# Error Reconstruction Functions of Non-Optimal Algorithms based on Atomic Functions

Yair Olvera, Vladimir Kazakov and Sviatoslav Africanov

***Abstract*—**A non-optimal algorithm based on atomic function is investigated for the Sampling – Reconstruction Procedure (SRP) of Gaussian random process realizations. The reconstruction error function of this non-optimal algorithm is compared with the reconstruction error functions obtained in the optimal algorithm, represented by the conditional mean rule, and in another non-optimal algorithm based on Balakrishnan's theorem. Results show that the application of atomic functions has disadvantages reflected in a larger magnitude of the reconstruction error.

***Keywords*—**Atomic functions, Conditional mean rule, Non-optimal algorithm, Reconstruction error function.

## I. Introduction

THERE have been several publications focused on the statistical description of the reconstruction of realizations that compose a random process through the set of their samples. This problem is called by the Sampling–Reconstruction Procedure (SRP). The most known work regarding this problem was done by A. Balakrishnan in his theorem (BT) [1]. He stated that any realization of a stationary random process $x(t)$ with a power spectrum $S(\omega)$ restricted by the boundary frequency $\omega_b$ can be reconstructed by:

$$\hat{x}(t)=\lim_{N\to\infty}\sum_{j=-N}^{N}x(T_j)\frac{\sin\omega_b(t-\Delta T_j)}{\omega_b(t-\Delta T_j)}, \qquad (1)$$

here, $x(T_j)$ is a sample in the time $T_j$, $\Delta T$ is the sampling interval determined by $\Delta T=\pi/\omega_b$, and $2N+1$ is the number of samples taking into account in the reconstruction algorithm. In (1) it is possible to establish a base function $\phi_j(t)$ for any sample $x(T_j)$. This base function is equal to the $\sin c$ function, and it has the form:

$$\phi_j(t)=\frac{\sin\omega_b(t-\Delta T_j)}{\omega_b(t-\Delta T_j)}=\sin c(\cdot). \qquad (2)$$

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A. Balakrishnan [2] wrote that "regardless of statistics, whether Gaussian or not, the best mean-square estimate of $x(t)$ from $x(T_j)$ is linear" and it is determined by (1). This point of view is practically not refuted in the literature. Furthermore, the reconstruction error is equal to zero for all types of the processes with a restricted power spectrum.

There may be observed some restrictions and disadvantages of Balakrishnan's theorem [3]: 1) It does not take into consideration the most important characteristic of each random process, which is its probability density function (pdf); 2) Its conditions are not feasible because the sampled process is singular, and the number of samples is equal to infinity; 3) The sampled process must be stationary; 4) Among many statistical characteristics of a sampled process, it limits its focus on a single numerical value: the boundary frequency $\omega_b$ of the power spectrum $S(\omega)$; 5) Other important statistic characteristics, like pdf, the covariance function $K(t_1,t_2)$ or the type of the power spectrum $S(\omega)$ where $(\omega<\omega_b)$, do not influence the outcome, whether in the reconstruction function or in the reconstruction error function; 6) It proclaims that the realizations of all types of stationary random processes can be optimally reconstructed by the unique linear algorithm with the base function $\sin c$; 7) Regardless which types of random process is considered, the reconstruction error function is equal to zero, and the reconstruction is based on the same function; 8) The formula (2) determines the impulse response of the linear non-realizable filter, and the summing procedure (1) demands infinite delay.

Over recent years, in order to overcome these drawbacks, a particular methodology has been extensively studied. It is based on the *conditional mean rule* (CMR). The application of this method in the statistical SRP description of random process realizations, provides a possibility to establish certain significant conclusions: 1) Every random process must have its own reconstruction algorithm and its own reconstruction error function; 2) BT is valid for Gaussian processes only; 3) The linear reconstruction algorithm is valid for Gaussian processes only; 4) If the Gaussian process has a limited spectrum and the number of samples is finite, the $\sin c$ function is not the optimal base function; 5) Within SRP description of Gaussian realizations, the reconstruction error function does not depend on the values of samples, but it depends only in concerning the axis of time; 6) BT is a particular case of the CMR algorithm for Gaussian processes with a finite spectrum.

Furthermore, an application of CMR provides the possibility of analyzing the SRP in the following cases: 1) The pdf of the sampled process is taken into account; 2) The number of samples can be arbitrary; 3) The sample location on the axis of time can be arbitrary; 4) The type of covariance function, or power spectrum can be arbitrary; 5) The sampled process can be non-stationary or stationary; 6) If the sampled process is continuous and non-Gaussian, its reconstruction function is a non-linear function regarding the set of samples, and its reconstruction error function depends on the values of the samples; 7) The sampled process can be an output process of any non-linear non-inertial converters; 8) The sampled process can be an output non-stationary process of some systems varying in time; 9) The sampled process can be a process with jumps; 10) The samples can be noised; 11) The samples can have some jitters; 12) The sampled Gaussian process can be multidimensional; 13) The algorithm can be applied in the SRP description of Gaussian fields and fields with jumps.

