Simulation of the highest insured catastrophe losses using quantile function

V. Pacáková, P. Jindrová

***Abstract*—**Catastrophe modelling and simulations are risk management tools using computer technology to help insurers, reinsurers and risk managers better assess the potential losses caused by natural and man-made catastrophes. This article aims to present methods for modelling and simulation of extreme insured losses using quantile function based on data caused the world natural catastrophes in time period 1970-2014, published in Swiss Re Sigma No2/2015. Our interest focuses particularly on the extreme observations in the upper tail of loss distributions. We have shown that it is possible to simulate the losses in upper tail of distribution without simulating the central values. This advantage will be used for simulation a few values of the highest insured losses in the world's natural catastrophes in the future.

***Keywords*—**Extreme claims, quantile function, Pareto distribution, simulation, Weibull distribution.

I. Introduction

The occurrences of catastrophic events are becoming more frequent (Fig.1) and also grow indemnity of insurance and reinsurance companies at these events (Fig.2).

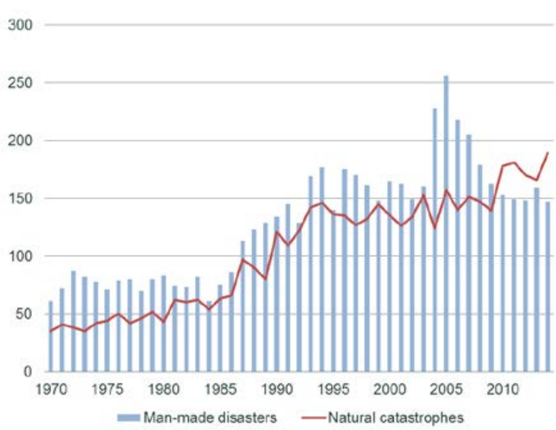

Fig.1 Number of catastrophic events, 1970-2014
Source: SwissRe economic Research&Consulting and Cat Perils

The enormous impact of catastrophic events on our society is deep and long. Not only we need to investigate the causes of such events and develop plans to protect against them, but also we have to resolve the resulting huge financial losses.

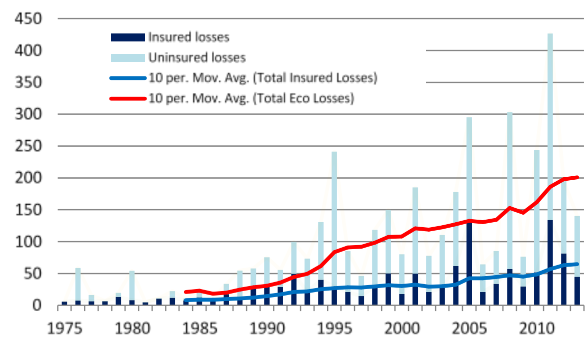

Fig.2 Natural catastrophes insured vs uninsured losses, 1975–2014, in 2014 USD billions
Source: SwissRe economic Research&Consulting and Cat Perils

From these facts it follows the need of knowledge the probability models for prediction of consequences of the catastrophe events and thus select the best options to cover risks and correct setting premiums or reinsurance.

The modelling process evolved in the late 1980s as companies become increasingly aware of their exposure to catastrophic risks. After Hurricane Andrew in 1992 and Northridge earthquake in 1994, the use of catastrophe models took of as companies sought to more accurately analyze, write and price for natural catastrophe risk.

Developments of the financial consequences of disasters have a major impact on the global insurance market and forcing the insurance and reinsurance companies to seek for new approaches and ways to cover these risks. Are the valid concerns that the capacity of the world's insurance and reinsurance markets in the future will not be sufficient to cover these risks.

In the modelling of extreme losses statistical methods are commonly used for inference from historical data. Different approaches had been proposed for certain circumstances, for example Block Maxima Models and Excess over Threshold Method [5], [6]. In this article we will present method for modelling and simulation based quantile functions [2], [4], [9], [12], [13], [14].

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II. LOSS DISTRIBUTIONS

A. Selected probability models

The conditions under which claims are performed allow us to consider the claim amounts arising from natural catastrophes to be samples from specific heavy-tailed probability distributions. Such distributions are positively skewed and very often they have high probabilities in the upper tails. So they are described as long tailed or heavy tailed distributions [1], [3], [7], [8], [11], [17].

