Fuzzy stochastic approaches for analysing the serviceability limit state of load bearing systems

Zdeněk Kala, Abayomi Omishore

***Abstract*—** Different approaches for determining the degree of uncertainty in models for the analysis of the serviceability limit state of building structures are described. An example of fuzzy and stochastic analysis of the uncertainty of deformation of a load bearing steel truss structure is presented. The relationships between the force acting on the structure and the elastic deformation of the structure caused by this force are studied using numerical simulations on the computer. The static response and elastic deformation are evaluated using linear and geometrically nonlinear models. It is proven in the article that small uncertainties in the initial geometric imperfections can lead to relatively large uncertainty in the response of the model, on the basis of which the safety and reliability of the structure are assessed.

***Keywords*—**Stochastic, fuzzy, mathematical modelling, simulation, imperfection, reliability, serviceability, static, elastic.

I. Introduction

Theoretical research of limit states of load bearing elements is the basis for reliable design of structural systems of buildings [1, 2]. Safety and reliability are studied using static and dynamic models and numerical simulation of the response of building structures on computers [3]. Researchers create new mathematical models [4,5] and with the help of computer simulations investigate the structural and mechanical properties of new building materials [6-9].

Steel structures are fully competitive with concrete or wood, especially with regards to their usability as bearing elements of steel bridges or towers [10]. Steel does not crack due to settlement or creep like concrete. Steel is a fully recyclable material. Steel elements are highly variable and structures constructed from them are lightweight and allow for great flexibility in creating the layout and overall appearance of the building. Transportation, assembly and putting into operation of steel structures tend to to be very short.

Truss structures are of numerous types, an almost infinite number of shapes, and are preferred for bridging large spans [11]. Steel truss beams are suitable whenever it is necessary to realize openings in structures. Steel towers are used as supporting structures of high tension wires, wind turbines, billboards, etc. Transportation of the tower is not a problem because assembly can take place at the construction site. Another advantage of truss towers is that they are transparent. On the other hand, bars of steel truss structures are slender, which requires that increased attention has to be paid to loss of stability and initial imperfections [10].

Stability phenomena affect the limit states of slender members that are partly or entirely in compression. Research aimed at the sensitivity analysis of initial imperfections on the load carrying capacity helps to identify the effects of initial imperfections on the reliability of load bearing systems [12-15]. Initial imperfections from production are measured [16] and compared with tolerance standards so that the construction of structures can be realized from high quality elements. The probability of failure, admissible according to standard EN1990, is maximally 7.2E-5 [17].

The limit states of the Eurocode standards are usually verified using probability based reliability studies [18-20]. The reliability of steel structures is very sensitive to initial imperfections. Small initial curvature of the member axis [21] or deviation from the verticality of the load bearing system [22] can greatly affect the stress and reduce the load carrying capacity.

The ability of the bent beam to develop a plastic hinge is the most important precondition for the formation of the load carrying capacity reserve during permanent deformation, which is a phenomenon studied by the shakedown theory [23]. New practical applications of the shakedown theory, which replaces the exhausting conventional iterative analysis, were presented in [24].

Researches on the limit states of steel columns exposed to extreme loading were published, e.g. in [25]. Generally, permanent deformation due to exceptional loading states is undesirable and collapse occurs more frequently due to fatigue from repeated loading. Fatigue is a typical failure of walled load bearing elements. Initial unplaneness of slender walls is a frequent cause of the initiation and propagation of fatigue cracks in walled load bearing elements [14].

The theory of mathematical statistics and probability or the theory of fuzzy logic are the general basis for the analysis of the reliability and limit states of load bearing structures. Numerous optimization problems can be solved using soft computing techniques, see, e.g. [26-29]. Generally, the reliability analysis of structures constitutes a significant part of

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operational research, whose sub-disciplines include the analysis of decision-making problems based on multiple criteria [30-32]. The appropriate division of complex problems taking into account multiple criteria explicitly results in more informed and better decision-making.

The serviceability limit state is generally studied using stochastic methods on one element [18]. This approach however can not be considered sufficient, because the limit deformation in design standards are determined subjectively. The theory of fuzzy sets [33] and stochastic analysis based on virtual simulations [34, 35] are used in this article to study the serviceability limit state of a truss structure. The fuzzy and stochastic approach do not compete, but rather are in a partnership, in which each of the partner contributes a different methodology for solving the reliability problems of an interactive system [36].