All the advantages mentioned above provide the possibility to apply the CMR algorithm in several SRP of random processes and fields. (See for instance [3]-[13].) The results from research are related with two main practical problems:

1) for the given characteristics of the random process and for a given set of samples, it is necessary to obtain a reconstruction function and to evaluate the reconstruction error function;
2) for the given characteristics of the random process and for the required reconstruction error function, it is necessary to obtain a reconstruction function and to find the sampling interval.

The conditional mean rule obtains its own *optimal reconstruction algorithm* and optimal reconstruction error algorithm for any random process [3]-[13].

Besides the optimal algorithm, there is a great amount of non-optimal algorithms. For instance, algorithms with some arbitrary base functions $\phi(\cdot)$ can be considered. There is another SRP non-optimal problem. This problem is related to choose a set of orthogonal functions which can be applied in the reconstruction algorithm of type (1).

The Khurgin-Yakovlev's theorem is an example of this [14]. The main idea of this methodology is related to the usage of a set of derivatives of a given function instead of the set of samples of the same function. Generally, one can use all derivatives of an arbitrary order $P$. The method is valid for functions with a restricted spectrum by the boundary frequency $\omega_b$. The statistical expression of this theorem is:

$$g(t)=\sum_{k=0}^{P-1}\sum_{n=-\infty}^{\infty} g^{(k)}(P_n\Delta T)(t-P_n\Delta T)^k\frac{\left[\sin c(q)\right]^k}{k!}, \qquad (3)$$

where $g^{(k)}(P_n\Delta T)$ is the derivative of the order $k$ of the function $g(t)$ with $\Delta \le 0.5\omega_b$, and

$$q=\frac{\pi(t-P_n\Delta T)}{P_n\Delta T}, \qquad (4)$$

$$\sin c(q)=\frac{\sin(q)}{(q)}. \qquad (5)$$

It is quite possible to use this algorithm for the SRP of random processes as well. However, its effectiveness is not exceptional because it is necessary to send all derivative values in a separate channel. The method using a set of samples of a given function is more productive because the reconstruction error is smaller (see [3] and citations there).

There are some generalizations of Sampling Theorem (1) on the basis of so called atomic functions [15]-[16], [18]-[20]. There are algorithms of Strang – Fix and Levitan. In the last variant one can introduce some trigonometric series in order to use it in the description of the SRP of various functions [13]. Below, the attention on the direct SRP algorithm based on *atomic functions* is focused. In the next paragraph, some initial information about such functions is given.

## II. About Atomic Functions

The theory of these functions was first described by V. L. Rvachyov [15]. The application of this theory in SRP problems was suggested for *deterministic functions* [16], [19]. However, it is possible to use it in problems considering random processes too.

The atomic functions are characterized by one principal property: the analytical expression for the derivative of any order of a function has the form of the analytical expression of the original function. Although the amplitudes, durations, and signs of the produced functions by the derivatives could be different. The atomic functions $h_\beta(x)(\beta>2)$ are the solutions of the differential equation [15]-[16], [18]-[20]:

$$y'(x)=\frac{\beta^2}{2}\left[y(\beta x+1)-y(\beta x-1)\right]. \qquad (6)$$

The Fourier transformation for the function $h_\beta(x)$ is:

$$F_\beta(\rho)=\prod_{k=1}^{\infty}\sin c\left(\frac{\rho}{\beta^k}\right). \qquad (7)$$

This function tends to zero at points $2\pi k;(k\neq 0)$. In Fig. 1 the functions $h_\beta(x)$ are presented for different values of $\beta$. While Fig. 2 shows the normalized first order derivative for each type of function expressed in Fig. 1. Fig. 3 displays the non-normalized derivatives of the function $h_2(x)$ only. In these graphs, the curves' forms of the different order derivatives are the same; however, the amplitudes, the durations and the signs vary. In [14], [15] more types of atomic functions are investigated in detail.

Any random realization has many ways to be reconstructed. This means that an alternative methodology can be used for the optimal methodology, accordingly to the properties of each realization. This parallel technique can hold some other statistical parameters than the ones employed by the optimal

technique to make the reconstruction. Due to this, the reconstruction is not adequate, hence it is called *non-optimal reconstruction algorithm*. Balakrishnan's theorem with the limited number of samples $N$, is an example of a non-optimal algorithm. Furthermore, the conditional mean rule algorithm can be considered as a non-optimal algorithm if it does not take into account the appropriate parameters. The main reason to apply such algorithms is to have a simpler methodology.

