Analysis and proposal motion of robotic manipulator system for monitoring luggage compartment including a dynamic analysis

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Abstract— Article deals with motion control manipulator, whose motion states are derived and subsequently calculated by based on the equations of motion. By using these calculations are determined effects of the moving parts of the manipulator, including the effect on individual moving arms, and dynamic load prospective drives. The resulting potential and kinetic energy of system then used for derived and calculated equation of motion. Base on that is created conception and 3D structural model. This autonomous robotic system model find use for monitoring and identification suspicious objects in luggage compartments of transport vehicles.

Keywords— Robotics, manipulator, motional equations, kinematics, dynamics

I. Introduction

Autonomous serviced robotic systems can be used in particular scanning and identification of suspicious objects in cabin luggage compartments of vehicles. Such spaces, as in vehicles type train, plane located above the passenger's heads and require viewing, where possible, identify and remove dangerous objects.

Based on the structure of this type of service robot, is primarily implemented manipulator, allowing the movement of the camera and a distance sensor as an element of external sensory system. Also for supporting any sensors of chemical compounds or explosives for the autonomous movement of the effector on the premises. Inner sensory system is commonly formed beside the angular position sensor of any electrical or hydraulic actuators mobile system, especially a system of sensors for controlling the movement of possible states of the manipulator.

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Give cause for the emergence of this project was the absence of potential manipulator arm, allowing the dimensions and design integration directly into the luggage space.

The main general aim of the works, one of which this text the first part, is methodology of motional equations derivation of manipulator and achieved results, necessary for motion system proposal and the control.

Next works will be dedicated to complete manipulator movement, its motional equations and created 3D structural model based on the obtained parameters.

II. Manipulator body motional equations

A. Statement of a problem

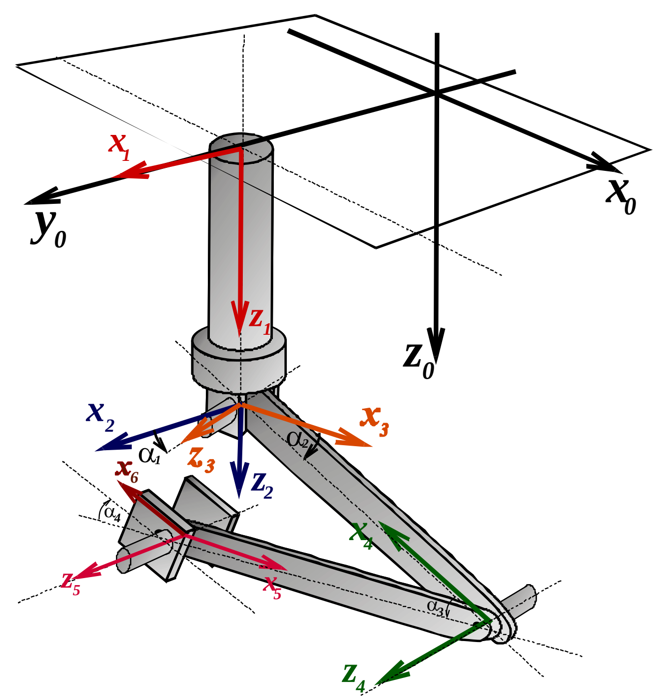

Fig. 1 manipulator parameters in the framework the coordinate system

On Fig.1 manipulator body presented, situated in plane as a system with six freedom degree.

Task is to derive the motional equations depending on the position of the arms towards the origin of the coordinate system, forming angles **α_1**, **α_2** and **α_3**. The end effector with the momentum of inertia relative to the axis **Z_5** is neglected in this case.

B. *Determination transformation and kinematic matrix*

For the compilation equations of motion is first necessary to find kinematic transformation matrix **$^{i-1}T_i$** for **i=1,2,3,4** equation of physical element **dm1** first arm and for **i=1,2,3,4,5** of physical element **dm2** by second arm, where basic matrix is:

$$^{i-1}\mathbf{T}_i = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad (1)$$

