Discrete Prediction and Sliding Mode Control for Multivariable Systems

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***Abstract*—This paper deals with the development of two strategies of control, for multivariable systems, combining Sliding Mode Control (SMC) and Model Predictive Control (MPC). This is an extension of our previous works synthesized in the case of single input single output. The first proposed controller is the Sliding Mode Control with Predictive Sliding Function (SMC-PSF) and the second proposed controller is the Predictive Sliding Mode Control (PSMC). These types of scheme improve the performances of the SMC and the MPC. Simulation results demonstrate that the (SMC-PSF) and (PSMC) give better performances, for multivariable systems, in terms of strong robustness to external disturbance and parameters variation, chattering elimination and fast convergence, in comparison with the SMC. In comparison between each other, the SMC-PSF in better then the PSMC, at the presence of hard parameter variation.**

***Keywords*—Multivariable systems, Sliding Mode Control, Model Predictive Control, Predictive Sliding function, Chattering phenomenon.**

I. INTRODUCTION

The growth in the complexity of modern industrial systems make difficult the design of an exact mathematical model and the development of a suitable control. In fact, these systems are non linear, multivariable, also with external disturbances, parameter uncertainties and time delays. Therefore Sliding Mode Control (SMC) and Model based predictive control(MPC) are excellent candidates to utilize as a control law for these systems.[1].

For a large class of systems, the SMC is particularly interesting due to its ability to deal with non linearities, uncertainties, modeling errors and disturbances [4]. The main idea behind SMC is to synthesize a discontinuous control input to force the states trajectories to reach a specific surface called the sliding surface in finite time and to stay on it. However, in spite of the robustness of the sliding mode control, the chattering phenomenon, caused by the discontinuous term of the control law, is still the main problem of the SMC which consists in a sudden and rapid variation of the control signal leading to undesirable results [1].

Many approaches have been proposed to solve this problem such as high order sliding mode control [2], [3].

On the other hand, in recent years, model based predictive control(MPC) has received a lot of attention in the control theory and applications. It has been successfully implemented in many industrial applications, showing good performances. The basic idea of MPC is to calculate a sequence of future control signals in such a way that it minimizes a multistage cost function defined over a prediction horizon. The index to be optimized is a difference between the predictive system output and predictive reference sequence over the prediction horizon plus a quadratic function measuring control effort [4], [5], [6]. Nevertheless the control law is model dependent, so a perfect model is required to guarantee the success of MPC control strategies. Because of the finite horizon, the stability and the robustness of the process is difficult to analyze and guarantee, especially when constraints are present [7], [8].

As a solution, we have proposed in [9], [10], [11], [12], [13], [14] the predictive sliding mode controller (PSMC) which combine the design of SMC and MPC for single input single output systems. This combination improves the performances of the two control laws and overcome most of their specific drawbacks.

In other works, we have proposed another combination, which, is consisting on a sliding mode controller, where the optimal sliding function is allowed by a model predictive control block based on a specific objective [10], [15]. The main idea of this work is to extend our previous works, concerning the Sliding Mode Controler with predictive sliding function (SMC-PSF) and the predictive sliding mode controller (PSMC), to multivariable systems.

The paper is organized as follows: Section II gives the synthesis of the classical discrete sliding mode control for multivariable systems. The synthesis of the multivariable sliding mode controll with predictive sliding function is presented in section III. The Multivariable Predictive Sliding Mode Control is synthesized in section IV. In the following section, the two proposed controller are tested on a simulation example, and compared to SMC control and with each other. Finally, section V draws conclusions of the paper.

II. THE CLASSICAL DISCRETE MULTIVARIABLE SLIDING MODE CONTROL

Consider a discrete multivariable system subjected to external disturbances and parameters variation, defined by [16]:

$$\begin{cases} x(k+1) = (A+\Delta A)x(k) + (B+\Delta B)(u(k)+v(k)) \\ y(k) = Hx(k) + Du(k) \end{cases} \tag{1}$$

where:

$x(k) \in \Re^n$ is the state vector at the instant k, $u(k) \in \Re^m$ is

the input vector at the instant k, $y(k) \in \Re^p$ is the output vector at the instant k, $v(k) \in \Re^m$ is the disturbance input vector at the instant k.
The matrices $A \in \Re^{n\times n}$, $B \in \Re^{n\times m}$, $H \in \Re^{p\times n}$ and $D \in \Re^{p\times m}$ are the nominal model matrices.
$\Delta A \in \Re^{n\times n}$ and $\Delta B \in \Re^{n\times m}$ are the parameter uncertainties matrices.

The system (1) can be presented by the following form:

$$\left\{ \begin{array}{l} x(k+1) = Ax(k) + Bu(k) + w(k) \\ y(k) = Hx(k) + Du(k) \end{array} \right. \tag{2}$$

with:

$$w(k) = \Delta Ax(k) + \Delta Bu(k) + (B + \Delta B)\, v(k) \tag{3}$$

where $w(k) \in \Re^n$.
The sliding function is defined as [17]:

$$S(k) = Cx(k) = [s_1(k) \cdots s_m(k)]^T \tag{4}$$

where the dimension of the matrix C are (m, n).
The sliding function vector is chosen in order to verify the following reaching law [18], [19]:

$$S(k+1) = \Phi S(k) - \begin{bmatrix} m_1 sign(s_1(k)) \\ m_2 sign(s_2(k)) \\ \vdots \\ m_m sign(s_m(k)) \end{bmatrix} \tag{5}$$

where Φ is a diagonal matrix with (m, m) dimension and verifying $0 \leq \Phi_{i,i} < 1$ and $m_i > 0$ for $i \in [1 \quad m]$.
and sign is the signum function defined as :

$$sign(s_i(k)) = \left\{ \begin{array}{lll} -1 & if & s_i(k) < 0 \\ +1 & if & s_i(k) > 0 \end{array} \right. \quad ; \quad i \in [1 \quad m]$$

Thus, using equation (5), the control law ensuring the quasi-sliding mode is calculated as follows [20]:

$$u(k) = (CB)^{-1} \left(-CAx(k) + \Phi S(k) - \begin{bmatrix} m_1 sign(s_1(k)) \\ m_2 sign(s_2(k)) \\ \vdots \\ m_m sign(s_m(k)) \end{bmatrix} \right) \tag{6}$$

(CB) is inversible.

