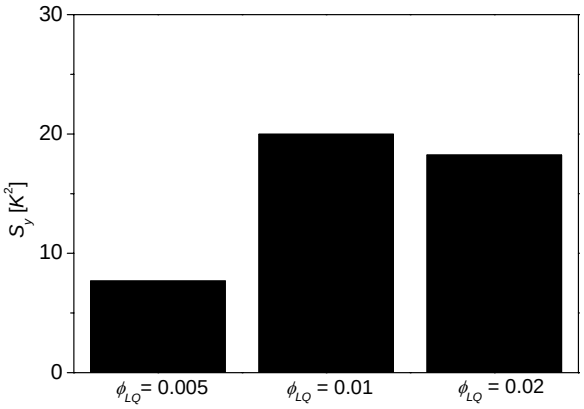

Fig. 16 Values of the quadratic criterion S_y for various values of ϕ_{LQ}, LQ approach

VI. Conclusion

The paper presents two modifications of the adaptive control applied on the control of the reactant temperature inside the tubular chemical reactor as a typical nonlinear system with distributed parameters. The nonlinear system is described by the external linear model in the general form parameters of which are estimated recursively during the control which fulfills the "adaptivity" of the system. The difference between these two modifications is in the choice of the stable polynomial in the Diophantine equation. The first method uses simple Pole-placement method with spectral factorization and the second modification is based on the LQ approach again together with the spectral factorization of the polynomial in the denominator of the ELM. Both methods have tuning parameters which can affect the quality of control, mainly the speed of the control and the overshoots. Obtained

simulation results have shown the usability of the adaptive control for controlling of such complex nonlinear systems. Obtained results were also discussed and quantified by the quadratic criterions that summarize changes of the input variable and the control error. The choice of the best controller setting always depends on the main purpose of the control, e.g. if the minimal control error or the changes of the input variable. These changes of the input variable are important mainly from the practical point of view. The future work could be focused on the verification of the obtained results on the real chemical reactor.

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