II. THE BAR SYSTEM

The presented example illustrates a typical problem in which the input variables of the computational model are burdened with uncertainty that can not be eliminated by measuring, but is always partially reflected in the decision-making process of engineers.

A. *Geometry of the truss structure*

Fig. 1 shows a system comprised of six members. Changes in the stiffness of the structure caused by deviations of the cross-sectional area are simulated in the analysis of the resistance of the system to force F, see Fig. 2. The real cross-sectional area may differ from the nominal value as a result of the technology of rolling in the smelting plant, weakening due to corrosion during operation of the structure or as a result of numerous other factors.

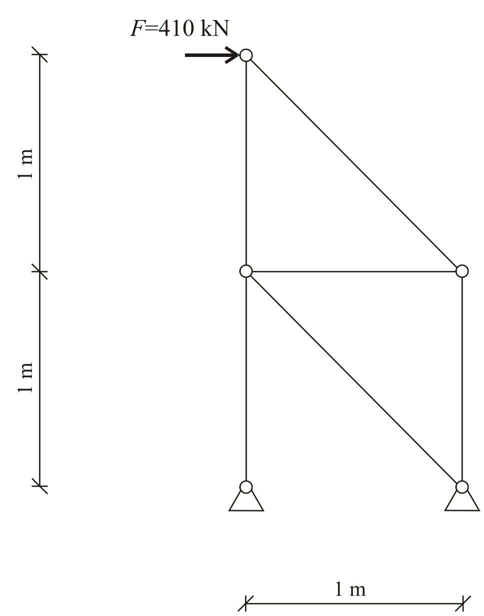

Fig. 1 geometry of the truss structure

B. *Cross-section geometry*

Fig. 2 illustrates a hot-rolled hollow steel cross-section. This cross-section is either only under compression or tension in the truss structure depending on which member is analysed. It is a European cross-section, which is classified in the Eurocode 3 standard as a Class 1 cross-section. Class 1 cross-sections are those that can form a plastic hinge with the rotation capacity required for plastic hinges.

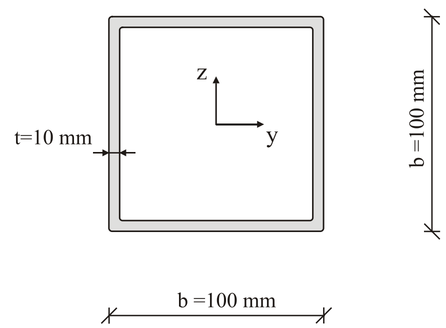

Fig. 2 rolled hollow cross-section

The cross-section illustrated in Fig. 2 je is a Class 1 cross-section, which is defined in the Eurocode 3 standard for member subject to compression by the relation

$$c / t \leq 33\varepsilon \tag{1}$$

where $c \approx b\text{-}2t$=100-2·10=80, ε=1.0 for steel grade S235 with yield strength f_y=235 MPa. Local stability phenomena such as buckling of slender compressed walls can be neglected for Class 1 cross-sections. Since there is no need to model local stability, there is also no need to analyse and model the shapes of initial imperfections of slender walls and solve loss of stability of the members in interaction with their buckling [37].

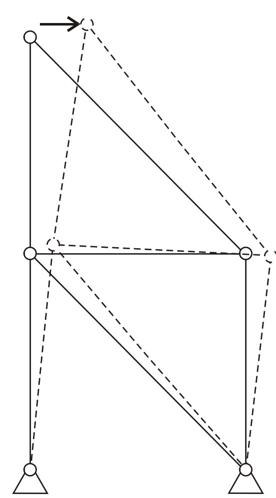

Fig. 3 horizontal deformation of the truss structure

III. BEAM FINITE ELEMENT ANALYSIS

The serviceability limit state of the truss structure was studied using the beam finite element (FE) method. Deformation at the point of load action was the monitored output variable whose uncertainty was analysed.

A. Linear FE Analysis

Linear static analysis of the dependence of the deformation on the load was evaluated first, see Fig. 3. The force of 410 kN produces an axial stress of 234.96 MPa in the left column. Stress in the other bars are lower. The limit value of stress in the steel was considered acc. to [38] as 235 MPa. The truss structure is designed economically with maximum utilization and does not exceed the ultimate limit state of the standard [38].