III. Synthesis of discrete multivariable sliding mode control with predictive sliding function

A block diagram of the SMC-PSF is shown in Figure1, where the primary loop is a Model Predictive Control (MPC) and the secondary loop is a Sliding Mode Control (SMC)[10], [15].

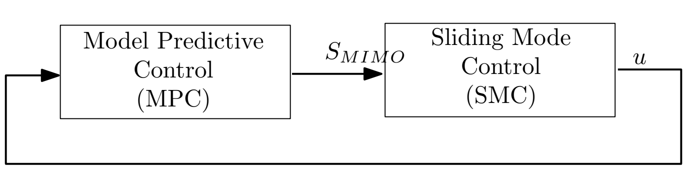

Fig. 1. SMC-PSF Controller bloc diagram.

The main purpose is to apply the discrete sliding mode control for multivariable systems which the sliding function is given optimally by the Model predictive control, based on a specific objective
We consider, now, the sliding mode control problem for multivariable system (1). The objective is to design sliding mode controller with predictive sliding function taking the reaching law (5). Define the vector $\Delta U_{eq}(k+1)$ as:

$$\Delta U_{eq}(k+1) = \begin{bmatrix} \partial u_{eq}(k+1) \\ \partial u_{eq}(k+2) \\ \vdots \\ \partial u_{eq}(k+M) \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} u_{eq}(k+1) - u_{eq}(k) \\ u_{eq}(k+2) - u_{eq}(k+1) \\ \vdots \\ u_{eq}(k+M) - u_{eq}(k+M-1) \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{7}$$

Or, the equivalent control vector is given by:

$$u_{eq}(k) = (CB)^{-1} \left[\Phi S(k) - CAx(k) \right]$$

So the vector $\Delta U_{eq}(k+1)$ can be written as:

$$\Delta U_{eq}(k+1) = \begin{bmatrix} (CB)^{-1}\Phi \left[S(k+1) - S(k)\right] \\ (CB)^{-1}\Phi \left[S(k+2) - S(k+1)\right] \\ \vdots \\ (CB)^{-1}\Phi \left[S(k+N) - S(k+N-1)\right] \end{bmatrix} - \begin{bmatrix} (CB)^{-1}CA \left[x(k+1) - x(k)\right] \\ (CB)^{-1}CA \left[x(k+2) - x(k+1)\right] \\ \vdots \\ (CB)^{-1}CA \left[x(k+N) - x(k+N-1)\right] \end{bmatrix} \tag{8}$$

or, we have:

$$\begin{cases} x(k+2)-x(k+1)=A(x(k+1)-x(k)) \\ \qquad +B(u_{eq}(k+1)-u_{eq}(k)) \\ x(k+3)-x(k+2)=A^2(x(k+1)-x(k)) \\ \qquad +AB(u_{eq}(k+1)-u_{eq}(k)) \\ \qquad +B(u_{eq}(k+2)-u_{eq}(k+1)) \\ \vdots \\ x(k+M)-x(k+M-1)=A^{M-1}(x(k+1)-x(k)) \\ \qquad +A^{M-2}B(u_{eq}(k+1)-u_{eq}(k)) \\ \qquad +A^{M-3}B(u_{eq}(k+2)-u_{eq}(k+1)) \\ \qquad +\cdots \\ \qquad +B(u_{eq}(k+M-1)-u_{eq}(k+M-2)) \\ \vdots \\ x(k+N)-x(k+N-1)=A^{N-1}(x(k+1)-x(k)) \\ \qquad +A^{N-2}B(u_{eq}(k+1)-u_{eq}(k)) \\ \qquad +A^{N-3}B(u_{eq}(k+2)-u_{eq}(k+1)) \\ \qquad +\cdots \\ \qquad +A^{N-M-1}B(u_{eq}(k+M)-u_{eq}(k+M-1)) \end{cases}$$

Then the vector $\Delta U_{eq}(k+1)$ can be presented by:

$$\Delta U_{eq}(k+1)=\begin{bmatrix} (CB)^{-1}\Phi\left[S(k+1)-S(k)\right] \\ (CB)^{-1}\Phi\left[S(k+2)-S(k+1)\right] \\ \vdots \\ (CB)^{-1}\Phi\left[S(k+N)-S(k+N-1)\right] \end{bmatrix} - \begin{bmatrix} (CB)^{-1}CA \\ (CB)^{-1}CA^2 \\ \vdots \\ (CB)^{-1}CA^N \end{bmatrix}(x(k+1)-x(k)) - \Psi_{Mimo}\Delta U_{eq}(k+1) \tag{9}$$

So, the equation (9) can be written as:

$$\Delta U_{eq}(k+1)=\Delta S_{\Phi_{MIMO}}(k+1)-\Pi_{Mimo}\left[x(k+1)-x(k)\right] - \Psi_{Mimo}\Delta U_{eq}(k+1) \tag{10}$$

with:

$$\Delta S_{\Phi_{MIMO}}=(k{+}1)\begin{bmatrix} (CB)^{-1}\Phi\left[S(k+1)-S(k)\right] \\ (CB)^{-1}\Phi\left[S(k+2)-S(k+1)\right] \\ \vdots \\ (CB)^{-1}\Phi\left[S(k+N)-S(k+N-1)\right] \end{bmatrix}$$

$$\Pi_{MIMO}=\begin{bmatrix} (CB)^{-1}CA \\ (CB)^{-1}CA^2 \\ \vdots \\ (CB)^{-1}CA^N \end{bmatrix}$$

and

$$\Psi_{MIMO}=\begin{pmatrix} 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ TAB & 0 & \cdots & \cdots & \cdots & 0 \\ TA^2B & TAB & 0 & \cdots & \cdots & 0 \\ TA^3B & TA^2B & TAB & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ TA^NB & TA^{N-1}B & \cdots & TA^{N-M}B & \cdots & 0 \end{pmatrix}$$

with $T=(CB)^{-1}C$.