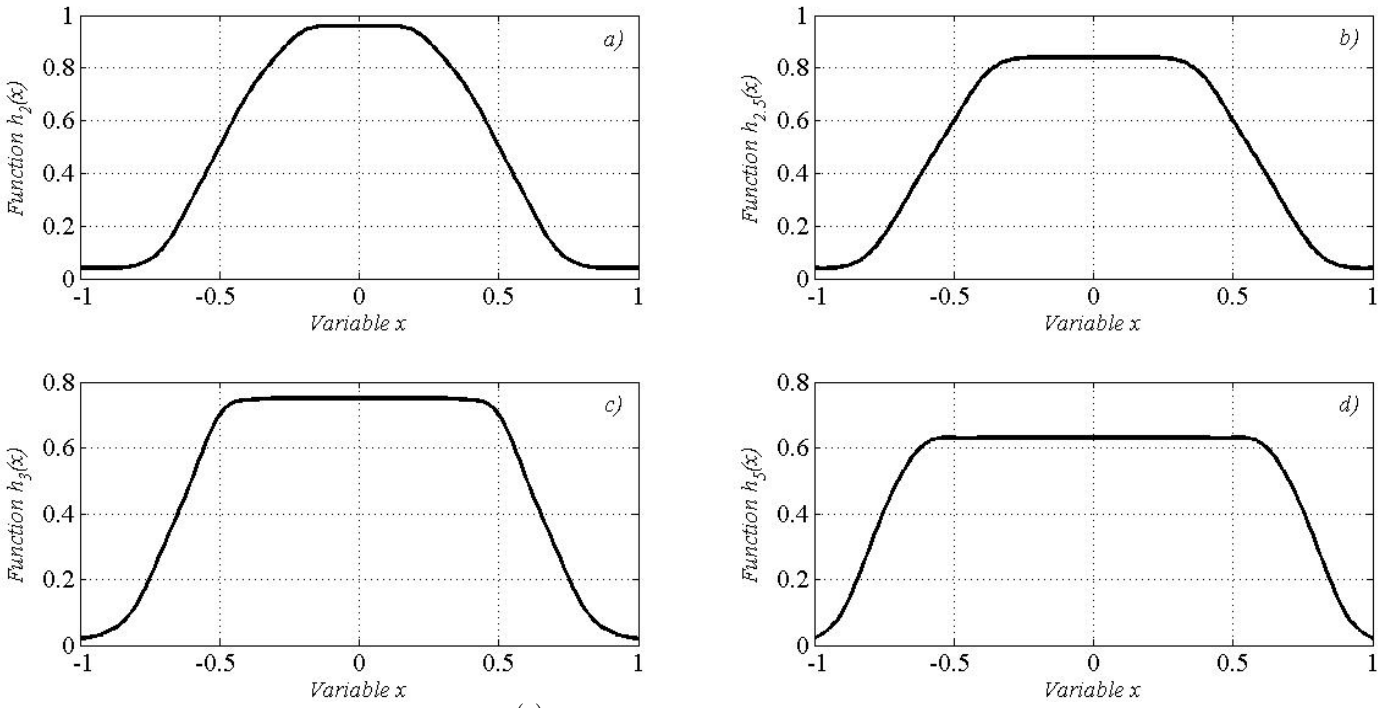


Fig. 1 Function $h_\beta(x)$ for: a) $\beta = 2$, b) $\beta = 2.5$, c) $\beta = 3$ and d) $\beta = 5$.