The distributions used in this article include 2-parametric Pareto [10], [14] and 3-parametric Weibull [8], [15], which are particularly appropriate for modelling of insured losses in natural catastrophes. These distributions are used as appropriate models in case when we need to obtain well-fitted upper tail. The simple form of their quantile functions allow to simulate the highest catastrophic losses.

Pareto Distribution (*2-parameter*s)

The Pareto cumulative distribution function of the losses X_a that exceed known threshold a is [10], [12], [15]:

$$F_a(x) = p = 1 - \left(\frac{a}{x}\right)^b, \quad x \geq a \tag{1}$$

Probability density function (PDF)) is in the form

$$f_a(x) = \frac{b \cdot a^b}{x^{b+1}}, \quad x \geq a \tag{2}$$

The quantile function *QF* we can derive by inverting the CDF (1) to the form

$$Q(p) = \frac{a}{(1-p)^{1/b}} \tag{3}$$

Weibull Distribution (*3-parameter*s)

The cumulative distribution function is given by formula [8], [15]

$$F(x) = p = 1 - \exp\left[-\left(\frac{x-\theta}{\beta}\right)^\alpha\right], \quad x > \theta, \alpha > 0, \beta > 0 \tag{4}$$

with parameters: shape $\alpha > 0$, scale $\beta > 0$, threshold θ.

Probability density function (PDF) is in the form

$$f(x) = \frac{\alpha}{\beta^\alpha}(x-\theta)^{\alpha-1}\exp\left[-(x-\theta)/\beta\right]^\alpha \tag{5}$$

For $0 < p < 1$ quantile function as the inverse distribution function is

$$Q(p) = F^{-1}(p) = \theta + \beta \cdot \left[-\ln(1-p)\right]^{\frac{1}{\alpha}} \tag{6}$$

B. Distribution Fitting

The Maximum Likelihood (ML) method [3], [12] is the most often used to estimate the parameters of the selected probability distributions. This method can be applied in a wide range of situations and the parameters obtained by ML generally have very good properties compared to estimates obtained by other methods (e. g. method of moments, method of quantile). In this article procedure *Distribution Fitting* in Statgraphics Centurion XV package will be used to obtain the maximum likelihood estimators.

Kolmogorov-Smirnov test (K-S test) was chosen from seven different goodness-of-fit tests, which offers the *Distribution Fitting* procedure.

Kolmogorov-Smirnov test (*K-S* test) compares the empirical cumulative distribution function $F_n(x)$ of the data to the fitted cumulative distribution function $F(x)$. The test statistic is given by formula

$$d_n = \sup_x \left|F_n(x) - F(x)\right| \tag{7}$$

The empirical *CDF* $F_n(x)$ is expressed as follows:

$$F_n(x) = \begin{cases} 0 & x \leq x_{(1)} \\ \dfrac{j}{n} & x_{(j)} < x \leq x_{(j+1)} \qquad j = 1, 2, ..., n-1 \\ 1 & x > x_{(n)} \end{cases} \tag{8}$$

where data are sorted from smallest to the largest in sequence $x_{(1)} \leq x_{(2)} \leq ...... \leq x_{(n)}$.

C. Simulation Using Quantile Function

The *Quantile Function*, *QF*, denoted by $Q(p)$, expresses the *p*-quantile x_p as a function of p: $x_p = Q(p)$, the value of x for which $p = P(X \leq x_p) = F(x_p)$.

The definitions of the *QF* and the *CDF* can by written for any pairs of values (x, p) as $x = Q(p)$ and $p = F(x)$. These functions are simple inverses of each other, provided that they are both continuous increasing functions. Thus, we can also write $Q(p) = F^{-1}(x)$, and $F(x) = Q^{-1}(p)$ [2], [13].