Following matrix, and calculations demonstrates the procedure for first and second arm element **dm1, dm2 .** After substitution:

$$^{0}\mathbf{T}_1 = \begin{bmatrix} \cos 90^0 & -\cos 0^0 \sin 90^0 & \sin 0^0 \sin 90^0 & y_T \cos 90^0 \\ \sin 90^0 & \cos 0^0 \cos 90^0 & -\sin 0^0 \cos 90^0 & y_T \sin 90^0 \\ 0 & \sin 0^0 & \cos 0^0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & y_T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{1}\mathbf{T}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{bmatrix}; {}^{2}\mathbf{T}_3 = \begin{bmatrix} -\sin\alpha_1 & 0 & \cos\alpha_1 & 0 \\ -\cos\alpha_1 & 0 & -\sin\alpha_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$$^{3}\mathbf{T}_4 = \begin{bmatrix} -\cos\alpha_2 & -\sin\alpha_2 & 0 & \ell_1 \cos\alpha_2 \\ \sin\alpha_2 & -\cos\alpha_2 & 0 & -\ell_1 \sin\alpha_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{4}\mathbf{T}_5 = \begin{bmatrix} -\cos\alpha_3 & \sin\alpha_3 & 0 & \ell_2 \cos\alpha_3 \\ -\sin\alpha_3 & -\cos\alpha_3 & 0 & \ell_2 \sin\alpha_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Coordinates physical element **dm1** of first arm in his coordinate system **O_{x4y4z4}** is:

$$[x \quad 0 \quad 0 \quad 1]^T$$

Following the determination of absolute coordinates physical element **dm1** of first arm, in global coordinate of systém **O_{x0y0z0}**

The resulting transformation matrix **$^3T_4$** is:

$$^{0}\mathbf{T}_4 = {}^{0}\mathbf{T}_1(y_T) \cdot {}^{1}\mathbf{T}_2(h) \cdot {}^{2}\mathbf{T}_3(\alpha_1) \cdot {}^{3}\mathbf{T}_4(\alpha_2) =>$$

$$^{0}\mathbf{T}_4 = \begin{bmatrix} -\cos\alpha_1 \cos\alpha_2 & -\cos\alpha_1 \sin\alpha_2 & \sin\alpha_1 & \ell_1 \cos\alpha_1 \cos\alpha_2 \\ \sin\alpha_1 \cos\alpha_2 & \sin_1 \sin\alpha_2 & \cos\alpha_1 & y_T - \ell_1 \sin\alpha_1 \cos\alpha_2 \\ -\sin\alpha_2 & \cos\alpha_2 & 0 & h + \ell_1 \sin\alpha_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

Global position of element **dm1**:

$$\begin{bmatrix} X_{dm1} \\ Y_{dm1} \\ Z_{dm1} \\ 1 \end{bmatrix} = {}^{0}\mathbf{T}_4 \cdot \begin{bmatrix} x \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \ell_1 \cos\alpha_1 \cos\alpha_2 \\ y_T - \ell_1 \sin\alpha_1 \cos\alpha_2 \\ h + \ell_1 \sin\alpha_2 \\ 1 \end{bmatrix} \quad (4)$$

Vector absolute speed of element **dm1** is then:

$$\vec{v}_{dm1} = \begin{bmatrix} (\ell_1 - x) \cdot (\omega_1 \cdot \sin\alpha_1 \cos\alpha_2 - \omega_2 \cdot \cos\alpha_1 \sin\alpha_2) \\ v_y + (\ell_1 - x) \cdot (\omega_2 \sin\alpha_1 \sin\alpha_2 - \omega_1 \cos\alpha_1 \cos\alpha_2) \\ (\ell_1 - x) \cdot \omega_2 \cos\alpha_2 \\ 0 \end{bmatrix} \quad (5)$$

Quadrat of size speed **dm1**:

$$|\vec{v}_{dm1}|^2 = (\ell_1 - x)^2 \cdot \{\omega_2^2 + \omega_1^2 \cdot \cos^2 \alpha_2\} + \\ + 2(\ell_1 - x) \cdot v_y \cdot (\omega_2 \cdot \sin\alpha_1 \sin\alpha_2 - \omega_1 \cos\alpha_1 \cos\alpha_2) + v_y^2 \quad (6)$$

Then apply for the first drive:

$$|\vec{v}_{p1}|^2 = \ell_1^2 \cdot \{\omega_2^2 + \omega_1^2 \cdot \cos^2 \alpha_2\} + \\ + 2\ell_1 \cdot v_y \cdot (\omega_2 \cdot \sin\alpha_1 \sin\alpha_2 - \omega_1 \cos\alpha_1 \cos\alpha_2) + v_y^2 \quad (7)$$

Coordinates physical element **dm2** of second arm in his coordinate system **O_{x5y5z5}** is:

$$[x \quad 0 \quad 0 \quad 1]^T$$

Following the determination of absolute coordinates physical element **dm2** of second arm, in global coordinate of systém **O_{x0y0z0}**