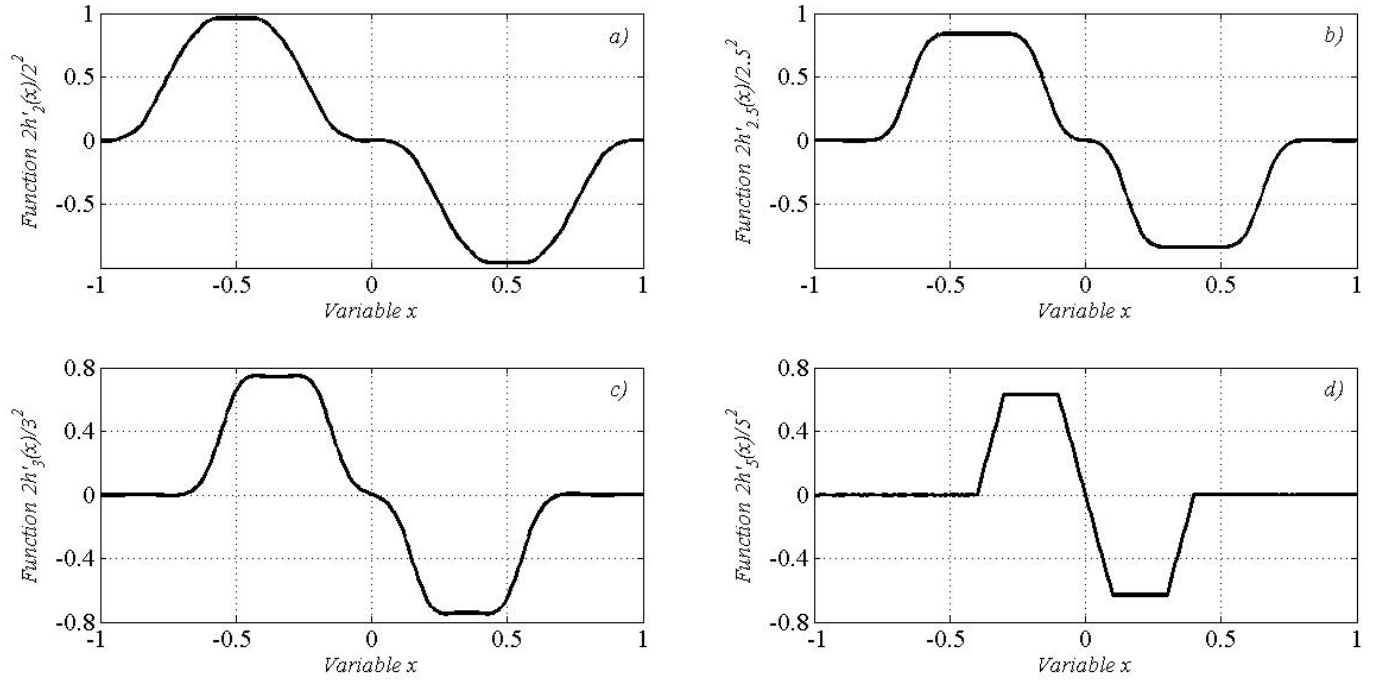


Fig. 2 Normalized derivative of the function $h_\beta(x)$ for: a) $\beta = 2$, b) $\beta = 2.5$, c) $\beta = 3$ and d) $\beta = 5$.

This document is dedicated to the investigation of a new non-optimal algorithm based on atomic functions. Besides, it is necessary to make a comparison of the reconstruction quality of the three most interesting variants: the optimal algorithm based on the conditional mean rule, Balakrishnan's non-optimal algorithm, and the non-optimal algorithm based on

atomic functions. The number of samples is finite in all cases.

The investigation is centered on the statistical SRP description of Gaussian random process realizations.

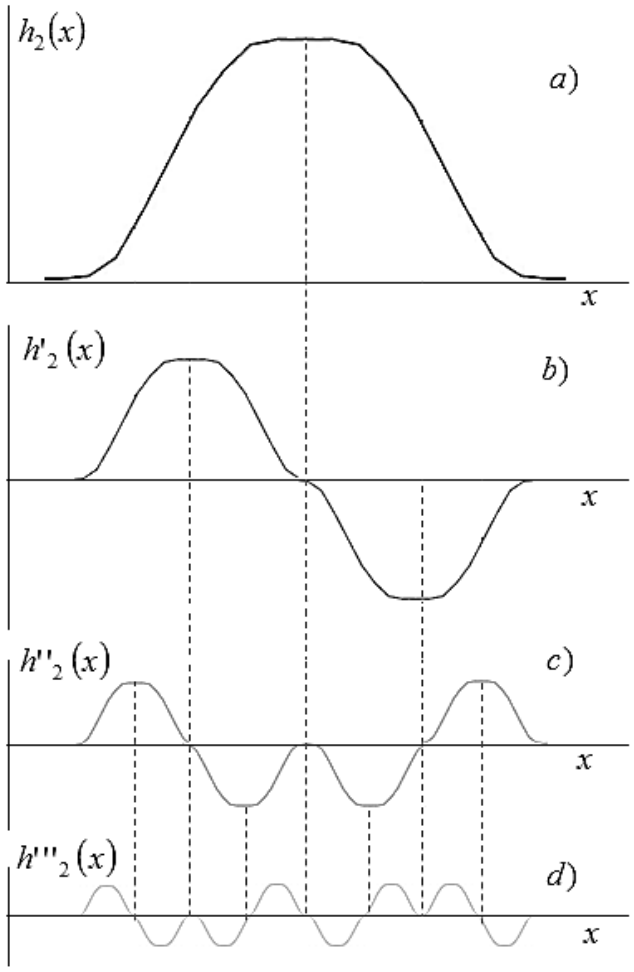


Fig. 3 Non-normalized derivatives of the function $h_2(x)$.

## III. The Optimal Reconstruction Algorithm

The optimal algorithm is based on the conditional mean rule. The reconstruction of all random process realizations can be obtained only by knowing their complete statistical description.

The main idea of this methodology has been proposed in [4] (see also [3], [7], [12]). First, a random process $x(t)$ characterized by its multidimensional probability functions $w_m[x(t_1), x(t_2),..., x(t_m)]$ has to be considered. One realization of this process is sampled in time instants $T = \{T_1, T_2,..., T_N\}$. Therefore, there is a set of samples $X, T = x(T_1), x(T_2),..., x(T_N)$, in which the number of samples $N$, and their times of occurrence $T$ are arbitrary. It means that the probability density function, all initial and central moment functions are conditional forthwith.

The conditional mean function $\tilde{m}(t) = \langle x(t) | X, T \rangle$ is used as reconstruction function. The quality of the reconstruction is evaluated by the conditional variance function $\tilde{\sigma}^2(t) = \langle [x(t) - \tilde{m}(t)]^2 | X, T \rangle$ or reconstruction error function. Both characteristics $\tilde{m}(t)$ and $\tilde{\sigma}^2(t)$ can be found on the basis of the conditional multidimensional pdf $w_{N+1}(x(t) | X, T)$ of the given process. It is clear that the sampled realization cannot be known exactly; nevertheless, but with this rule it is possible to reconstruct the realization. This rule also provides the minimum estimation reconstruction error for realizations with an arbitrary pdf.