We denoted a set of ordered sampling data of losses by

$$x_{(1)}, x_{(2)}, ..., x_{(r)}, ..., x_{(n-1)}, x_{(n)}.$$

The corresponding random variables are being denoted by

$$X_{(1)}, X_{(2)}, ..., X_{(r)}, ..., X_{(n-1)}, X_{(n)}.$$

Thus $X_{(n)}$ for example is the random variable representing the largest observation of the sample of *n*. The *n* random variables are referred as the *n order statistics*. These statistics play a major role in modelling with quantile function $Q(p)$.

Consider first the distribution of the largest observations $X_{(n)}$ with distribution function denoted as $F_{(n)}(x) = p_{(n)}$. The probability

$$F_{(n)}(x) = p_{(n)} = P\left(X_{(n)} \le x\right)$$

is also probability that all *n* independent observations on X are less than or equal to this value *x*, which for each one is *p*. By the multiplication law of probability

$$p_{(n)} = p^n \text{ so } p = p_{(n)}^{1/n} \text{ and } F(x) = p = p_{(n)}^{1/n}.$$

Inverting $F(x)$ to get the quantile function we have

$$Q_{(n)}\left(p_{(n)}\right) = Q\left(p_{(n)}^{1/n}\right) \tag{9}$$

So the quantile function of the largest observation is thus found from the original quantile function by very simple calculation.

For the general *r*-th order statistic $X_{(r)}$ calculation becomes more difficult. The probability that the *r*-th larges observations is less than some value *z* is marked as

$$p_{(r)} = F_{(r)}(z) = P\left(X_{(r)} \le z\right)$$

This is also probability that at least *r* of the *n* independent observations is less or equal to *z*. The probability of *s* observations being less than or equal to *z* is p^s, where $p = F(z)$ is given by the binomial expression [2]

$$P(s\ \ observations \le z) = \binom{n}{s} p^s \, (1-p)^{(n-s)}$$

and

$$p_{(r)} = \sum_{s=r}^{n} \binom{n}{s} p^s \, (1-p)^{(n-s)}$$

If it can be inverted, then we can write

$$p = BETAINV\left(p_{(r)},\ r,\ n-r+1\right).$$

From the last two expressions we get

$$Q_{(r)}\left(p_{(r)}\right) = Q\left(BETAINV\left(p_{(r)}, r, n-r+1\right)\right) \tag{10}$$

$BETAINV(\cdot)$ is a standard function in packages such as Excel. Thus, the quantiles of the order statistics can be evaluated directly from the distribution $Q(p)$ of the data. The quantile function thus provides the natural way to simulate values for those distributions for which it is an explicit function of *p* [2].

D. Simulation of the extreme values

In a number of applications of quantile functions in general insurance interest focuses particularly on the extreme observations in the tails of the data. Fortunately it is possible to simulate the observations in one tail without simulating the central values. We will present here how to do this.

Consider the right-hand tail. The distribution of the largest observation has been shown to be $Q\left(p^{1/n}\right)$. Thus the largest observation can be simulated as $x_{(n)} = Q\left(u_{(n)}\right)$, where $u_{(n)} = v_n^{1/n}$ and v_n is a random number from interval [0, 1]. If we now generate a set of transformed variables by

$$\begin{aligned} u_{(n)} &= v_n^{1/n} \\ u_{(n-1)} &= \left(v_{n-1}\right)^{\frac{1}{n-1}} \cdot u_{(n)} \\ u_{(n-2)} &= \left(v_{n-2}\right)^{\frac{1}{n-2}} \cdot u_{(n-1)} \end{aligned} \tag{11}$$

where v_i, $i = n, n-1, n-2, ...$ is simply simulated set of independent random uniform variables, not ordered in any way. It will be seen from their definitions that $u_{(i)}$, $i = n, n-1, n-2, ...$ form a decreasing series of values with $u_{(i-1)} < u_{(i)}$.

In fact, values $u_{(i)}$ form an ordering sequence from a uniform distribution. Notice that once $u_{(n)}$ is obtained, the relations have the general form

$$u_{(m)} = \left(v_m\right)^{\frac{1}{m}} \cdot u_{(m+1)}, \quad m = n-1, n-2, ...$$

The order statistics for the largest observations on *X* are then simulated by

$$\begin{aligned} x_{(n)} &= Q\left(u_{(n)}\right) \\ x_{(n-1)} &= Q\left(u_{(n-1)}\right) \\ x_{(n-2)} &= Q\left(u_{(n-2)}\right) \\ &\vdots \end{aligned} \tag{12}$$

In most simulation studies of *n* observations are generated and the sample analyses *m* times to give an overall view of their behavior. A technique that is sometimes used as an alternative to such simulation is to use a simple of ideal observations, sometimes called a *profile*. Such a set of ideal observations could be the medians M_r, $r = 1, 2, ..., n.$.

