Allocation of students into academic programs using mathematical programming

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Abstract**—**A mathematical programming model is built to optimize the allocation of students into academic programs of a department. The mathematical programming model takes into account the limits of space capacity, financial allocation, the number of instructors and affirmative action quotas as goal constraints that are required to be fulfilled. Each constraint has a priority level and a weight attached. This goal programming model is then applied to the School of Mathematical Sciences, Universiti Kebangsaan Malaysia. The results of the preemptive goal programming model are then compared to that of the weighted non-preemptive goal programming model and current allocation using the weighted mean absolute percentage error. The successful application demonstrates the ability of the mathematical programming model to comply with the student intake requirement and goal constraints of the academic programs.

Keywords**—**Affirmative, constraints, priority, weighted mean.

I. Introduction

GOAL programming has been used extensively in many areas such as in management for Malaysian crops [1]-[4], portfolio of Malaysian stock market [5], management of tourism activities [6], library acquisition and funding allocation [7]-[8], food product distribution [9] and bakery production [10]. Currently preemptive goal programming models are being applied in minimization of energy consumption on multiprocessor platforms [11], fuzzy investment decisions [12], flood flow model [13], and joint decision making of inventory [14].

Earlier modeling approaches in institutions of higher learning tend to be directed towards aggregate planning of human, financial, and physical resources in the higher levels of academic administration planning [15]-[19]. Current research in education are development of self-regulation of online education [20], theory and practice in laboratory [21], learning in global education framework [22], multichoice goal programming for course planning [23], introducing programming coursework to non-computer savvy students [24], e-learning evaluation [25], menu planning model for schools [26], e-activities in pre-university education [27], incorporating students' views to promote computer learning [28], and knowledge management in higher education [29]. However the main academic thrusts of the institutions are left out. Some departmental level modeling techniques dealing with faculty-course assignment required the development of complex utility functions to express faculty preferences for certain courses [30]-[33]. The required time consuming modeling efforts, and the complexity and the time necessary to develop utility functions of faculty preferences could however limit their application when used on a practical reoccurring basis on departmental level in an organization of higher education [34].

In order to emphasize the thrust of academic institutions, academic administrators have to determine the number of students to be enrolled based on the expertise of academic staff, student capacity of each program, admission policies and create a racial balance in each program based on the affirmative action policy to be dealt with every semester. Administrators' decisions should indicate the thrust of the academic faculty, limited infrastructure, and the affirmative action requirement for government funded public universities.

In this paper, a preemptive goal programming model is developed which will optimize the allocation of students into academic programs taking into account the expertise of academic staff, student capacity of each program, admission policies and financial allocations. It is further refined to create a racial balance in each program based on the affirmative action policy and provide a fair distribution of student-to-faculty ratio. Weights will be used to apportion the students into academic programs in the department that will reflect the research thrust of the department. The weighted deviations are then given priority levels in the objective function to emphasize the ranking of goals. Error analysis is established for the preemptive model based on the deviations from the aspired levels and then compared against those of the non-preemptive model and current allocation using a weighted mean absolute percentage error (MAPE) analysis [35].

II. Model Development

Listed below are the input parameters, constraints and the objective function of the model in allocating students of a department, the School of Mathematical Sciences, to its three academic programs of mathematics, statistics and actuarial science, for its three years undergraduate study.

A. Input Parameters

c_j = capacity of first year students in program j

r_j = student to faculty ratio required for program j

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q_j = minimum ratio of native students over the total students entering program j
z_j = number of drop-out native students from program j
t_j = total capacity of students in program j proportionate to the number of classes
e_j = number of students enrolling into year two
h_j = number of students enrolling into year three

B. Variables

x_j = number of native students admitted into program j
y_j = number of non-native students admitted into program j
a_j = total number of first year students in program j
d_j = total number of students enrolled in program j
f_i = total number of students in department i
l_j = number of faculty required for program j
X = total number of first year native students admitted into the department
Y = total number of non-native students admitted into the department
A = total number of first year students admitted into the department.

C. Constraints

The constraints involved in the School of Mathematical Sciences with three programs are as follows.

$$\sum_{j=1}^{M} x_j = X_i, \quad \sum_{j=1}^{M} y_j = Y_i, \quad \sum_{j=1}^{M} a_j = A_i, \tag{1}$$

where $M = 3$, $X_i = 134$, $Y_i = 88$ and $A_i = 222$.
For $j = 1,\ldots, 3$, we have

$$a_j - x_j - y_j = 0, \tag{2}$$

$$a_j + d_{1j}^{-} - d_{1j}^{+} = c_j, \tag{3}$$

where c_1, c_2 dan c_3 are 90, 80 and 70 respectively.

$$d_j + d_{2j}^{-} - d_{2j}^{+} = t_j, \tag{4}$$

where t_1, t_2 dan t_3 are 260, 220 and 190.