Considering that the realization sampled is Gaussian, their conditional characteristics are [17]:

$$\tilde{m}(t) = m(t) + \sum_{i=1}^{N}\sum_{j=1}^{N} K(t, T_i) a_{ij} [x(T_j) - m(T_j)], \quad (8)$$

$$\tilde{\sigma}^2(t) = \sigma^2(t) - \sum_{i=1}^{N}\sum_{j=1}^{N} K(t, T_i) a_{ij} K(T_j, t), \quad (9)$$

where $m(t)$ and $\sigma^2(t)$ are the mathematical expectation and the variance of the initial process $x(t)$ respectively; $K(\cdot)$ is the covariance function, and $a_{ij}$ represents the elements of the inverse covariance function. If it is assumed that the process $x(t)$ is stationary with $m(t) = 0$, $\sigma^2(t) = 1$, and $K(t - T_i)$, equations (8), and (9) would change to:

$$\tilde{m}(t) = \sum_{i=1}^{N}\sum_{j=1}^{N} K(t - T_i) a_{ij} [x(T_j) - m(T_j)], \quad (10)$$

$$\tilde{\sigma}^2(t) = 1 - \sum_{i=1}^{N}\sum_{j=1}^{N} K(t - T_i) a_{ij} K(T_j - t). \quad (11)$$

From (10) and (11) it can be seen that the reconstruction function is a linear function of the samples, and the reconstruction error function does not depend on the samples.

## IV. The Non-Optimal Reconstruction Algorithm

Now the non-optimal algorithm based on atomic functions shall be discussed. To make the reconstruction of any realization of a given random process with its restricted power spectrum, it is possible to use the Fourier transformation (7) of the atomic functions [15]-[16], [18]-[20]. This is due to the zeros of (7) located periodically. In addition to this, the functions obtained by (7) tend to zero in the infinite faster than other functions [16]. Considering this methodology, the reconstruction function $\hat{m}(t)$ for any random processes with a limited power spectrum by a boundary frequency on the basis of the samples $x(\Delta T_k)$ is:

$$\hat{m}(t) = \sum_{k=-\infty}^{\infty} x(\Delta T_k) F_\beta \left[ \frac{\beta\pi}{\Delta T_k} (t - \Delta T_k) \right], \quad (12)$$

where $F_\beta(\rho)$ is given by (7), and [16]:

$$\beta > 2\,;T \le \frac{\pi}{\omega_b}\frac{\beta-2}{\beta-1} \quad \text{or} \quad T < \frac{\pi}{\omega_b}\,;\beta \ge \frac{2-T\omega_b/\pi}{1-T\omega_b/\pi}. \tag{13}$$

Then the equation (12) changes to:

$$\hat{m}(t) = \sum_{k=-\infty}^{\infty} x(\Delta T_k) \prod_{n=1}^{\infty} \sin c\left(\frac{\beta\pi}{\Delta T_k \beta^n}(t-\Delta T_k)\right). \tag{14}$$

Expression (14) can be interpreted as the disintegration of the function $x(t)$ on the displacement - narrowing's basis of the imaginary Fourier functions of the atomic function $h_2(\cdot)$. The problem in the calculation of the last equation is the number of terms in the product. For its application, it can be restricted by a finite amount $M$ on the right side, being indicated as [16]:

$$\hat{m}(t) = \sum_{k=-\infty}^{\infty} x(\Delta T_k) \prod_{n=1}^{M} \sin c\left(\frac{\beta\pi}{\Delta T_k \beta^n}(t-\Delta T_k)\right), \tag{15}$$

when:

$$\beta\left(1-\beta^{-M}\right) > 2 \quad ; \quad \Delta T = \frac{\pi}{\omega_b}\frac{\beta\left(1+\beta^{-M}\right)-2}{\beta-1}. \tag{16}$$

The minimum values are determined by the solution of the equation $\beta\left(1+\beta^{-M}\right)=2$. When $M=1$, the WKS series is obtained. If $M=\infty$, the series (15) is transformed into (14).

To calculate the reconstruction error, let introduce the base function $\varphi_j(t)$:

$$\phi_j(t) = \prod_{j=1}^{M} \sin c\left(\frac{\beta\pi}{\Delta T_j \beta^j}\left(t-\Delta T_j\right)\right). \tag{17}$$

Then, assuming a finite number of samples $N$, the reconstruction error function is [5]:

$$\begin{aligned}\hat{\sigma}^2(t) &= 1 - 2\sum_{j=1}^{N} R\left(t-\Delta T_j\right)\varphi\left(t-\Delta T_j\right) \\ &+ \sum_{j=1}^{N}\sum_{i=1}^{N} R\left(\Delta T_j - \Delta T_i\right)\varphi\left(t-\Delta T_j\right)\varphi\left(t-\Delta T_i\right).\end{aligned} \tag{18}$$

In this algorithm, an equal situation equivalent to (10) and (11) occurs: the reconstruction function depends on the values of the samples, and the error reconstruction function does not relies on them.