III. Problem Solution

The publication [16], Swiss Re Sigma No 2/2015 in Table 10, page 41, provides data about 40 the most costly insurance catastrophic losses (1970- 2015) in million USD, 2014 prices. These data are the basis for our modelling and simulation of a few the highest values of catastrophe losses. The data are ranging from 3410 to 78 638 million USD in 2014 prices (Table 1).

Table 1: 40 the most costly insurance losses (1970-2015)

3410	4818	7681	15783
3501	5125	8241	16157
3839	5426	8458	16836
3882	5740	8682	22258
3899	5780	8730	22355
4010	6134	9813	25104
4123	6449	10087	26990
4200	6456	11339	36079
4492	6959	12240	36828
4765	7418	15234	78638

Source: Swiss Re Sigma No 2/2015

A. *Simulation by Pareto Quantile Function*

First we want to verify whether the 2-parameters Pareto distribution defined by (1) fits the data in Table 1 adequately by selecting Goodness-of-Fit Tests in *Distribution Fitting* procedure of Statgraphics Centurion XV package [15]. The first step is parameters estimation by maximum likelihood method [3], [12]. The estimated parameters of the fitted Pareto distribution are shown in Table 2. According to our parameter markers by (1) - (3) can be written: *est* $a = 3410$ and *est* $b = 1.04777$.

Table 2 Parameters of Pareto Fitted Distribution

Pareto (2-Parameter)
b - shape = 1.04777
a - lower threshold = 3410.0

Table 3 shows the results of test run to determine whether the most costly insured catastrophe losses can be adequately fit by a 2-parameter Pareto distribution (1).

Since the smallest P-value = 0.858776 amongst the tests performed is greater than to 0.05 we do not reject the hypothesis that losses come from a 2-parameters Pareto distribution with 95% confidence.

Table 3 Results of Kolmogorov-Smirnov Test

	Pareto (2-Parameter)
DPLUS	0.0576431
DMINUS	0.0955203
DN	0.0955203
P-Value	0.858776

We can also by Quantile plot and Quantile-Quantile or *Q-Q* plot assess visually how well the 2-parameter Pareto distribution with ML estimated parameters in Table 2 fits the data.

The Quantile Plot (Fig. 3) shows the fraction of observations at or below *x*, together with the cumulative distribution function of the fitted distribution. To create the plot, the data are sorted from smallest to largest and plotted at the coordinates. Ideally, the points will lie close to the line for the fitted distribution, as is the case in the plot at Fig. 3.

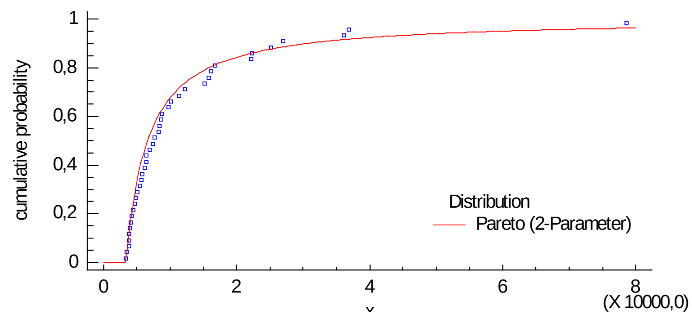

Fig.3 Quantile plot

The Quantile-Quantile plot (Fig. 4) shows the fraction of observations at or below *x* plotted versus the equivalent percentiles of the fitted distribution. The fitted Pareto distribution has been used to define the x-axis. The fact that the points lie close to the diagonal line confirms the fact that the Pareto distribution provides good fit for the data, but deviates away from the data at the highest values of *x*. Evidently,, the tail of the Pareto distribution is too fat.