Equation (10) can be written as:

$$(I+\Psi_{Mimo})\Delta U_{eq}(k+1)=\Delta S_{\Phi_{MIMO}}(k+1) - \Pi_{Mimo}\left[x(k+1)-x(k)\right] \tag{11}$$

$\Delta U_{eq}(k+1)$ can be given by:

$$\Delta U_{eq}(k+1)=(I+\Psi_{Mimo})^{-1}\Delta S_{\Phi}(k+1) - \Pi_{Mimo}\left[x(k+1)-x(k)\right]] \tag{12}$$

So:

$$\Delta U_{eq}(k{+}1)=K_{MIMO}\Delta S_{\Phi}(k{+}1)+L_{MIMO}\left[x(k+1)-x(k)\right] \tag{13}$$

with:

$$\begin{cases} K_{MIMO}=(I+\Psi_{Mimo})^{-1} \\ L_{MIMO}=-(I+\Psi_{Mimo})^{-1}\Pi_{Mimo} \end{cases} \tag{14}$$

To find the predictive function vector, the following corresponding optimization cost function is defined:

$$j_{SMC-PSF}(k)=\sum_{j=1}^{N} q_j\left[\delta S_{\Phi}(k+j)-\delta S_r(k+j)\right]^2 + \sum_{l=1}^{M} g_l\left[\delta u_{eq}(k+l-1)\right]^2 \tag{15}$$

where $\delta S_r(k+j)$ is the increment of the sliding mode references trajectories vector, $\delta S_{\Phi}(k+j)$ is the increment of the predictive sliding function vector, multiplied by the term $(CB)^{-1}\Phi$, q_j and g_l are weight coefficients.
In order to simplify the synthesis of the controller, we consider $q_j=q$ and $g_l=g$. So, the following corresponding optimization cost function (15) is written by:

$$j_{SMC-PSF}(k)=\sum_{j=1}^{N} q\left[\delta S_{\Phi}(k+j)-\delta S_r(k+j)\right]^2 + \sum_{l=1}^{M} g\left[\delta u_{eq}(k+l-1)\right]^2 \tag{16}$$

Rewrite equation (16) in vector form:

$$J_{SMC-PSF}(k)=\left\|\Delta S_{\Phi_{MIMO}}(k)-\Delta S_{r\_MIMO}(k)\right\|_Q^2 + \left\|\Delta U_{eq}(k)\right\|_G^2 \tag{17}$$

where

$$\Delta S_{r\_MIMO}(k+1)=\left[\delta S_r(k+1),\delta S_r(k+2),...,\delta S_r(k+N)\right]^T$$

$$G=[gI_m,gI_m,...,gI_m]$$

$$Q=[qI_m,qI_m,...,qI_m]$$

Since the control objective is to keep states on the sliding surface, the desired sliding mode reference trajectory vector is approximated by the predictive sliding function vector and should verify:

$$S_r(k) = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{(m\times 1)} \text{, so} \qquad \Delta S_{r\_MIMO} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{(mN\times 1)}$$

So, the optimization cost function can be written as:

$$J_{SMC-PSF}(k) = \|\Delta S_{\Phi_{MIMO}}(k)\|_Q^2 + \|\Delta U_{eq}(k)\|_G^2 \quad (18)$$

Rewrite equation(13), $J_{SMC-PSF}(k)$ can be written as:

$$\begin{aligned} J_{SMC-PSF}(k) &= \Delta S_{\Phi_{MIMO}}(k)^T\ Q\ \Delta S_{\Phi_{MIMO}}(k) + \\ &[K_{MIMO}\Delta S_{\Phi_{MIMO}}(k) + L_{MIMO}(x(k) - x(k-1))]^T \\ &G\ [K_{MIMO}\Delta S_{\Phi_{MIMO}}(k) + L_{MIMO}(x(k) - x(k-1))] \end{aligned} \quad (19)$$

The optimal sequence of the increment of the predictive sliding function vector is obtained by minimizing the cost function $J_{SMC-PSF}$:

$$\frac{\partial J_{SMC-PSF}(k)}{\partial \Delta S_{\Phi_{MIMO}}(k)} = 0 \quad (20)$$

So, the increment of the predictive sliding function vector can be calculated as:

$$\Delta S_{\Phi_{MIMO}}(k) = -(GK_{MIMO}^T K_{MIMO} + Q)^{-1} GK_{MIMO}^T L_{MIMO}(x(k) - x(k-1)) \quad (21)$$

We suppose that $\delta S_\Phi(k)$ is the vector of the m first elements of the vector $\Delta S_{\Phi_{MIMO}}$, so, the predictive sliding function vector $S_\Phi(k)$ is given as:

$$S_\Phi(k) = S_\Phi(k-1) + \delta S_\Phi(k) \quad (22)$$

with: $S_\Phi(k) = (CB)^{-1}\Phi S(k)$
So, the control law $u_{eq}(k)$ is given by the equation:

$$u_{eq}(k) = (CB)^{-1}[\Phi S(k) - CAx(k)] \quad (23)$$

Then:

$$u_{eq}(k) = S_\Phi(k) - (CB)^{-1}CAx(k) \quad (24)$$

Or, we have:

$$u_{dis}(k) = -(CB)^{-1} \begin{bmatrix} m_1 sign(s_1(k)) \\ m_2 sign(s_2(k)) \\ \vdots \\ m_m sign(s_m(k)) \end{bmatrix} \quad (25)$$

So:

$$u(k) = u_{eq}(k) + u_{dis}(k) \quad (26)$$

IV. Synthesis of Discrete Multivariable Predictive Sliding Mode Control

The principle of the Discrete Predictive Sliding Mode Controller (DPSMC) is given by the block diagram shown in Figure2, where the primary loop is a Sliding Mode Control (SMC) and the secondary loop is a Model Predictive Control (MPC)[9], [13].

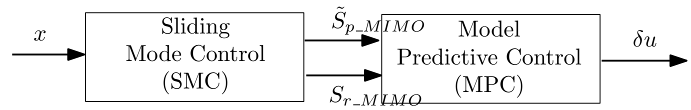

Fig. 2. DPSMC Controller bloc diagram.