## V. Comparison between both Reconstruction Algorithms

If a realization from a Gaussian Markovian process is chosen. It is formed on the output of an one-stage integrated *RC* circuit driven by Gaussian white noise $n(t)$. The normalized covariance function $R(\tau)$ and the power spectrum density $S(\omega)$ are:

$$R(\tau) = \exp\left(-\alpha|\tau|\right), \tag{19}$$

$$S(\omega) = \frac{2\alpha}{\alpha^2+\omega^2}. \tag{20}$$

As the covariance time $\tau_c$ is unitary, then $\alpha = 1$. The Gaussian Markovian process is not differentiable because the variance of its derivative is equal to infinity. Restricting the spectrum (20) in the boundary frequency $\omega_b$, the characteristics of the given process change radically. The resulting process is Gaussian non-Markovian and infinitely differentiable. So, the normalized covariance function now is expressed by:

$$R(\tau) = \left(\int_{-\omega_b}^{\omega_b} \frac{d\omega}{\alpha^2+\omega^2}\right)^{-1} \int_{-\omega_b}^{\omega_b} \frac{e^{j\omega\tau}\,d\omega}{\alpha^2+\omega^2}. \tag{21}$$

The first term in (21) ensures the normalization for the power spectrum of the obtained realization with any value of the boundary frequency $\omega_b$. Fig 4 shows the normalized covariance function $R(\tau)$ for different values of $\omega_b$. When $\omega_b$ is equal to infinity, the curve is similar to the curve obtained with a one-stage *RC* filter. Also, if $\omega_b$ has a smaller value the curve tends to zero slower.

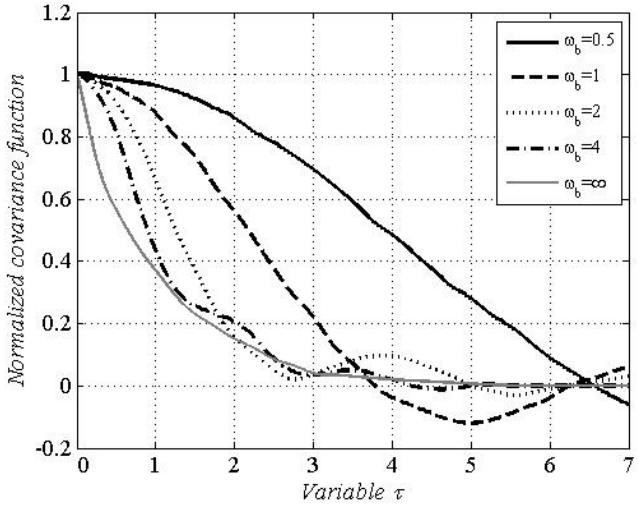


Fig. 4 Normalized covariance function of a Gaussian process with different values of $\omega_b$.

In order to have a complete comparison of the efficiency of the optimal algorithm and the non-optimal algorithm, another type of non-optimal algorithm has to be introduced. This

algorithm is based on Balakrishnan's theorem [1] when the number of samples $N$ is finite. The reconstruction function is represented by (1). Balakrishnan's theorem mentions that the reconstruction error is equal to zero if the number of samples is infinity. However, if the number of samples is finite the error exists and it is different to zero. In this way, the reconstruction error function has a similar formulation to the one used on (18). The difference is the base function that each one non-optimal algorithm uses.

Although the main purpose is to know how good the quality of the reconstruction of the realization is, it is necessary to know the type of base functions for each mentioned algorithms first. They are: optimal algorithm (conditional mean rule), case *A* of the non-optimal algorithm (Balakrishnan's theorem with a finite set of samples), and case *B* of the non-optimal algorithm (atomic functions). Fig. 5 shows all these base functions. All curves have a value of zero at the sampling points, except at the initial point because it corresponds to the multiplication of the sample with the base function. The curves' forms are different. In the non-optimal algorithm, the curve of the case *A* tends to zero more slowly as consequence of its large peak values. The curve of the case *B*, which was calculated with $\beta = 2.1$ because it is a near value to the limit value which is $\beta = 2$, tends to zero fastest. It means that in this case, the influence between the samples is minimal. The base function for $\beta = 4$ is also given. When the value of $\beta$ increases, the amplitudes of the oscillations increase as well. However, all base functions of the case *B* of the non-optimal algorithm tend to zero faster than the optimal algorithm and the case *A* of the non-optimal algorithm.

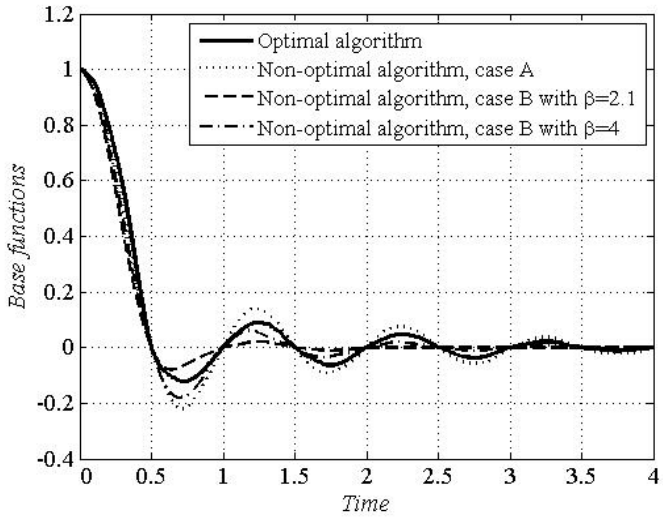


Fig. 5 Comparison of base functions.