B. Nonlinear FE Analysis

High sensitivity to changes in the geometry can lead to different dependencies of load vs. deformation, which must be studied using the geometrically nonlinear FE solution [39]. The basic difference is that the equilibrium conditions are written for the structure with deformations from the load and not for the unloaded structure. The application of the geometrically nonlinear solution to study a truss structure was published in [40]. This solution has been applied in numerous reliability studies, see, e.g. [21, 41]. The basis of the geometrically nonlinear solution is increasing the load step by step and eliminating errors of the solution using Newton-Raphson incremental method. The load was divided into sixty incremental steps.

C. Stability analysis

Stability analysis is part of the linear FE analysis in which it is necessary to check the reliability of the compressed bars and their resistance to buckling. Stability analysis is used to determine the size of the load at which loss of stability occurs. A particularity of the truss structure shown in Fig. 1 is that buckling of the diagonal members occurs simultaneously. The bar buckles at the moment the determinant of the matrix of the tangential stiffness of the nonlinear solution changes sign. If two bars buckle simultaneously, change in sign does not occur. It is therefore necessary to introduce slightly different stiffness parameters of the diagonal bars to ensure that a change in sign occurs. To ensure the stability analysis, it suffices to specify the stiffness parameters of the diagonal bars with such little difference that the values of the buckling loads F_{cr1} and F_{cr2} differ only by 4 N. For the real buckling load of an ideal structure without imperfection, it must hold that $F_{cr1}=F_{cr2}\approx 3385.44$ kN, see Fig. 4.

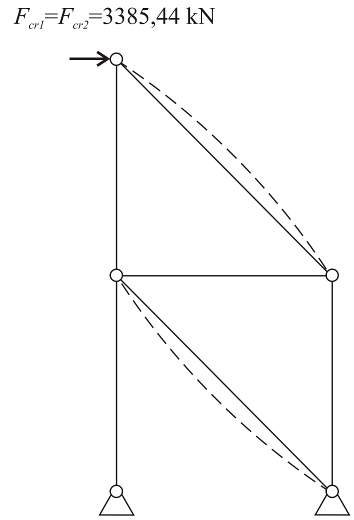

Fig. 4 first and second buckling modes

The right column buckles at the moment the force reaches magnitude F_{cr3}, see Fig. 5. It may be noted that the forces F_{cr1}, F_{cr2}, F_{cr3} are related to the buckling of the bars without relation to the stress, which would be very high from the critical forces. From the comparison of the values of F_{cr1} and F_{cr2} with F=410 kN it is apparent that buckling of the diagonal members occur when the load is more than eight times greater than load F. The fact that forces F_{cr1}, F_{cr2}, F_{cr3} are much higher than force F is an important finding that strength and not the stability of steel is decisive for safety and reliability.

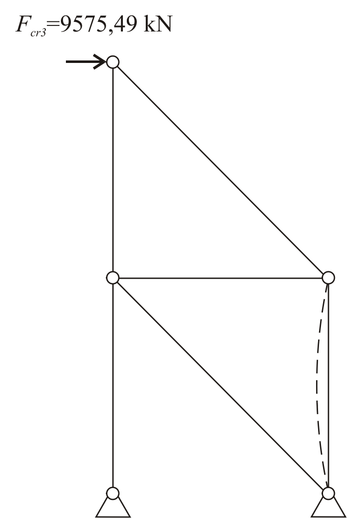

Fig. 5 the third buckling mode

Buckling of the diagonal members is of greater significance than buckling of the right vertical column in the stability analysis. The buckling length of each diagonal member is 1.414 m. The slenderness of each diagonal member is 38.869, which is relatively small. Therefore, it can be concluded that the effect of stability phenomena on the behaviour of the truss structure is relatively small.

IV. Serviceability Limit State

The serviceability limit state is defined by the maximum allowable deformation. If the deformation from the load is greater than the allowable deformation, the structure is classified as only for limited use or as unusable. The permissible deformations are specified in standards, the authors of which are groups of people in social organizations, such as the standards developing organization. The structure is satisfactory if the maximum deflection, which is obtained from static analysis, is smaller than the deflection specified by the standards.

The maximum allowable deformations in standards are specified subjectively and correspond to the purpose for which the structure was constructed. The serviceability limit state thus differs significantly from the ultimate limit state, which is determined by the physical properties of structures, such as yield strength, Young's modulus. The physical properties of materials are influenced by man, especially the chemical composition and production technology. Nevertheless, the realization of these properties are random variables.