The main purpose is to approximate the predictive sliding functions vector S_{p_MIMO} to the sliding reference functions vector S_{r_MIMO}, penalizing at the same time the variation in the control signal.
We consider, now, the sliding mode control problem for multivariable system (1). The objective is to design a predictive sliding mode controller taking the reaching law (5). The reference sliding mode trajectory is chosen as:

$$\begin{cases} S_r(k+1) = \Phi S_r(k) - \begin{bmatrix} m_1 sign(s_{r1}(k)) \\ m_2 sign(s_{r2}(k)) \\ \vdots \\ m_m sign(s_{rm}(k)) \end{bmatrix} \\ S_r(k) = S(k) \end{cases} \quad (27)$$

We consider that $w(k)$ is equal to null matrix.
The sliding functions vector at the instant $k+1$, $k+2$ and $k+3$ can be written as:

$$\begin{aligned} S(k+1) &= Cx(k+1) \\ &= CAx(k) + CB(u(k) - u(k-1)) + CBu(k-1) \\ &= CAx(k) + CB\delta u(k) + CBu(k-1) \end{aligned}$$

$$\begin{aligned} S(k+2) &= Cx(k+2) \\ &= CA^2x(k) + CB\delta u(k+1) + CB\delta u(k) \\ &\quad + CAB\delta u(k) + CBu(k-1) + CABu(k-1) \\ &= CA^2x(k) + CB\delta u(k+1) \\ &\quad + C(A+I)B\delta u(k) + C(A+I)Bu(k-1) \end{aligned}$$

$$\begin{aligned} S(k+3) &= Cx(k+3) \\ &= CA\left[A\left[Ax(k) + Bu(k)\right]\right] + CABu(k+1) \\ &\quad + CBu(k+2) \\ &= CA^3x(k) + CB\delta u(k+2) + C(A+I)B\delta u(k+1) \\ &\quad + C(A^2 + A + I)B\delta u(k) + C(A^2 + A + I)Bu(k-1) \end{aligned}$$

Then, $S(k+p)$ can be calculated as:

$$\begin{aligned} &S(k+p) = CA^p x(k) + CB\delta u(k+p-1) \\ &+C(A+I)B\delta u(k+p-2) + \cdots + C\left[\sum_{j=0}^{p-1} A^j\right] B\delta u(k) \\ &+C\left[\sum_{j=0}^{p-1} A^j\right] Bu(k-1) \end{aligned} \quad (28)$$

where:
$\delta u(k) = u(k) - u(k-1)$; I is the identity matrix with the dimension $n \times n$.

We introduce, then the predictive sliding functions vector of multivariable system S_{p_MIMO} as:

$$S_{p\_MIMO}(k+1) = \begin{bmatrix} S(k+1) \\ S(k+2) \\ \vdots \\ S(k+N) \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} s_1(k+1) \\ s_2(k+1) \\ \vdots \\ s_m(k+1) \end{bmatrix} \\ \begin{bmatrix} s_1(k+2) \\ s_2(k+2) \\ \vdots \\ s_m(k+2) \end{bmatrix} \\ \vdots \\ \begin{bmatrix} s_1(k+N) \\ s_2(k+N) \\ \vdots \\ s_m(k+N) \end{bmatrix} \end{bmatrix} \tag{29}$$

With N is prediction horizon.
Equation (29) can be described as follows::

$$\begin{array}{c} S_{p\_MIMO}(k+1) = \Gamma_{MIMO}x(k) + \Omega^F_{MIMO}\Delta U(k) \\ +\Omega^P_{MIMO}u(k-1) \end{array} \tag{30}$$

where:

$$\Delta U(k) = \left[\delta u(k), \delta u(k+1), \cdots, \delta u(k+M-1), \underbrace{0 \quad, \cdots, \quad 0}_{m\times(N-M+1)}\right]$$

With M is control horizon.

$$\Gamma_{MIMO} = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^N \end{bmatrix} \tag{31}$$

$$\Omega^F_{MIMO} = \begin{bmatrix} CB & 0 & \cdots & \cdots & 0 \\ C(A+I) & CB & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ C(\sum\limits_{j=0}^{M-1} A^j)B & C(\sum\limits_{j=0}^{M-2} A^j)B & \cdots & \cdots & CB \\ C(\sum\limits_{j=0}^{M} A^j)B & C(\sum\limits_{j=0}^{M-1} A^j)B\cdots & \cdots & \cdots & C(A+I)B \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ C(\sum\limits_{j=0}^{N-1} A^j)B & C(\sum\limits_{j=0}^{N-2} A^j)B & \cdots & \cdots & C(\sum\limits_{j=0}^{N-M} A^j)B \end{bmatrix}$$

$$\Omega^P_{MIMO} = \begin{bmatrix} CB \\ C(A+I)B \\ \vdots \\ C(\sum\limits_{j=0}^{M-1} A^j)B \\ \vdots \\ C(\sum\limits_{j=0}^{N-1} A^j)B \end{bmatrix} \tag{32}$$

In practice, to make correction to the future predictive sliding function vector $S_{p\_MIMO}(k+p)$, we introduce the error between the sliding functions vector $S(k)$ and the predictive sliding functions vector $S(k/k-p)$. Therefore, the predictive sliding functions vector is given as follows:

$$\begin{array}{l} \tilde{S}_{p\_MIMO}(k+p) = S(k+p) + h_p e(k) \\ = CA^p x(k) + CB\delta u(k+p-1) + C\,(A+I)\,\delta u(k+p-2) \\ + \cdots + C\left[\sum\limits_{j=0}^{p-1} A^j\right] B\delta u(k) + C\left[\sum\limits_{j=0}^{p-1} A^j\right] Bu(k-1) + h_p e(k) \end{array} \tag{33}$$

h_p is a correct coefficient.
The equation (33), can be given as:

$$\tilde{S}_{p\_MIMO}(k+1) = S_{p\_MIMO}(k+1) + H_p E(k) \tag{34}$$

where:

$$\begin{array}{l} \tilde{S}_{p\_MIMO}(k+1) = \left[\tilde{S}_p(k+1), \tilde{S}_p(k+2), ..., \tilde{S}_p(k+N)\right]^T \\ H_p = diag\left[h_1 I_m, h_2 I_m, ..., h_N I_m\right] \\ E(k) = S_v(k) - S_{mp}(k) \\ S_v(k) = [S(k), S(k), ..., S(k)] \\ S_{mp}(k) = [S(k/k-1), S(k/k-2), ..., S(k/k-N)]^T \end{array}$$