The resulting transformation matrix **$^4T_5$** is:

$$^{0}\mathbf{T}_5 = {}^{0}\mathbf{T}_1(y_T) \cdot {}^{1}\mathbf{T}_2(h) \cdot {}^{2}\mathbf{T}_3(\alpha_1) \cdot {}^{3}\mathbf{T}_4(\alpha_2) \cdot {}^{4}\mathbf{T}_5(\alpha_3) =>$$

$$^{0}\mathbf{T}_5 = \begin{bmatrix} \cos\alpha_1 \cos(\alpha_2 - \alpha_3) & \cos\alpha_1 \sin(\alpha_2 - \alpha_3) & \sin\alpha_1 & -\cos\alpha_1 [\ell_2 \cdot \cos(\alpha_2 - \alpha_3) - \ell_1 \cdot \cos\alpha_2] \\ -\sin\alpha_1 \cos(\alpha_2 - \alpha_3) & -\sin\alpha_1 \sin(\alpha_2 - \alpha_3) & \cos\alpha_1 & y_T + \sin\alpha_1 [\ell_2 \cdot \cos(\alpha_2 - \alpha_3) - \ell_1 \cdot \cos\alpha_2] \\ \sin(\alpha_2 - \alpha_3) & -\cos(\alpha_2 - \alpha_3) & 0 & h - \ell_2 \cdot \sin(\alpha_2 - \alpha_3) + \ell_1 \cdot \sin\alpha_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

Global position of element **dm2**:

$$\begin{bmatrix} -\ell_2 \cos\alpha_1 \cos(\alpha_2 - \alpha_3) + \ell_1 \cdot \cos\alpha_1 \cos\alpha_2 \\ y_T + \ell_2 \sin\alpha_1 \cos(\alpha_2 - \alpha_3) - \ell_1 \sin\alpha_1 \cos\alpha_2 \\ h - \ell_2 \cdot \sin(\alpha_2 - \alpha_3) + \ell_1 \sin\alpha_2 \\ 1 \end{bmatrix} \quad (9)$$

Vector absolute speed of element **dm2** is then:

$$\vec{v}_{dm2}=\begin{bmatrix} \omega_1\cdot\left[(\ell_2-x)\sin\alpha_1\cos(\alpha_2-\alpha_3)-\ell_1\cdot\sin\alpha_1\cos\alpha_2\right]+ \\ \omega_2\cdot\left[(\ell_2-x)\cos\alpha_1\sin(\alpha_2-\alpha_3)-\ell_1\cdot\cos\alpha_1\sin\alpha_2\right]- \\ -\omega_3\cdot\left[(\ell_2-x)\cos\alpha_1\sin(\alpha_2-\alpha_3)\right] \\ v_y+\omega_1\cdot\left[(\ell_2-x)\cos(\alpha_1)\cdot\cos(\alpha_2-\alpha_3)-\cos(\alpha_1)\cdot\cos(\alpha_2)\cdot\ell_1\right]- \\ \omega_2\cdot\left[(\ell_2-x)\sin\alpha_1\sin(\alpha_2-\alpha_3)-\ell_1\cdot\sin\alpha_1\sin\alpha_2\right]- \\ +\omega_3\cdot\left[(\ell_2-x)\cos\alpha_1\sin(\alpha_2-\alpha_3)\right] \\ -\omega_2\cdot\left[(\ell_2-x)\cos(\alpha_2-\alpha_3)-\ell_1\cdot\cos(\alpha_2)\right]+\omega_3\cdot\left[(\ell_2-x)\cos(\alpha_2-\alpha_3)\right] \\ 1 \end{bmatrix} \quad (10)$$

Quadrat of size speed **dm2**:

$$\begin{aligned} |\vec{v}_{dm2}|^2 &= (\ell_2-x)^2\cdot(\omega_2-\omega_3)^2+ \\ &+(\ell_2-x)^2\cdot\omega_1^2\cdot\cos^2(\alpha_2-\alpha_3)- \\ &-2\ell_1(\ell_2-x)\cdot\omega_1^2\cdot\cos\alpha_2\cdot\cos(\alpha_2-\alpha_3)- \\ &-2\ell_1(\ell_2-x)\cdot(\omega_2^2-\omega_2\cdot\omega_3)\cos\alpha_3+ \\ &+2(\ell_2-x)\cdot v_y\cdot\omega_1\cdot\cos\alpha_1\cos(\alpha_2-\alpha_3)- \\ &-2(\ell_2-x)\cdot v_y\cdot\omega_2\cdot\sin\alpha_1\sin(\alpha_2-\alpha_3)+ \\ &+2(\ell_2-x)\cdot v_y\cdot\omega_3\cdot\sin\alpha_1\sin(\alpha_2-\alpha_3)+ \\ &+\ell_1\omega_1\cdot\cos\alpha_2\cdot(\ell_1\omega_1\cdot\cos\alpha_2-2\cdot v_y\cdot\cos\alpha_1)+ \\ &+\ell_1\omega_2(\ell_1\omega_2-2\cdot v_y\cdot\sin\alpha_1\sin\alpha_2)+v_y^2 \end{aligned} \quad (11)$$