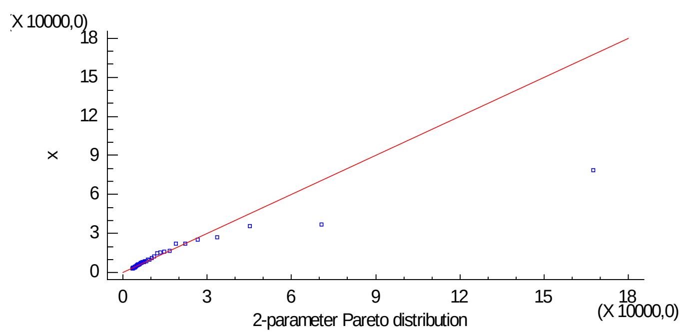

Fig.4 Quantile-Quantile plot

Table 4 *CDF* of the Pareto (2-parameters) distribution

x	*Lower Tail Area (<)*	*Upper Tail Area (>)*
10000	0.676084	0.323916
50000	0.940011	0.0599889
100000	0.970983	0.0290175
200000	0.985964	0.0140362
300000	0.990822	0.00917793

Table 4 shows the values of the cumulative distribution function at 5 selected values of *x*. *Lower tail area* is the probability that the catastrophe insured losses are less than or equal to *x*, *upper tail area* is the probability that losses are greater than *x*. So for example the probability that the catastrophe losses exceed the value of 200 000 million USD is 0,014, or so about 1.4% world catastrophe losses exceed 200 000 mil. USD.

Table 5 contains the selected percentiles of the Pareto distribution, which is well fitted model for the most costly insured catastrophe losses.

Table 5 Selected quantiles of the fitted Pareto distribution

Lower Tail Area (<=)	*Pareto (2-Parameter)*
0.75	12804.5
0.8	15843.6
0.9	30701.5
0.95	59492.8
0.99	276417.0

If will not change conditions of the occurrence of these events on the globe, will not change even their distribution. Then 20% of the most costly insurance losses will exceed 15843.6 million USD, 10% will exceed 30701.5 million USD, 1% will exceed 276 417 million USD.

Knowing the probability model and its parameters, we can use quantile function (3) and by simulation procedure described in part II-D we can simulate a few, for example five the highest possible values among 40 the most costly insurance world catastrophe losses.

Table 6 presents the steps of simulation by (11) and (12) the highest five possible values (in million USD) in the world natural catastrophes which we can find in the last column denoted as $Q(u)$. So the highest simulated loss is 82 421.36 million USD, the second highest is 70505.06 million USD etc.

Table 6 Process of simulation $Q(u)$ by Pareto quantile function

v	n	$v^{1/n}$	u	$Q(u)$
0.23549	40	0.964494	0.964494	82481.36
0.77309	39	0.993423	0.958150	70505.06
0.55488	38	0.984619	0.943413	52865.58
0.90776	37	0.997388	0.940949	50758.02
0.33132	36	0.969781	0.912514	34880.10

The first column in Table 7 contains the medians and two last columns show the boundaries for each order statistic. For example the highest possible insured loss is with probability 0.95 from value 24 991.87 million USD to value 18 066 831.58 million USD and 0.5% of losses may even exceed the value of 18 066 831.58 million USD if losses are by Pareto distributed.

Table 7 Quantiles of selected order statistics

Q(BETAINV(0.5))	Q(BETAINV(0.995))	Q(BETAINV(0.005))
164 921.29	18 066 831.58	24 991.87
70 901.33	993 661.54	18 346.01
45 453.34	318 235.24	15 002.34
33 581.25	163 674.11	12 884.65
26 690.92	103 499.70	11 390.54

Visualized results of the simulation process by 2 Parameters Pareto distribution with parameters in Table 2 we can see at Fig.5 and Fig.6.