Knowing that:

$$S(k/k-p) = CA^p x(k-p) + \sum_{j=1}^{p} CA^{j-1}Bu(k-j) \tag{35}$$

The following corresponding optimization cost function is defined by:

$$\begin{array}{l} j_{DPSMC} = \sum\limits_{j=1}^{N} q_j \left[\widetilde{S}_p(k+j) - S_r(k+j)\right]^2 \\ + \sum\limits_{l=1}^{M} g_l \left[\delta u(k+l-1)\right]^2 \end{array} \tag{36}$$

where $S_r(k+j)$ is the sliding mode references trajectories vector, q_j and g_l are weight coefficients.
In order to simplify the synthesis of the controller, we consider $q_j = q$ and $g_l = g$. So, the following corresponding optimization cost function (36) is written by:

$$j_{DPSMC} = \sum_{j=1}^{N} q\left[\tilde{S}_p(k+j) - S_r(k+j)\right]^2 + \sum_{l=1}^{M} g\left[\delta u(k+l-1)\right]^2 \tag{37}$$

The equation (37) can be rewritten as:

$$\begin{array}{l} J_{DPSMC} = \left\|\tilde{S}_{p\_MIMO}(k+1) - S_{r\_MIMO}(k+1)\right\|^2_Q \\ \quad + \left\|\Delta U(k)\right\|^2_G \\ = \left[\Gamma_{MIMO}x(k) + \Omega^F_{MIMO}\Delta U(k) + \Omega^P_{MIMO}u(k-1)\right. \\ \left. + H_p E(k) - S_{r\_MIMO}(k+1)\right]^T Q\left[\Gamma_{MIMO}x(k)\right. \\ + \Omega^F_{MIMO}\Delta U(k)) + \Omega^p_{MIMO}u(k-1) + H_p E(k) \\ - S_{r\_MIMO}(k+1)] + \Delta U(k)^T G\Delta U(k) \end{array} \tag{38}$$

where

$$\begin{array}{l} S_{r\_MIMO}(k+1) = [S_r(k+1), S_r(k+2), ..., S_r(k+N)]^T \\ G = [gI_m, gI_m, ..., gI_m] \\ Q = [qI_m, qI_m, ..., qI_m] \end{array}$$

The optimal control law can be obtained by:

$$\frac{\partial J_{DPSMC}}{\partial \Delta U(k)} = 0$$

So,

$$\begin{array}{l}\Delta U(k) = -((\Omega^F_{MIMO})^T \Omega^F_{MIMO} + G)^{-1} (\Omega^F_{MIMO})^T \\ {[\Gamma_{MIMO} x(k) + H_p E(k) + \Omega^p_{MIMO} u(k-1)} \\ -S_{r\_MIMO}(k+1)]\end{array} \quad (39)$$

Only the m present increment of control input signals vector are implemented, the next time increment of control signals vector $\delta u(k)$ will be calculated recursively by:

$$\delta u(k) = [1, 1, ..., 1, 0, ...0]^T \, \Delta U(k) \quad (40)$$

So, we have:

$$u(k) = u(k-1) + \delta u(k) \quad (41)$$

V. Simulation results

To evaluate the robustness of the control laws (equations 6, 26 and 39) in presence of constant or periodic disturbances and parameters uncertainties, we consider a discrete multivariable process described by the following equation:

$$x(k+1) = (A + \Delta A)x(k) + (B + \Delta B)(u(k) + v(k))$$

where:

$$A = \begin{bmatrix} 0 & 1 \\ 0.24 & 0.2 \end{bmatrix} \quad ; \quad B = \begin{bmatrix} 1.5 & 0 \\ 0 & 1 \end{bmatrix}$$

The retained synthesis parameters are:

$$C = \begin{bmatrix} 0.6667 & 0 \\ 0 & 1 \end{bmatrix}$$

and $m_1 = 0.01$, $m_2 = 0.01$, $\Phi = [0.01 \quad 0; 0 \quad 0.01]$, $N = 10$, $M = 5$, $H_p = 0.001I(N,N)$, and $G = 0.001I(N,N)$

The sliding functions vector is given by:

$$S(k) = Cx(k) = C = \begin{bmatrix} 0.6667 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} = \begin{bmatrix} s_1(k) \\ s_2(k) \end{bmatrix}$$

A. *Case of constant disturbances*

The results presented in this section are obtained with the presence of constant disturbances which are given by:

$$v(k) = \begin{bmatrix} 0.15 \\ 0.2 \end{bmatrix} \quad , \quad \forall k \geq 100$$

The parameters variation are given by:

$$\Delta A = 0.1 \begin{bmatrix} 5\sin(-\frac{2k\pi}{10}) & 6\sin(-\frac{2k\pi}{10}) \\ 3\sin(-\frac{2k\pi}{10}) & 3\sin(-\frac{2k\pi}{10}) \end{bmatrix}$$

$$\Delta B = 0.1 \begin{bmatrix} 2\sin(-\frac{2k\pi}{10}) & 5\sin(-\frac{2k\pi}{10}) \\ 3\sin(-\frac{2k\pi}{10}) & 5\sin(-\frac{2k\pi}{10}) \end{bmatrix}, \forall k \geq 300$$

The evolution of the states $x_1(k)$ and $x_2(k)$, the control inputs $u_1(k)$ and $u_2(k)$ and the sliding mode functions $s_1(k)$ and $s_2(k)$ with SMC-PSF and SMC are given, respectively, in Figures 3 to 7.