While stochastic analysis is sufficient to study the ultimate limit state, the serviceability limit state can be studied using both stochastic and fuzzy analysis. Given the subjectivity of the maximum allowable deformations, fuzzy approaches are fully relevant for evaluating the usability of structures. In advanced problems both types of uncertainties can be assigned according to their relevance to the studied problem [42].

V. Fuzzy Analysis

The notion of fuzzy logic was first presented in 1965 by professor Lotfi A. Zadeh in the article [33], where the basic concept of fuzzy logic, i.e. fuzzy sets, was defined. The term fuzzy can be defined as unclear, blurred, hazy, uncertain, or vague. This is fully reflected in what the theory of fuzzy set deals with, i.e. describing reality with its imprecisions and uncertainties.

Fuzzy description is well suited for the description of the vagueness with which we determine structural deformation using FEM computational models. The deformation can be evaluated as a fuzzy number, which is assigned a truth value in the range of 0 to 1, inclusive of both limits. Fuzzy logic allows in connection with the word "deformation" the mathematical expression of terms like "little", "enough" or "a lot", etc. Specifically, it allows us to express partial membership to a set. Fuzzy logic uses a degree of membership as a mathematical model of vagueness, while probability is a mathematical model of unfamiliarity.

A. Fuzzy numbers as input quantities

Fuzzy uncertainties of the cross-sectional areas of the members of the truss structure were considered during the analysis. The cross-sectional area A_i of the i-th bar was introduced as a fuzzy number, see Fig. 6.

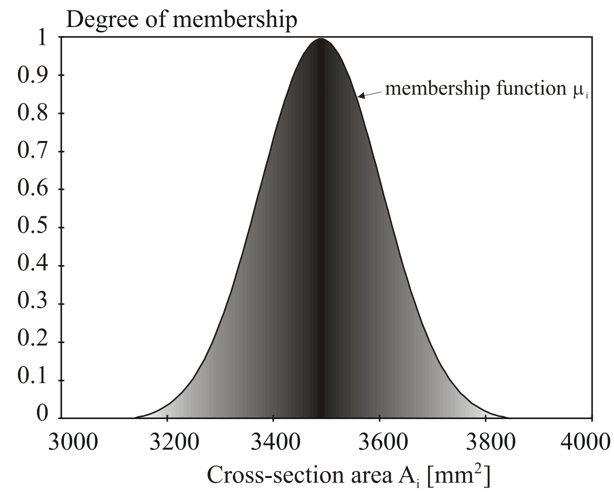

Fig. 6 fuzzy number of cross-section area A_i

Fig. 6 illustrates the fuzzification of the cross-sectional area, which was performed with consideration to deviations of the hot-rolled cross-section from experimental research [16]. The kernel of the fuzzy number in Fig. 6 is the nominal value of the cross-sectional area 3490 mm^2 as is specified by the manufacturer. The degree of membership of the fuzzy number shown in Fig. 6 defines the degree of truth with which the cross section is the hot-rolled hollow cross-section depicted in Fig. 2. If the cross-sectional area is less than 3136 mm^2 or on the other hand greater than 3841 mm^2 then the degree of truth that the cross-section is the cross-section depicted in Fig. 2 is zero. On the other hand, if the cross-sectional area is in the interval of 3136 mm^2 to 3841 mm^2, then it is with a greater or lesser degree of truth the cross-section depicted in Fig. 2.

B. Fuzzy Analysis numbers as input quantities

Fuzzy analysis was performed using the general extension principle [42]. The general extension principle is one of the most basic concepts of the fuzzy set theory, put forward by prof. L. Zadeh to enrich the theory [33]. It is used to extend operations and functions originally defined as functions of real variables in fuzzy sets. Let μ_i be the membership function of fuzzy number A_i. Then the membership function μ_u of fuzzy number u is defined as

$$\mu_u = \bigvee_u \left(\mu_1(A_1) \wedge \ldots \wedge \mu_i(A_i) \wedge \ldots \wedge \mu_n(A_n)\right) \tag{2}$$

The symbol $\bigvee$ in equation (2) is defined as the supremum. The supremum in (2) was introduced as an alternative to the concept of the greatest element, however, unlike the greatest element it can be found in more sets, for e.g., open intervals of real numbers do not contain a greatest element but have a supremum. If a set of real numbers has a maximum, then it also has a supremum, for which it holds that the maximum is equal to the supremum. The supremum can be understood as the generalisation of the concept of the greatest element not only on the set of real numbers, but generally in all sets.