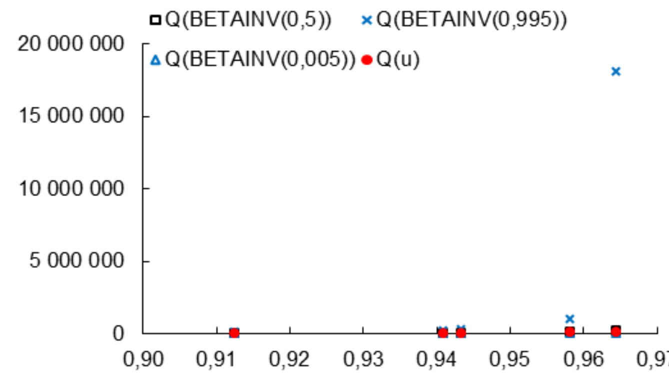

Fig.5 Graphical results of simulation of five the most costly insurance losses using Pareto *QF*

Because the largest value of Q(BETAINV(0.995)) on Fig.5 is too high compared to other values, Fig.6 does not have this value.

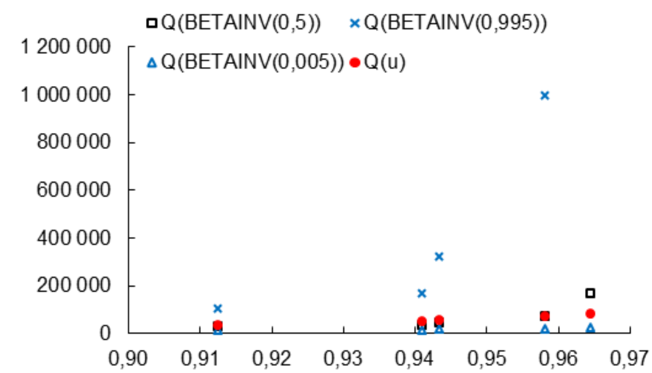

Fig.6 Graphical results of simulation of five the most costly insurance losses except the highest one using Pareto *QF*

B. Simulation by Weibull Quantile Function

Analogous procedure of probability modelling and extreme losses simulation as using Pareto quantile function in part III-A we have repeated for the Weibull 3-parameters distribution (4).

Table 8 Parameters of Fitted Weibull Distribution

Weibull (3-Parameter)
α - shape = 0.723827
β - scale = 7185.26
θ - lower threshold = 3410.0

The estimated parameters of the fitted Weibull distribution are shown in Table 8. The results of Goodness-of-Fit test that the 3-parameters Weibull distribution fits the losses adequately has shown Table 9. Since the smallest P-value = 0,984947 is greater than to 0.05, we do not reject the hypothesis that data in Table 1 come from a 3-parameters Weibull distribution with 95% confidence.

Table 9 Results of Kolmogorov-Smirnov Test

	Weibull (3-Parameter)
DPLUS	0.0723181
DMINUS	0.0719318
DN	0.0723181
P-Value	0.984947

We can also assess visually that the 3-parameters Weibull distribution with parameters in Table 8 fits very well to the most costly insured catastrophe losses (Fig. 7, Fig. 8).

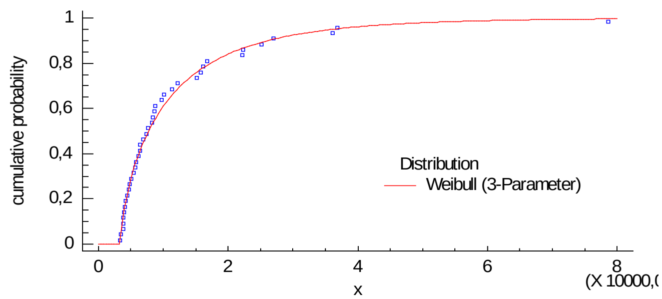

Fig.7 Quantile plot

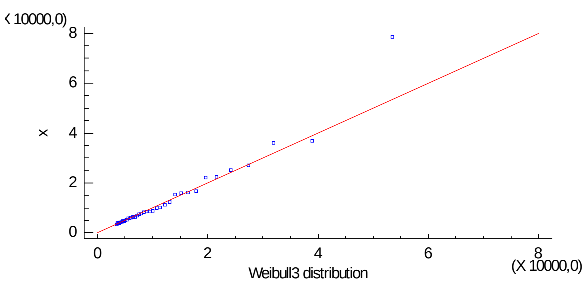

Fig.8 Quantile-Quantile plot

Comparing the P-Values in Table 3 and Table 9 and *Q-Q* plots at Fig. 4 and Fig. 8 we can observe better fit of the Weibull distribution on empirical data in Table 1, even on the upper tail of distribution. The fact that the Weibull distribution not overestimates the largest insured catastrophe losses is positive for simulation of the highest possible insured catastrophe losses.