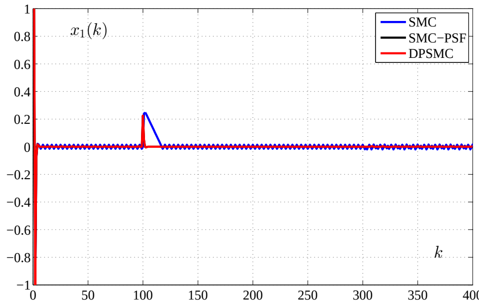

Fig. 3. Evolution of the state $x_1(k)$.

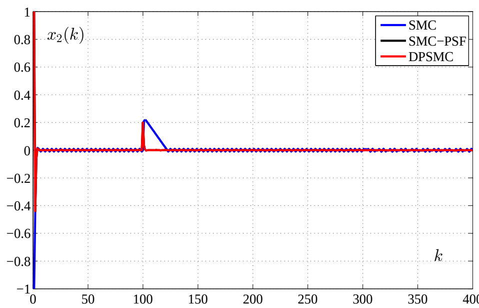

Fig. 4. Evolution of the state $x_2(k)$.

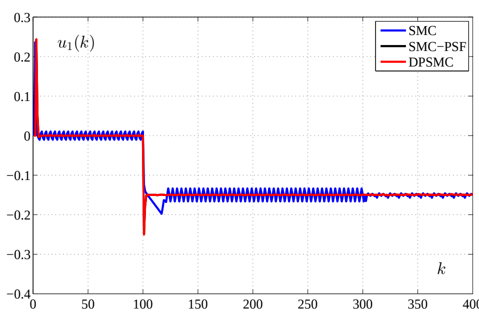

Fig. 5. Evolution of the control signal $u_1(k)$.

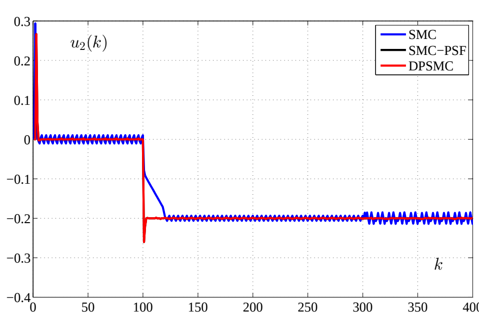

Fig. 6. Evolution of the control signal $u_2(k)$.

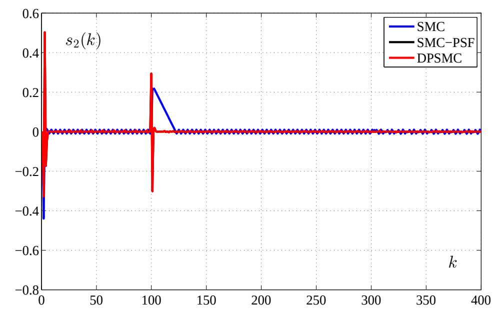

Fig. 7. Evolution of the sliding function $s_2(k)$.

It can be seen that the performances of multivariable SMC-PSF and PSMC are better then the SMC, not only, for rejecting constant disturbances, but also, for eliminating chattering.
In fact, without disturbances and parameters uncertainties, the results of SMC, PSMC and SMC-PSF are comparable. But, in presence of constant disturbances $(k \geq 100)$, we find that the proposed control laws ensure good performances in term of rejection of external disturbances and fast convergence.
When we add parameters uncertainties, at the instant $(k \geq 300)$, the oscillation encountered, in the case of classical SMC, are reduced.

B. Case of periodic disturbances

The results presented in this section are obtained with the presence of disturbances, whose evolutions are given in figures 8 and 9, and with the following parameters variation:

$$\Delta A = 0.1 \begin{bmatrix} 5\sin(-\frac{2k\pi}{10}) & 6\sin(-\frac{2k\pi}{10}) \\ 3\sin(-\frac{2k\pi}{10}) & 3\sin(-\frac{2k\pi}{10}) \end{bmatrix}$$

$$\Delta B = 0.1 \begin{bmatrix} 2\sin(-\frac{2k\pi}{10}) & 5\sin(-\frac{2k\pi}{10}) \\ 3\sin(-\frac{2k\pi}{10}) & 5\sin(-\frac{2k\pi}{10}) \end{bmatrix}, \forall k \geq 300$$

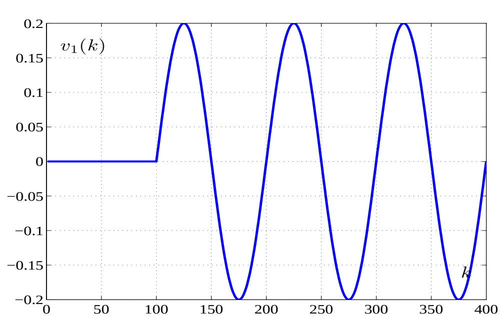

Fig. 8. Evolution of the disturbances $v_1(k)$.

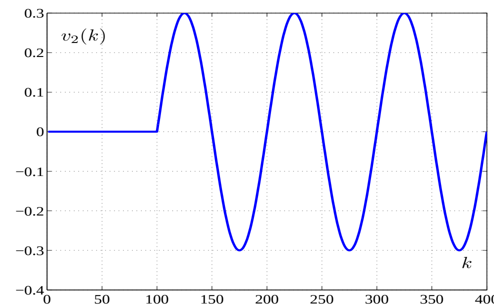

Fig. 9. Evolution of the disturbance $v_2(k)$.

The evolution of the states $x_1(k)$ and $x_2(k)$, the sliding mode functions $s_1(k)$ and $s_2(k)$ and the control inputs $u_1(k)$ and $u_2(k)$,with SMC-PSF, PSMC and SMC are given, respectively, in figures 10 to 15.

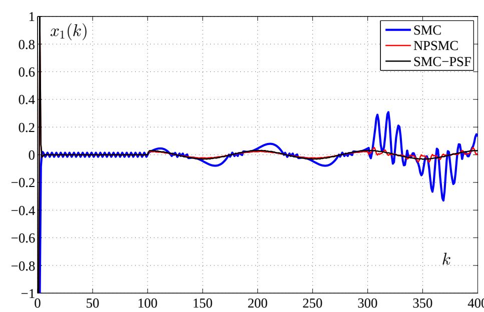

Fig. 10. Evolution of the state $x_1(k)$.