Fig. 6 illustrates the reconstruction error function for the case *B* of the non-optimal algorithm (atomic functions) for various values of $N$ and $\beta = 2.1$. As it can be seen in Fig. 6, the unit of measurement on the *x*-axis is the boundary frequency $\omega_b$, not the time. Obviously, when the number of samples increases, the reconstruction error decreases. The maximum error is obtained at the half of the sampling intervals $t = \Delta T/2$. It is worth mentioning that $\Delta T = \pi/\omega_b$, in this way the reconstruction error also depends on the separation between the samples.

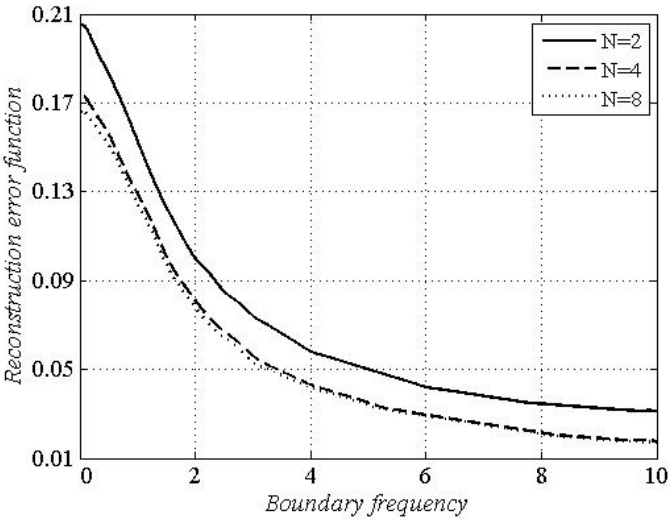


Fig. 6 Reconstruction error functions using atomic functions with $N$ and $\beta = 2.1$.

Now, the effectiveness of the non-optimal algorithm using atomic functions shall be compared with the reconstruction error of the optimal algorithm and the case *A* of the non-optimal algorithm. In Fig. 7, the reconstruction error functions are presented for all methodologies mentioned above with $N = 2$; in Fig. 8 the curves are calculated with $N = 4$; and in Fig. 9 the number used of samples is $N = 8$.

In the case *B* of the non-optimal algorithm, when values of the boundary frequency are $\omega_b < 2$, the magnitude of the error increases with respect to the error of the optimal algorithm if the number of samples $N$ grows. When the boundary frequency is $\omega_b > 2$ the difference with the optimal error decreases gradually (aside from the situation when $N = 2$ and $\beta = 4$). The curves of the case *A* in the non-optimal algorithm take a similar behavior like the curves of the optimal algorithm while the number of samples grows.

When $N = 2$ the error in case *A* is the biggest. When $N = 4$ the error in case *A* is smaller than the error in case *B* with $\beta = 2.1$, but bigger than the error in case *B* with $\beta = 4$. When $N = 8$ the error in case *A* is the smallest in all non-optimal cases. The physical interpretation of this effect is related to the special property of atomic functions. They tend to zero very quickly, thus, the influence between the neighboring samples for making the reconstruction operation is small. For that reason, with $N = 2$ the influence is minimal and the error grows considerably. Among more samples, the influence rises, and the error decreases.

Clearly, the magnitude of the error in the optimal algorithm is smaller than the non-optimal algorithm. Notwithstanding, this is a natural effect due to the structure of its functions and the statistical properties that each one uses.

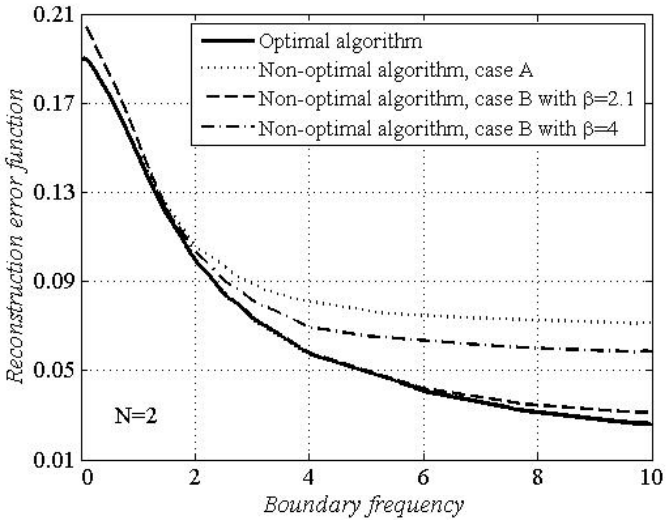


Fig. 7 Comparison of the reconstruction error function with $N = 2$.