The input fuzzy numbers of cross-sectional areas are depicted in Fig. 6. It was assumed that the fuzzy uncertainty of each cross-section is the same for all bars. Displacement *u* of the top hinge of the truss structure was the considered output variable, see Fig. 7. Since the cross-sectional areas are fuzzy numbers the displacement is also a fuzzy number.

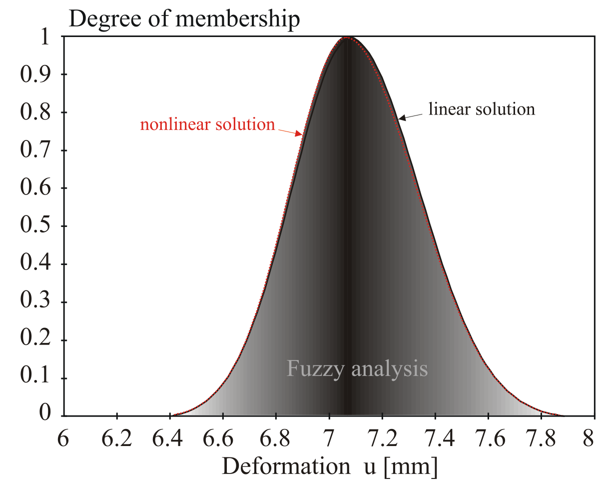

Fig. 7 fuzzy number of maximum horizontal displacement *u*

The so-called α–cuts were used to evaluate equation (2). On each α–cuts conditions of maximum and minimum of the realizations of the output displacement resulting from all possible combinations of input values were performed. All material and geometric characteristics, apart from the cross-sectional areas, were considered in the computational model by their nominal and characteristic values listed in the Eurocode 3 and other related standards.

Displacement *u* was calculated using the linear solution and the geometrically nonlinear solution. Initial imperfections of the members were taken into consideration in the geometrically non-linear solution. Initial curvatures of the compressed members were introduced in the shape of a half-wave of the sine function acc. to Fig. 4 and Fig. 5. The amplitudes of the initial curvatures of member axes were considered as one-thousandth of the member length, which is a common approach applied, for e.g., in [19-22].

It is apparent from Fig. 7. that the membership functions obtained from the linear and non-linear solution differ very little. Whilst the membership function in Fig. 6 is symmetrical, the output membership functions of *u* in Fig. 7 are not symmetrical. The fuzzy number is more blurred to the right of the kernel and the fuzzy set includes relatively higher values of deformation, which may be relevant when assessing the serviceability limit state.

This description of uncertainty, which normally would not be obtained from the deterministic study provides valuable insight into the behaviour of the structure in terms of the uncertainty in evaluating its usability.

VI. Stochastic Analysis

The second approach in analysing the uncertainty of deformation is based on the stochastic analysis of the influence of input random variables on the output. Each cross-sectional area was considered as a random variable with Gauss probability density function with mean value 3490 mm^2 and standard deviation 180 mm^2. These statistical characteristics mean that 95 realizations are found in the interval from 3136 mm^2 to 3841 mm^2, which is in agreement with the results of experimental research [16].

Analysis was performed using the Latin Hypercube Sampling method [34, 35]. Statistical analysis of the deformation was performed using ten thousand simulation runs. Nine input variables with random variability of cross-sectional area described in the preceding paragraph were used. Ten thousand observations of the deformation obtained from the linear FEM solution are depicted in Fig. 8 and Fig. 9.