The kinetic energy of swing:

$$W_{k1}=\frac{1}{2}\cdot m\cdot\dot{y}^2+\frac{1}{2}\cdot J_m\cdot\dot{\alpha}_1^2 \quad (12)$$

The kinetic energy of element **dm1** by first arm is:

$$dW_{km1}=\frac{1}{2}\cdot\frac{m_1}{\ell_1}\cdot|\vec{v}_{dm1}|^2\cdot dx \quad (13)$$

The kinetic energy of first arm W_{km1}:

$$W_{km1}=\frac{1}{2}\cdot m_1\cdot\left|\begin{matrix}\frac{1}{3}\cdot(\dot{\alpha}_2^2+\dot{\alpha}_1^2\cdot\cos^2\alpha_2)\cdot\ell_1^2+\ell_1\cdot\dot{y}\cdot \\ .(\dot{\alpha}_2\cdot\sin\alpha_1\sin\alpha_2-\dot{\alpha}_1\cos\alpha_1\cos\alpha_2)+\dot{y}^2\end{matrix}\right| \quad (14)$$

Resulting kinetic energy of first drive W_{kp1}:

$$W_{kp1}=\frac{1}{2}\cdot m_{p1}\cdot\begin{bmatrix}\ell_1^2\cdot(\dot{\alpha}_2^2+\dot{\alpha}_1^2\cdot\cos^2\alpha_2)+2\ell_1\cdot\dot{y}\cdot \\ (\dot{\alpha}_2\cdot\sin\alpha_1\sin\alpha_2-\dot{\alpha}_1\cos\alpha_1\cos\alpha_2)+\dot{y}^2\end{bmatrix} \quad (15)$$

The kinetic energy of element **dm2** by second arm is:

$$dW_{km2}=\frac{1}{2}\cdot\frac{m_2}{\ell_2}\cdot|\vec{v}_{dm2}|^2\cdot dx \quad (16)$$

The kinetic energy of second arm W_{km2}:

$$W_{km2}=\frac{1}{2}\cdot m_2\cdot\left\{\begin{matrix}\frac{1}{3}\left[(\dot{\alpha}_2-\dot{\alpha}_3)^2+\dot{\alpha}_1^2\cdot\cos^2(\alpha_2-\alpha_3)\right]\cdot\ell_2^2+ \\ +\ell_2\cdot\dot{y}\cdot\begin{bmatrix}\dot{\alpha}_1\cdot\cos\alpha_1\cos(\alpha_2-\alpha_3)- \\ -\dot{\alpha}_2\cdot\sin\alpha_1\sin(\alpha_2-\alpha_3)+\dot{\alpha}_3\cdot\sin\alpha_1\sin(\alpha_2-\alpha_3)\end{bmatrix}- \\ -2\cdot\dot{y}\cdot\ell_1\cdot\left[\dot{\alpha}_1\cdot\cos\alpha_1\cos\alpha_2-\dot{\alpha}_2\cdot\sin\alpha_1\sin\alpha_2\right]+ \\ +\ell_1^2\cdot(\dot{\alpha}_2^2+\dot{\alpha}_1^2\cdot\cos^2\alpha_2)+\dot{y}^2\end{matrix}\right\} \quad (17)$$

Resulting kinetic energy of second drive W_{kp2}:

$$W_{kp2}=\frac{1}{2}\cdot(m_{p2}+m_e)\cdot\left\{\begin{matrix}\ell_2^2\cdot\left[(\dot{\alpha}_2-\dot{\alpha}_3)^2+\omega_1^2\cdot\cos^2(\alpha_2-\alpha_3)\right]- \\ 2\ell_1\ell_2\cdot\begin{bmatrix}(\dot{\alpha}_2^2-\dot{\alpha}_2\cdot\dot{\alpha}_3)\cos\alpha_3+ \\ \omega_1^2\cdot\cos\alpha_2\cdot\cos(\alpha_2-\alpha_3)\end{bmatrix}+ \\ +2\ell_2\dot{y}\cdot\begin{bmatrix}\dot{\alpha}_1\cdot\cos\alpha_1\cos(\alpha_2-\alpha_3)- \\ \dot{\alpha}_2\cdot\sin\alpha_1\sin(\alpha_2-\alpha_3) \\ +\dot{\alpha}_3\cdot\sin\alpha_1\sin(\alpha_2-\alpha_3)\end{bmatrix}+ \\ +\ell_1^2\cdot(\dot{\alpha}_2^2+\dot{\alpha}_1^2\cdot\cos^2\alpha_2)- \\ 2\cdot\ell_1\dot{y}\cdot\begin{bmatrix}\dot{\alpha}_1\cdot\cos\alpha_1\cos\alpha_2 \\ -\dot{\alpha}_2\cdot\sin\alpha_1\sin\alpha_2\end{bmatrix}+\dot{y}^2\end{matrix}\right\} \quad (18)$$