$$x_j - z_j - q_j a_j + d_{3j}^{-} - d_{3j}^{+} = 0, \tag{5}$$

where q_1, q_2 dan q_3 are 0.80, 0.49 and 0.48.

$$d_j - a_j = e_j + h_j \tag{6}$$

where $e_j + h_j$ are 172, 134 and 121.

$$f = \sum_{j=1}^{3} d_j = 649. \tag{7}$$

$$r_j . l_j - d_j + d_{4j}^{-} - d_{4j}^{+} = 0, \tag{8}$$

where r_1, r_2 and r_3 are 14, 12 and 26 respectively.

$$l - \sum_{j=1}^{3} l_j = 0. \tag{9}$$

The students cost in a program does not vary much to other programs within the same department since they share the same equipment and facilities. Hence the budget cost constraint is redundant and thus omitted.

D. Objective Function

The criterion of optimization aims at maximizing the allocation of students accepted into the department by maximizing first year admission and enrollees in the department while minimizing affirmative action quota and number of faculty members.

$$\max \sum_{j=1}^{M} a_j = \sum_{j=1}^{M} x_j + y_j \tag{10}$$

$$\max \ f = \sum_{j=1}^{M} d_j \tag{11}$$

$$\min \sum_{j=1}^{M} x_j - q_j a_j \tag{12}$$

$$\min \sum_{j=1}^{M} l_j. \tag{13}$$

Note that the objective function in this case, has to be rewritten as a single function of deviations and prioritized accordingly.

$$\begin{aligned} Z = {} & P_1 \sum_{j=1}^{M} k_{1j}(d_{1j}^{-} + d_{1j}^{+}) + P_2 \sum_{j=1}^{M} k_{2j}(d_{2j}^{-} + d_{2j}^{+}) \\ & + P_3 \sum_{j=1}^{M} k_{3j}(d_{3j}^{-} + d_{3j}^{+}) + P_4 \sum_{j=1}^{M} k_{4j}(d_{4j}^{-} + d_{4j}^{+}). \end{aligned} \tag{14}$$

Note that the weights k_{ij} have values 1, 2 or 3. In our case the first priority goal P_1 was admission requirement, the second priority goal P_2 was the capacity requirements of each program, the third priority goal P_3 was the affirmative action ratio whilst the fourth goal P_4 was student-staff ratio. If priority is not taken into account, that is, the varied goals all have the same priorities, the preemptive goal programming model will be reduced to the non-preemptive goal programming model. The values of the weights of deviations are based on their rank, the higher the rank the higher would the value of the weight be.

$$\begin{aligned} & k_{11} = 2, k_{12} = 3, k_{13} = 1, k_{21} = 3, k_{22} = 2, k_{23} = 1, \\ & k_{31} = 1, k_{32} = 3, k_{33} = 2, k_{41} = 2, k_{42} = 1, k_{43} = 3. \end{aligned} \tag{15}$$

III. Analysis of Results

The output obtained for the preemptive goal programming model with regard to the enrollment into three academic programs of Mathematics, Statistics and Actuarial Science in the School of Mathematical Sciences is shown in Table 1.

Note that from the third column of the Table 1, the model suggest a mix of 39 native and 41 non-native students to be admitted into the statistics program in order to fulfil the admission capacity of 80 students. This is due to the highest priority and weightage given towards this requirement. Compare these values to that of the last column where the mix of 26 native and 28 non-native students will only fill up 54 of the 70 places available.

This situation arises because filling up the capacity of the actuarial program is given least priority, compared to that of mathematics or statistics. The fifth row displays the number of staff required in each program corresponding to the total

Table 1: Results of the Preemptive Model

Departmental programs	Math	Stats	Actuary
Number of first year native students	69	39	26
Number of first year non-native students	19	41	28
Number of first year students to be admitted	88	80	54
Number of academic staff in each program	19	18	7
Number of students in each program	260	214	175

Table 2: Results of the Non-Preemptive Model

Departmental programs	Math	Stats	Actuary
Number of first year native students	65	39	30
Number of first year non-native students	16	41	31
Number of first year students to be admitted	81	80	61
Number of academic staff in each program	18	18	7
Number of students in each program	253	214	182

Table 3: Error Calculations for the Goal Programming Models

Priority	Weights w	Aspiration X	Preemptive Model	Error	Non-Preemptive	Error	Current	Error
	2	90	88	2	81	9	82	8
1	3	80	80	0	80	0	78	2
	1	70	54	16	61	9	62	8
	3	260	260	0	253	7	254	6
2	2	220	214	6	214	6	212	8
	1	190	175	15	182	8	183	15
	1	0.80	0.784	0.016	0.802	0.002	0.805	0.005
3	3	0.49	0.488	0.002	0.489	0.001	0.487	0.003
	2	0.48	0.482	0.002	0.492	0.012	0.484	0.004
	2	14	13.68	0.32	14.06	0.06	13.37	0.63
4	1	12	11.89	0.11	11.89	0.11	11.78	0.22
	3	26	25.00	1	26.00	0	26.14	0.14

number of students in that particular program. The values of the deviational variables with their priorities and respective weights are listed below. Note that the objective value is 84.16.