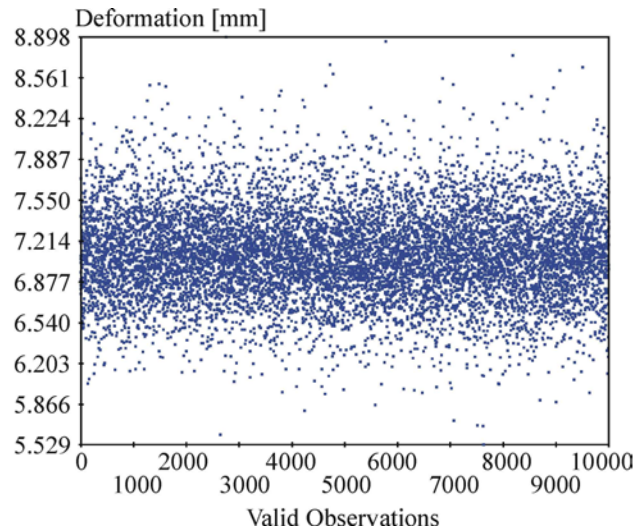

Fig. 8 runs of deformation from linear solution

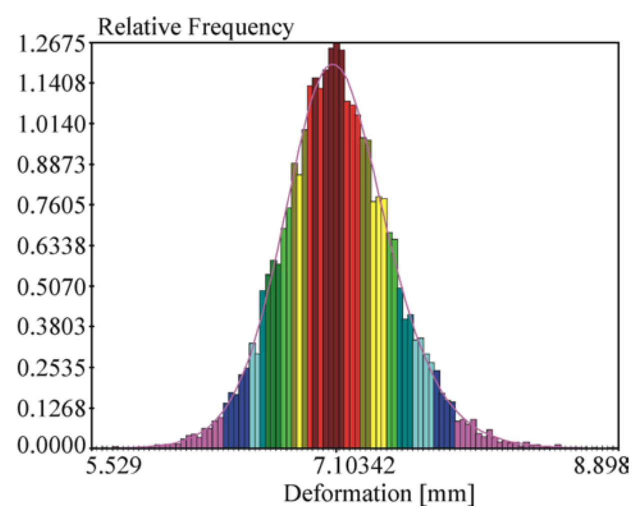

Fig. 9 histogram of deformation from linear solution

In the next study the deformation was calculated using the geometrically nonlinear solution. The aim of this analysis was to validate the assumption that the equilibrium conditions of these structures can be written for the unloaded structure. Ten thousand observations of the deformation obtained from the nonlinear FEM solution are depicted in Fig. 10 and Fig. 11.

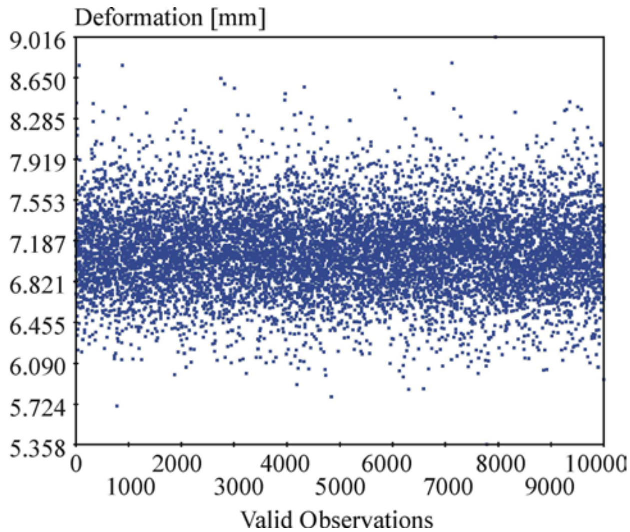

Fig. 10 deformation obtained from nonlinear solution

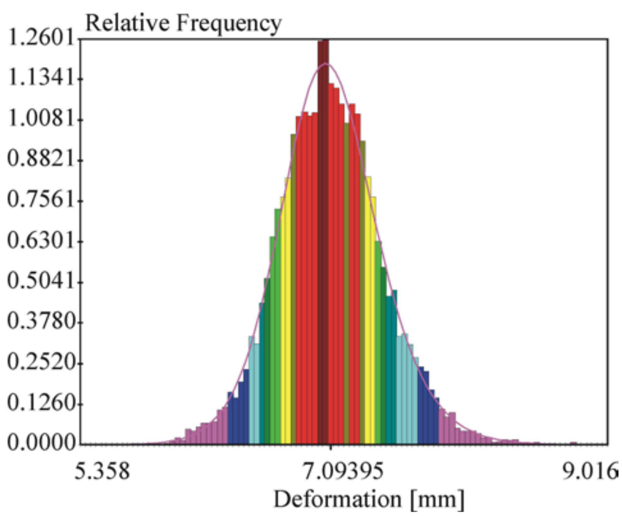

Fig. 11 histogram of deformation from nonlinear solution

Fig. 12 shows a comparison of the results of the linear and nonlinear solution approximated using two types of probability density functions. Hermite density function is a four-parameter distribution capable of taking into account the effect of skewness and kurtosis in the set of random realizations. Hermite probability density function was used for approximations in Fig. 9 and Fig. 11. The Gaussian approximation of results of the linear and nonlinear solutions are practically identical. The same is true for the Hermite approximations, see Fig. 12.