Potential energy of element **dm1** by first arm is:

$$W_{pm1}=-m_1\cdot g\cdot\left(h+\frac{1}{2}\ell_1\cdot\sin\alpha_2\right) \quad (19)$$

Potential energy of firs drive W_{pp1}:

$$W_{pp1}=-m_{p1}\cdot g\cdot\left[h+\ell_1\sin\alpha_2\right] \quad (20)$$

Potential energy of element **dm2** by second arm is:

$$W_{pm2}=-m_2\cdot g\cdot\left\lfloor h+\ell_1\cdot\sin\alpha_2-\frac{1}{2}\ell_2\sin(\alpha_2-\alpha_3)\right\rfloor \quad (21)$$

Potential energy of second drive and efector W_{pp2}:

$$W_{pp2}=-(m_{p2}+m_e)\cdot g\cdot\left[h+\ell_1\sin\alpha_2-\ell_2\cdot\sin(\alpha_2-\alpha_3)\right] \quad (22)$$

After determining the kinetic energy of the system, we can compile the equations of motion (25). But first, a description of the dynamics of the system using lagranger function (23).

Lagrangian:

$$\begin{aligned} L=&\overbrace{(m+m_1+m_{p1}+m_2+m_{p2}+m_e)}^{M}\cdot\dot{y}+\left(m_{p1}+\frac{1}{2}m_1\right)\cdot \\ &\ell_1\left[\dot{\alpha}_2^2\cdot(\sin\alpha_1\cdot\sin\alpha_2)-\dot{\alpha}_1^2\cdot(\cos\alpha_1\cdot\cos\alpha_2)\right]+m_2. \\ &.\left\{\begin{matrix}\frac{1}{2}\ell_2\begin{bmatrix}\dot{\alpha}_1\cdot\cos\alpha_1\cos(\alpha_2-\alpha_3)+\dot{\alpha}_2\cdot\sin\alpha_1\sin(\alpha_2-\alpha_3)- \\ -\dot{\alpha}_3\cdot\sin\alpha_1\sin(\alpha_2-\alpha_3)\end{bmatrix}- \\ -\ell_1(\dot{\alpha}_1\cos\alpha_1\cos\alpha_2-\dot{\alpha}_2\sin\alpha_1\sin\alpha_2)\end{matrix}\right\}+ \\ &+(m_{p2}+m_e)\cdot\left\{\begin{matrix}\ell_2\begin{bmatrix}\dot{\alpha}_1\cdot\cos\alpha_1\cos(\alpha_3-\alpha_2)+\dot{\alpha}_2\cdot\sin\alpha_1\sin(\alpha_3-\alpha_2)- \\ -\dot{\alpha}_3\cdot\sin\alpha_1\sin(\alpha_3-\alpha_2)\end{bmatrix}- \\ -\ell_1(\dot{\alpha}_1\cos\alpha_1\cos\alpha_2-\dot{\alpha}_2\sin\alpha_1\sin\alpha_2)\end{matrix}\right\} \end{aligned} \quad (23)$$

Where $\frac{\partial L}{\partial y}=0$

C. Motion equations by manipulator object

The most important step for the dynamic analysis of the manipulator is a derivation equations motion of system, describing the dynamic behavior of the manipulator. The equations of motion derived from a lagranger function (23).