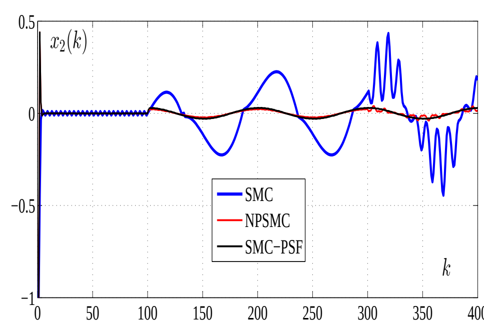

Fig. 11. Evolution of the state $x_2(k)$.

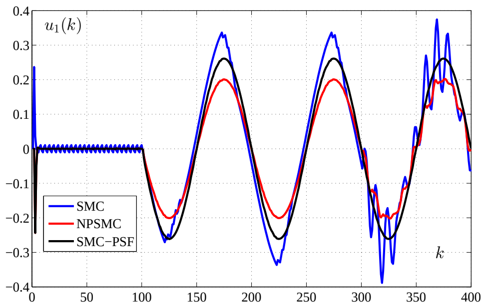

Fig. 12. Evolution of the control signal $u_1(k)$.

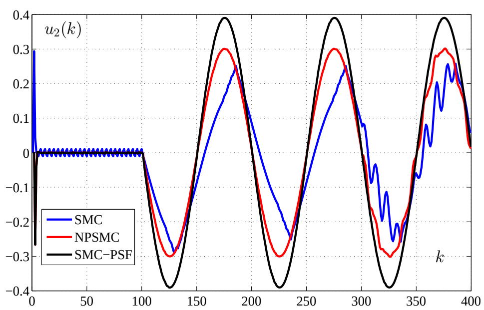

Fig. 13. Evolution of the control signal $u_2(k)$.

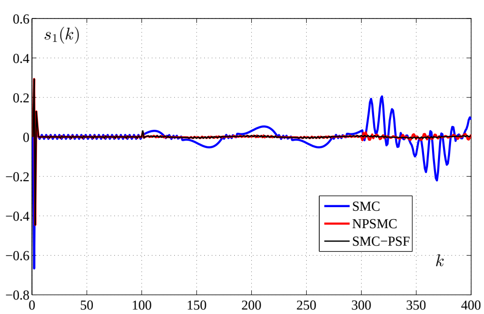

Fig. 14. Evolution of the sliding function $s_1(k)$.

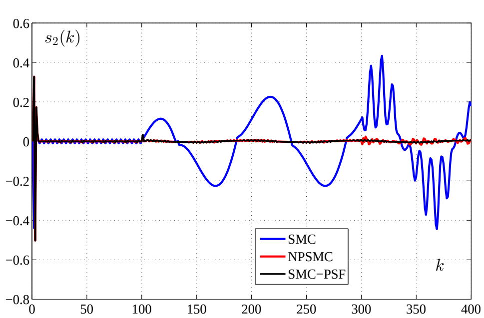

Fig. 15. Evolution of the sliding function $s_2(k)$.

A comparison between the SMC-PSF, PSMC and SMC, in the case of multivariable systems, reveals that the use of the new control strategies SMC-PSF and PSMC reduces the chattering problem effectively ($k \geq 300$).
Furthermore, the results obtained prove the capability of the proposed control laws to reduce periodic disturbances ($k \geq 100$) and parameter uncertainties ($k \geq 300$).

C. *Comparison between multivariable PSMC and SMC-PSF*

Comparing, only, between the PSMC and the SMC-PSF, we can deduce, of the previous figures, that at the presence of constant disturbances , results given by the two control laws are comparable.
At the presence of periodic disturbances, the evolution of the states $x_1(k)$ and $x_2(k)$, the sliding mode functions $s_1(k)$ and $s_2(k)$ and the control inputs $u_1(k)$ and $u_2(k)$,with SMC-PSF and PSMC are given, respectively, in figures 16 to 21.

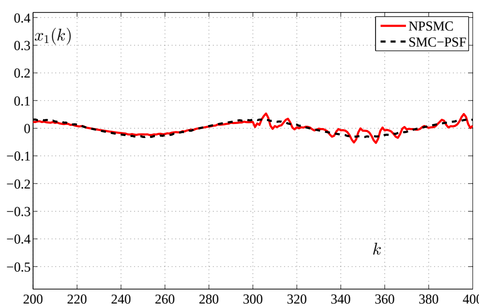

Fig. 16. Evolution of the state $x_1(k)$.

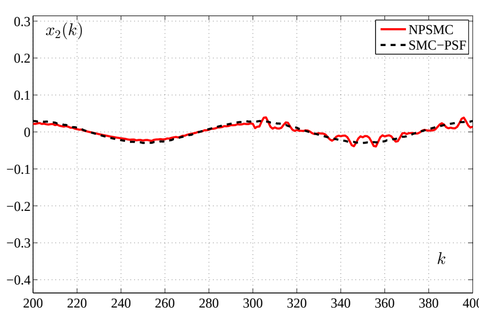

Fig. 17. Evolution of the state $x_2(k)$.

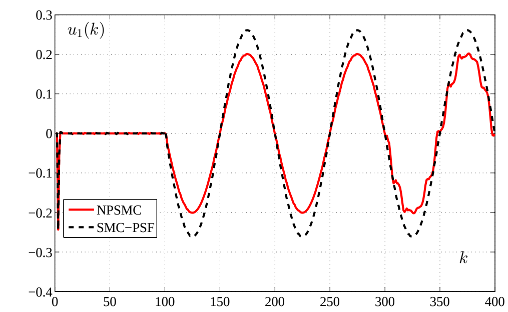

Fig. 18. Evolution of the control signal $u_1(k)$.