First priority is student admission, with declining weights in Statistics, Mathematics and Actuary. The corresponding deviational variables are $d_{12}^- = 0$, $d_{11}^- = 2$, $d_{13}^- = 16$. Note that the preemptive model ensures that admission into the Mathematics program is optimum. The Actuary program on the other hand has an increased underachievement of 16 students since admission into it is accorded the least weight.

Second priority is student capacity with declining weights in the Mathematics, Statistics and Actuary programs. The corresponding deviational variables are $d_{21}^- = 0$, $d_{22}^- = 6$, $d_{23}^- = 15$. Note that the preemptive model optimizes student capacity of the Mathematics program with the highest weightage as indicated by the value of d_{21}^-. The high underachievement d_{23}^- of the student capacity of the Actuary program is because it has the least weight.

Third priority is affirmative action with declining weights in the Statistics, Actuary and Mathematics program. The corresponding deviational variables are $d_{32}^- = 0.2$, $d_{32}^+ = 0$, $d_{33}^- = 0$, $d_{33}^+ = 0.08$, $d_{31}^- = 1.4$, $d_{31}^+ = 0$. The model indicated a small deviation from the affirmative action mix. It maintained that no more than two students will exceed the required mix of the affirmative action requirement.

Fourth priority is the student-faculty ratio with declining weights in the Actuary, Mathematics and Statistics programs, with $d_{43}^- = 0$, $d_{43}^+ = 7$, $d_{41}^- = 0$, $d_{41}^+ = 6$, $d_{42}^- = 0$, $d_{42}^+ = 2$. Note that the student-faculty ratio has been a little bit overachieved in both the programs of Actuary and Mathematics as indicated by d_{43}^+ and d_{41}^+. These mean that the student-faculty ratio is less than 26:1 and 14:1 respectively. In other words, an additional number of seven students in the Actuary program and an additional six students from the Mathematics program will be required to meet the ratio of those two programs.

Table 2, on the other hand displays the results obtained for the non-preemptive model when all goals have the same priorities.

Error analysis is established based on the error deviations from the aspired levels, of our models and those of current values as indicated in Table 3 by using the weighted Mean Absolute Percentage Error (MAPE) analysis [25].

For our preemptive model, the weighted MAPE

$$\frac{\sum w_i \frac{|e_i|}{X_i} \times 100}{\sum w_i} = \frac{0.618205386}{24} \times 100 = 2.576\ \%,$$

while for the non-preemptive model, the weighted MAPE

$$\frac{\sum w_i \frac{|e_i|}{X_i} \times 100}{\sum w_i} = \frac{0.582351921}{24} \times 100 = 2.426\ \%.$$

Comparing this value to the MAPE of the current practice which is 2.965 %, we note that the MAPE value for both of our models give better results which are closer to the aspiration values compared to that of the current allocation practice. If we are to categorize the MAPE values according to priorities, then the weighted MAPE values of the preemptive model can be found as in Table 4.

Table 4: MAPE values based on priorities

Priorities	Preemptive Model %	Current %
Student admission	4.5503	6.1177
Student capacity	2.2249	2.9800
Affirmative action	0.6905	0.6881
Student-staff ratio	2.8378	2.0748

Note that the first priority weighted MAPE value of the preemptive model is significantly lower by a third of the current value, while the second priority weighted MAPE value is reduced by a fourth of the current value. Consequently, there is a small percentage rise of the MAPE values of the second and third priority.

IV. Conclusion

We have successfully obtained the results of the preemptive and non-preemptive goal programming models and error analyses in the form of weighted Mean Absolute Percentage Error (MAPE) were conducted. It is shown that the MAPE values of our models are less than the MAPE values of the current practice. These show that our models adhered closely to the requirements of aspiration levels. Based on the results obtained, we were able to undertake an in-depth discussion on the deviation variables based on the given priorities and relate the findings to the weights and priority levels assigned to these variables. From the discussion of these deviational variables, we can verify that the results of the models conform to our requirement of fulfilling the highest priority goals in accordance to the corresponding weights of the three programs in the School of Mathematical Sciences. Thus we believe the mathematical programming models can be used for policy-making in the decision process of future allocation of students to academic programs of the department. The model can be further extended to allocation of students into academic faculties if additional constraints regarding budget allocation are taken into consideration.

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