Motion equation for **y**:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{y}}\right)-\frac{\partial L}{\partial y}=F_{ext}$$

Substituting then:

$$
\begin{aligned}
F_{ext} &= \overbrace{\left(m+m_1+m_{p1}+m_2+m_{p2}+m_e\right)}^{M}\cdot\ddot{y}+\\
&+\ddot{\alpha}_1\cdot\left\{\begin{array}{l}\left[\left(m_{p2}+m_e\right)+\dfrac{m_2}{2}\right]\ell_2\cos\alpha_1\cos\left(\alpha_3-\alpha_2\right)-\\ -\left[\left(m_{p2}+m_e+m_2+m_{p1}+\dfrac{m_1}{2}\right)\cdot\ell_1\cos\alpha_1\cos\alpha_2\right]\end{array}\right\}+\\
&+\ddot{\alpha}_2\cdot\left\{\begin{array}{l}\left[\left(m_{p2}+m_e\right)+\dfrac{m_2}{2}\right]\ell_2\sin\alpha_1\sin\left(\alpha_3-\alpha_2\right)+\\ +\left[\left(m_{p2}+m_e+m_2+m_{p1}+\dfrac{m_1}{2}\right)\cdot\ell_1\sin\alpha_1\sin\alpha_2\right]\end{array}\right\}+\\
&-\ddot{\alpha}_3\cdot\left\{\left[\left(m_{p2}+m_e\right)+\dfrac{m_2}{2}\right]\ell_2\sin\alpha_1\sin\left(\alpha_3-\alpha_2\right)\right\}+\\
&+\dot{\alpha}_1^2\left\{\begin{array}{l}\left(m_{p2}+m_e+m_2+m_{p1}+\dfrac{m_1}{2}\right)\ell_1\sin\alpha_1\cos\alpha_2-\\ -\left(m_{p2}+m_e+\dfrac{m_2}{2}\right)\ell_2\sin\alpha_1\cos\left(\alpha_3-\alpha_2\right)\end{array}\right\}+\\
&+\dot{\alpha}_2^2\left\{\begin{array}{l}\left(m_{p2}+m_e+m_2+m_{p1}+\dfrac{m_1}{2}\right)\ell_1\sin\alpha_1\cos\alpha_2-\\ -\left(m_{p2}+m_e+\dfrac{m_2}{2}\right)\ell_2\sin\alpha_1\cos\left(\alpha_3-\alpha_2\right)\end{array}\right\}-\\
&-\dot{\alpha}_3^2\left(m_{p2}+m_e+\frac{m_2}{2}\right)\ell_2\sin\alpha_1\cos\left(\alpha_3-\alpha_2\right)\\
&+2\dot{\alpha}_1\cdot\dot{\alpha}_2\left[\begin{array}{l}\left(m_{p2}+m_e+\dfrac{m_2}{2}\right)\ell_2\cos\alpha_1\sin\left(\alpha_3-\alpha_2\right)+\\ +\left(m_{p2}+m_e+m_2+m_{p1}+\dfrac{m_1}{2}\right)\ell_1\cos\alpha_1\sin\alpha_2\end{array}\right]-\\
&-2\dot{\alpha}_1\cdot\dot{\alpha}_3\left(m_{p2}+m_e+\frac{m_2}{2}\right)\ell_2\cos\alpha_1\sin\left(\alpha_3-\alpha_2\right)+2\dot{\alpha}_2\cdot\\
&.\dot{\alpha}_3\left(m_{p2}+m_e+\frac{m_2}{2}\right)\ell_2\cos\alpha_1\sin\left(\alpha_3-\alpha_2\right)
\end{aligned}
\tag{24}
$$

Example motion equation for **Mα1**, **Mα2** and **Mα3** are:

$$
\begin{aligned}
&\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\alpha}_1}\right)-\frac{\partial L}{\partial\alpha_1}=M_{\alpha_1},\ \frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\alpha}_2}\right)-\frac{\partial L}{\partial\alpha_2}=M_{\alpha_2}\\
&\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\alpha}_3}\right)-\frac{\partial L}{\partial\alpha_3}=M_{\alpha 3}
\end{aligned}
\tag{25}
$$

Following is a sample derivation of the equation for **Mα1**, **Mα2** and **Mα3**. The first step is an equation (23) into these derivatives $\dfrac{\partial L}{\partial\dot{\alpha}_3}\,a\,\dfrac{\partial L}{\partial\alpha_3}$.