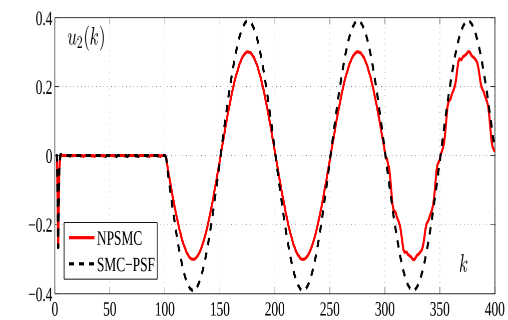

Fig. 19. Evolution of the control signal $u_2(k)$.

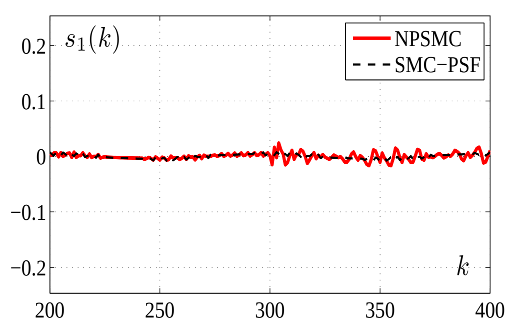

Fig. 20. Evolution of the sliding function $s_1(k)$.

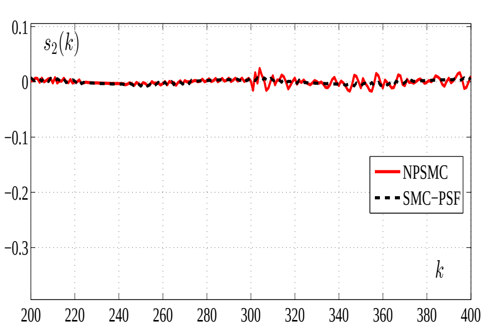

Fig. 21. Evolution of the sliding function $s_2(k)$.

With periodic disturbances, and without parameter variantion (for $100 \leq k \leq 300$), the PSMC reduce better disturbances then the SMC-PSF.
But at the presence of parameter variation (for $k \geq 300$), SMC-PSF is more able to to eliminate chattering. In fact, it can eliminates oscillation better, than PSMC.

VI. Conclusion

In this paper, a sliding mode controller with predictive sliding function and a Predictive Sliding Mode Controller, for multivariable systems are proposed. These two controllers combine the design technique of the SMC and the MPC.
These methods are tested on a multivariable system, and compared to the results given by the SMC controller. It is shown that mixing both control techniques, for multivariable systems, gives news controller with better robustness properties in rejecting disturbances, hard parameter variations and in eliminating the chattering problem.

Acknowledgment

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References

[1] P. Lopez and A.S. Nouri. *Thorie lmentaire et pratique de la commande par les rgimes glissants*. Mathmatiques et application, 2006.

[2] M. Mihoub, A.S. Nouri, and R. Ben Abdennour. Real-time application of discrete second order sliding mode control to a chemical reactor. *Control Engeneering Practice*, 17:1089–1095, 2009.

[3] A. Cavallo and C. Natale. High-order sliding control of mechnical systems: theory and experiments. *Automatica*, 12:1139–1149, 2004.

[4] D. Clarke and R. Scattolini. Constrained receding horizon predictive control. *IEEE Transaction automatic control*, 23 N2:347–354, 1991.

[5] E. F. Camacho and C. Bordon. *Model Predictive Control*. Springer, 2004.

[6] R. J. Culi and C. Bordon. Iterative nonlinear model preditive control, stability robustness and applications. *Chemical Engineering Practice*, 16:1023–1034, 2008.

[7] J. M. Maciejowski. Predictive control with constraints. In *Pretice Hall, Harlow*, 2011.

[8] D.W. Clarke, C.Mohtadi, and P.S.Tuffs. Generalised predictive control i: The basic algorithm. *Automatica*, 23:137–148, 1987.

[9] H. BenMansour, K. Dehri, and A. S. Nouri. New predictive sliding mode control for non minimum phase systems. *International Journal of Computer Applications IJCA*, 70 - N. 11:1–8, 2013.

[10] H. BenMansour, K. Dehri, and A. S. Nouri. New discrete sliding mode controller with predictive sliding function. *International Review of Automatic Control*, 6 N 4:1–10, 2013.

[11] H. BenMansour and A.S. Nouri. Discrete predictive sliding mode control of uncertain systems. In *Proccedings of the 9th International Multi-Conference on System, Signals and Devices Germany*, 2012.

[12] H. BenMansour and A. S. Nouri. Predictive sliding mode control for perturbed discrete delay time systems: Robustness analysis. In *International Conference on Electrical Engineering and Software Application ICEESA 2013*, 2013.

[13] H. BenMansour and A. S. Nouri. Discrete predictive sliding mode control of uncertain systems. In *Proceeding of the 9th International Multi-Conference on Systems, Signals and devices*, 2012.

[14] N. Abdennabi, H. Ben Mansour, and A. S. Nouri. A new sliding function for discrete predictive sliding mode control of time delay systems. *International Journal of Computer Applications (IJCA)*, 10(4):288–295, 2013.

[15] H. BenMansour, K. Dehri, and A. S. Nouri. Comparison between predictive sliding mode control and sliding mode control with predictive sliding function. In *Second International Conference on Electrical Engineering and Control Applications ICEECA, Constantine, Algeria.*, November, 18-20, 2014.

[16] R. BenAbdennour, P. Borne, M. Ksouri, and F. Msahli. *Identification et commande numrique des procds industriels*. Technip, Paris, France, 2001.

[17] K. Dehri. Sur le rejet des perturbations harmoniques par les rgimes glissant. Master's thesis, Ecole Nationale d'Ingnieurs de Gabs (ENIG), 2013.

[18] J. L. Chang. Discrete sliding-mode control of mimo linear systems. *Asian Journal of Control*, 2:10–15, 2002.

[19] G. Monsees. Discrete-time sliding mode control. Master's thesis, PhD thesis, Delft University of Technology, 2002.

[20] K. Dehri, Ltaief M., Nouri A. S., and Ben Abdennour R. Rejection of periodic disturbances with unknown frequency for multivariable systems. *International Journal on Sciences and Techniques of Automatic control and computer engineering*, 5, n1:1458–1471, 2011.