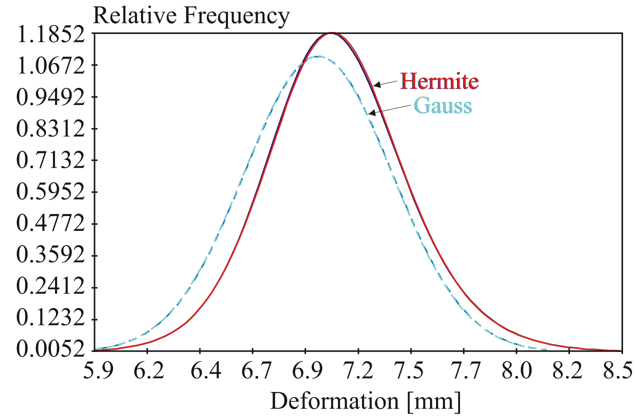

Fig. 12 approximation of Hermite and Gauss density functions

VII. Comparison of Fuzzy And Stochastic Analysis

The comparison of results of the fuzzy and stochastic analyses of the horizontal deformation of the top hinge is depicted in Fig. 13. Two scales, one for the relative frequency (stochastic analysis) and the other for the degree of membership (fuzzy analysis), were plotted to enable us plot two varying types of results. The advantage of fuzzy analysis is that the kernel corresponds to the result, which would be obtained for the minimum values of input fuzzy number A_i. The mean value obtained from the stochastic analysis is generally not equal to the kernel. The kernel is an approximate equivalent of the arithmetic mean of the statistical analysis with the specificity that it provides information on the result of the deterministic analysis with the degree of truth of one. Stochastic analysis does not have an equivalent of the fuzzy kernel. The advantage of stochastic analysis is particularly the possibility to evaluate quantiles from approximation functions of the probability density function. On the other hand, the fuzzy analysis leads directly to the output membership function of the horizontal deformation u and does not require seeking a suitable approximation function, as is the case for the statistical analysis based on simulation methods.

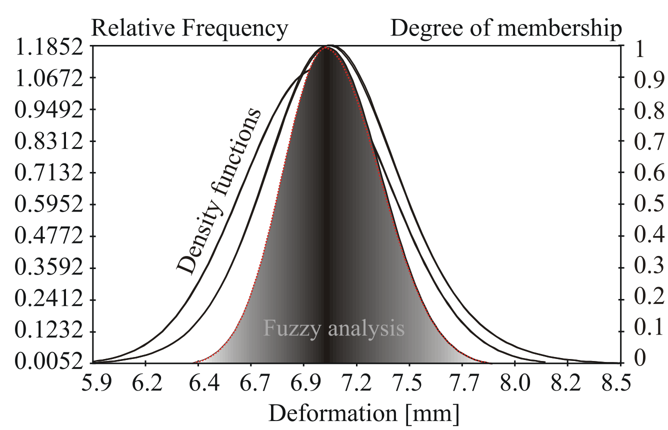

Fig. 13 Fuzzy and stochastic analysis of the deformation of the top hinge

VIII. Conclusion

The article deals with the fuzzy and stochastic methods of analysis of the serviceability limit state of building structures. The maximum deformation of the top hinge of the truss structure was evaluated as the output variable. Linear and nonlinear computer models based on the finite element method were used for the analysis. It was demonstrated that the influence of the applied linear and nonlinear solution was not as large as the influence of the uncertainty of cross-sectional areas. The presented study contributes to the discussions on the appropriateness of using fuzzy or stochastic approaches depending on the type of task. Fuzzy analysis is suitable in cases where the numerical values are determined subjectively. Stochastic analysis provides better results in cases where the uncertainty is purely random in character. Differentiation of fuzzy and random character of uncertainty of measurements is very difficult, as well as for all activities involving humans. Computational models are also very sensitive to parameters that are not random in nature, but without which the model cannot be adapted and used to find better solutions to practical problems, which arise in the sectors of optimization of the reliability of building structures.

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