After substituting is expressed **Mα1**:

$$
\begin{aligned}
M_{\alpha_1} &= \ddot{y}\cdot\left\{\begin{array}{l}\left[\left(m_{p2}+m_e\right)+\dfrac{m_2}{2}\right]\ell_2\cos\alpha_1\cos\left(\alpha_3-\alpha_2\right)-\\ -\left[\left(m_{p2}+m_e+m_2+m_{p1}+\dfrac{m_1}{2}\right)\cdot\ell_1\cos\alpha_1\cos\alpha_2\right]\end{array}\right\}+\\
&+\ddot{\alpha}_1\cdot\left\{\begin{array}{l}J_m+\left(m_{p2}+m_e\right)\left[\ell_1\cos\alpha_2+\ell_2\cos\left(\alpha_3-\alpha_2\right)\right]^2+\\ +\dfrac{m_2}{3}\cdot\ell_1^2\cos^2\left(\alpha_3-\alpha_2\right)+\left(m_2+m_{p1}+\dfrac{m_1}{3}\right)\cdot\ell_1^2\cos^2\alpha_2\end{array}\right\}-\\
&+\dot{\alpha}_1\cdot\dot{\alpha}_2\cdot\left\{\begin{array}{l}2\left[\left(m_{p2}+m_e\right)\ell_2^2+\dfrac{m_2}{3}\ell_1^2\right]\cdot\cos\left(\alpha_3-\alpha_2\right)\sin\left(\alpha_3-\alpha_2\right)+\\ +2\left(m_{p2}+m_e\right)\ell_1\ell_2\sin\cdot\left(2\alpha_2-\alpha_3\right)-\\ -2\left(m_{p2}+m_e+m_2+\dfrac{m_1}{3}+m_{p1}\right)\ell_1^2\cdot\cos\alpha_2\sin\alpha_2\end{array}\right\}\\
&+\dot{\alpha}_1\cdot\dot{\alpha}_3\cdot\left\{\begin{array}{l}-2\left[\dfrac{m_2}{3}\ell_1^2+\left(m_{p2}+m_e\right)\ell_2^2+\left(m_{p2}+m_e\right)\cdot\ell_1\ell_2\right]\cdot\\ .\cos\left(\alpha_3-\alpha_2\right)\sin\left(\alpha_3-\alpha_2\right)\end{array}\right\}
\end{aligned}
\tag{26}
$$

Substituting for **Mα2**:

$$
\begin{aligned}
M_{\alpha_2} &= \ddot{y}\cdot\left\{\begin{array}{l}\left(\dfrac{m_1}{2}+m_{p1}+m_2+m_{p2}+m_e\right)\ell_1\sin\alpha_1\sin\alpha_2+\\ +\left(m_{p2}+m_e+\dfrac{m_2}{2}\right)\cdot\ell_2\sin\alpha_1\sin\left(\alpha_3-\alpha_2\right)\end{array}\right\}\\
&+\ddot{\alpha}_2\cdot\left[\begin{array}{l}\left(m_{p2}+m_e\right)\left(\ell_2^2-2\ell_1\ell_2\cos\alpha_3\right)+\\ +\left(\dfrac{m_1}{3}+m_{p1}+\dfrac{4m_2}{3}+m_{p2}+m_e\right)\ell_1^2\end{array}\right]-\\
&-\ddot{\alpha}_3\cdot\left[\frac{m_2}{3}\ell_1^2+\left(m_{p2}+m_e\right)\left(\ell_2^2-\ell_1\ell_2\cos\alpha_3\right)\right]+\\
&+\dot{\alpha}_1^2\cdot\left\{\begin{array}{l}\ell_1^2\left(\dfrac{m_1}{3}+m_{p1}+m_2+m_{p2}+m_e\right)\cdot\cos\alpha_2\sin\alpha_2-\\ -\left[\ell_1^2\left(\dfrac{m_1}{3}+m_2\right)+\ell_2^2\left(m_{p2}+m_e\right)\right].\\ .\cos\left(\alpha_3-\alpha_2\right)\sin\left(\alpha_3-\alpha_2\right)-\\ -\left(m_{p2}+m_e\right)\ell_1\ell_2\sin\left(2\alpha_2-\alpha_3\right)\end{array}\right\}+\\
&+\dot{\alpha}_2\dot{\alpha}_3\cdot 2\left(m_{p2}+m_e\right)\cdot\ell_1\ell_2\cdot\sin\alpha_3-\\
&-\dot{\alpha}_3^2\cdot\left(m_{p2}+m_e\right)\cdot\ell_1\ell_2\cdot\sin\alpha_3-\\
&-g\cdot\left(\frac{m_1}{2}+m_2+m_{p2}+m_e+m_{p1}\right)\cdot\\
&.\ell_1\cdot\cos\alpha_2+g\cdot\ell_2\cdot\left(\frac{m_2}{2}+m_{p2}+m_e\right)\cos\left(\alpha_3-\alpha_2\right)
\end{aligned}
\tag{27}
$$