Table 10 *CDF* of the Weibull (3-Parameters) distribution

X	*Lower Tail Area (<)*	*Upper Tail Area (>)*
10000.0	0.609108	0.390892
20000.0	0.839985	0.160015
50000.0	0.979128	0.0208718
60000.0	0.98837	0.0116304
100000.0	0.998583	0.00141745

Table 10 presents tail areas for the fitted 3-parameters Weibull distribution. Comparing with Table 4 the output indicates that the upper tail of the Weibull probability model is not so fat as it is in case of Pareto distribution.

Table 11 presents selected high quantiles for the fitted 3-parameters Weibull distribution. For example, the output indicates that the value of the fitted Weibull distribution below which we would find an area equal to 0.99 is 62 668.1million USD, which is much lower value than is the same quantile of Pareto distribution in Table 5.

Table 11 Selected quantiles of the fitted Weibull distribution

Lower Tail Area (<=)	*Weibull (3-Parameter)*
0.75	14692.9
0.8	17276.7
0.9	26153.7
0.95	36125.5
0.99	62668.1

By the same sequence of the steps as mentioned in section II-*D* and was applied for the Pareto distribution in section III-*A* we have simulated five the highest possible insured catastrophe losses using quantile function of the fitted Weibull distribution. The Table 12 obtains the results of the simulation and Fig. 9 presents the results of simulation in graphical form.

Table 12 Process of simulation $Q(u)$ for Weibull distribution

v	n	$v^{1/n}$	u	$Q(u)$
0.23549	40	0.964494	0.964494	41400.19
0.77309	39	0.993423	0.958150	38840.09
0.55488	38	0.984619	0.943413	34273.20
0.90776	37	0.997388	0.940949	33642.14
0.33132	36	0.969781	0.912514	27998.02

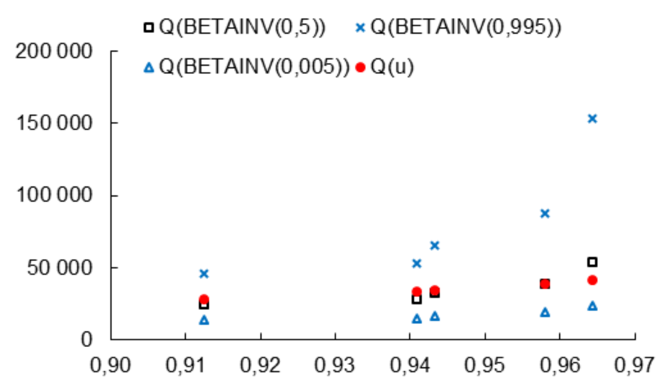

Fig.5 Graphical results of simulation of five the most costly insurance losses using Weibull *QF*

IV. Conclusion

The worldwide insurance industry has been rocked by the increasing catastrophes in recent years and increased demand for catastrophe cover (e.g., per occurrence excess of loss reinsurance), leading to a capacity shortage in catastrophe reinsurance. Catastrophe events in last years are associated with increases in premiums for some lines of business. These market developments are particularly important for non-proportional reinsurance because this coverage is designed to cover the tail of the loss distribution and is triggered only when losses are unexpectedly high.

Modelling the loss distributions in non-life insurance is one of the problem areas, where obtaining a good fit to the upper tail is of major importance. That is of particular relevance in non-proportional reinsurance if we required choosing or pricing a high-excess layer. Long tailed distributions as Pareto or Weibull play a central role in this matter and an important role in quotation in non-proportional reinsurance.

The results of the probability modelling and simulations based on 40 the most costly insurance losses in the world natural catastrophes in time period 1970-2014 using Pareto and Weibull quantile functions provide valuable information for insurance and non-proportional reinsurance of catastrophe losses. These results may be useful for setting priorities and premiums in non-proportional reinsurance or in case of Largest Claims Reinsurance LCR(5).

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