And substituting for **Mα3**:

$$
\begin{aligned}
M_{\alpha 3} &= -\ddot{y}\cdot\left\{\left(\frac{m_2}{2}+m_e+m_{p2}\right)\cdot\ell_2\cos\alpha_1\sin\left(\alpha_3-\alpha_2\right)\right\}+\\
&+\ddot{\alpha}_2\cdot\left[\left(m_{p2}+m_e\right)\left(\ell_1\ell_2\cos\alpha_3-\ell_2^2\right)-\frac{m_2}{3}\ell_1^2\right]+\\
&+\ddot{\alpha}_3\cdot\left[\left(m_{p2}+m_e\right)\ell_2^2-\frac{m_2}{3}\ell_1^2\right]+\\
&+\dot{\alpha}_1^2\left[\left(m_{p2}+m_e\right)\ell_2^2+\frac{m_2}{3}\ell_1^2\right]\cos\left(\alpha_3-\alpha_2\right)\sin\left(\alpha_3-\alpha_2\right)+\\
&+\dot{\alpha}_1^2\left[\left(m_{p2}+m_e\right)\right]\ell_1\ell_2\cos\alpha_2\sin\left(\alpha_3-\alpha_2\right)-\\
&-\dot{\alpha}_2^2\left(m_{p2}+m_e\right)\cdot\ell_1\ell_2\sin\alpha_3
\end{aligned}
\tag{28}
$$

D. The structural design of the manipulator

Based on the equations of motion within the simulation was performed complete project management of physical states, and were created by the physical and simulation models information interaction. This was detected effects inertial masses whole moving object, their interaction and its effects on individual homogenates moving arms including the dynamic loading drives. On that basis, was created by 3D design of one of the possible forms of the manipulator (Fig.2), including the selection of appropriate materials and drives.

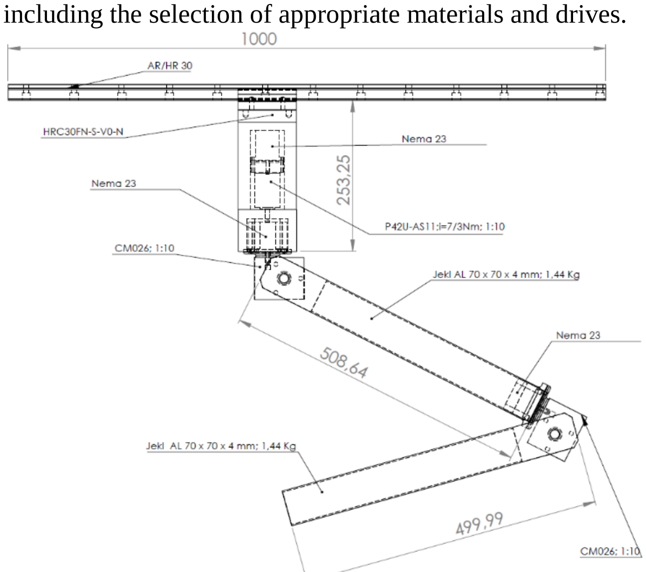

Fig. 2 proposal body of manipulator manned movable elements

III. Conclusion

The paper is concerned with utilization equations of motion on based on which are obtained kinematic and dynamic properties of an object usable for subsequent simulations.

This contribution forms the first part of the works concerned on created manipulator autonomous serviced robotic system for luggage compartments. 3D graphic model on Fig. 3 shows one possible form of the final construction of the manipulator.

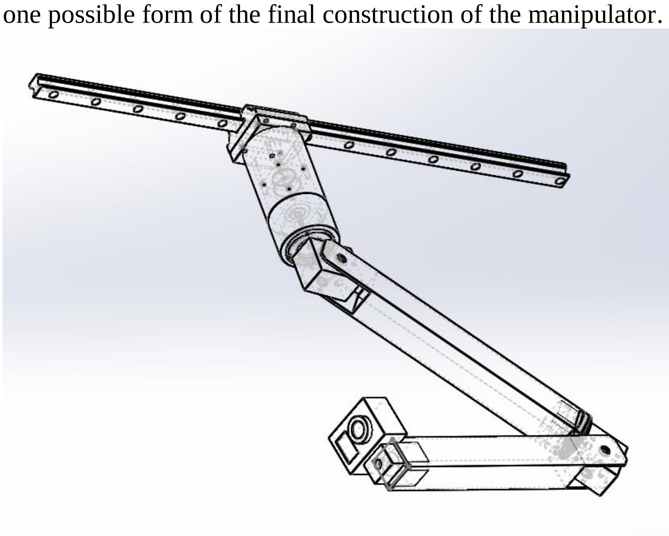

Fig. 3 the final form 3D graphic form of the manipulator systems

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