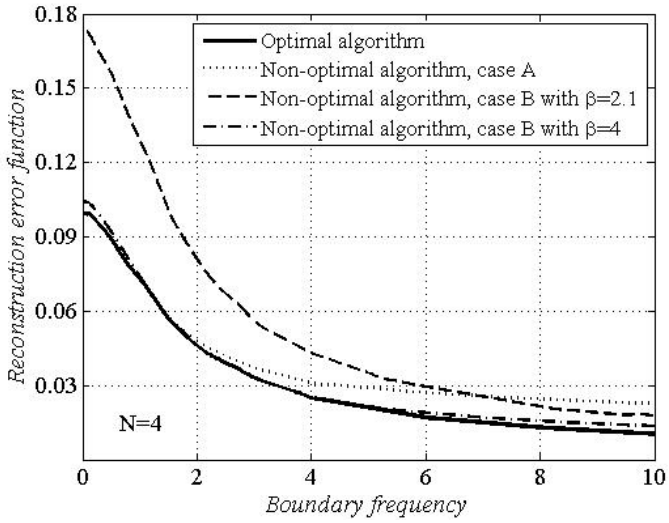


Fig. 8 Comparison of the reconstruction error function with $N = 4$.

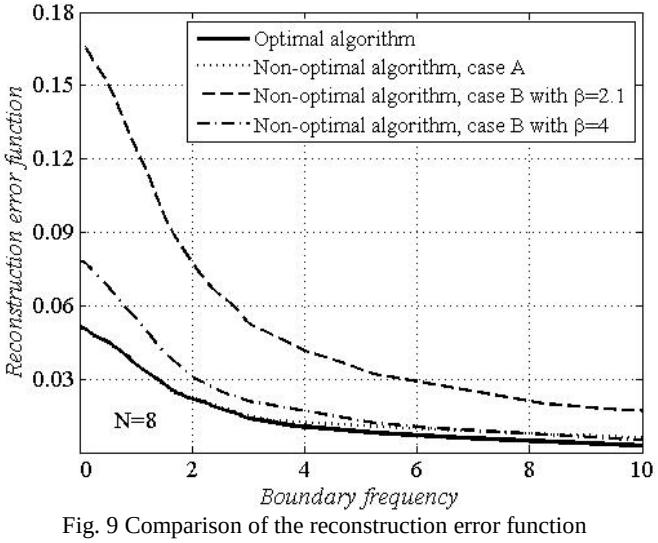


Fig. 9 Comparison of the reconstruction error function with $N = 8$.

## VI. Conclusion

Three different reconstruction algorithms are analyzed to describe the Sampling - Reconstruction Procedure of realizations of a Gaussian process with a finite power spectrum. Both main principal (the reconstruction function and the reconstruction error function) are calculated. Overall, the smallest reconstruction error is obtained by the conditional mean rule, also called optimal algorithm. Then, it is followed by the non-optimal algorithm based on the Balakrishnan's theorem. Finally, the biggest reconstruction error is obtained with the non-optimal algorithm based on atomic functions. The non-optimal functions do not depend on the covariance function for making the reconstruction operation. They rely on the boundary frequency only. The algorithm on the basis of the conditional mean rule takes into account the covariance function of the sampled process, and for this reason the optimal algorithm has the best characteristics of the reconstruction procedures.

## References

[1] A. V. Balakrishnan, A note on the sampling principle for continuous signals, *IRE Transactions on Inf Theory*, Vol. IT-3, 1957, pp. 143-146.

[2] A. V. Balakrishnan, On the problem of time jitter in Sampling, *IRE Transactions on Inf Theory*, Vol. IT-3, April,1962, pp.226-236.

[3] V. A. Kazakov, Sampling-reconstruction procedures of Gaussian process realizations – Chapter 9 in book: *Probability: Interpretation, Theory and applications*, Edited by Y. Shmaliy, Nova Science Publishers Inc., USA, 2012, pp. 269 -297.

[4] V. Kazakov, Regeneration of samples of random processes following non-linear inertialess convertions, *Telecommunication and Radio engineering*, Vol. 43, No. 10, 1988, pp. 94-96.

[5] V. Kazakov, S. A. Africanov, M. A. Belyaev, Comparison of two algorithms of the realization restoration using random number of counts, *Radioelectronics and Communications Systems*, Vol. 37, No. 4, 1994, pp. 43-45.

[6] V. Kazakov, M. A. Belyaev, Sampling-reconstruction procedure for non-stationary Gaussian processes based on conditional expectation rule, *Sampling Theory in Signal and Image Processing*, Vol. 1, No. 2, 2002, pp. 135-153.

[7] V. Kazakov, The sampling-reconstruction procedure with a limited number of samples of stochastic processes and fields on the basis of the conditional mean rule, *Electromagnetic Waves and Electronic Systems*, Vol. 10, No. 1-2, 2005, pp. 98-116.

[8] V. Kazakov, D. Rodriguez, Sampling-Reconstruction Procedure of Gaussian Processes with two Jitter Sources, *WSEAS Transactions On Electronics*. Issue 2, Vol. 2, April 2005, pp. 53-57.

[9] V. Kazakov, D. Rodriguez, Reconstruction of Gaussian regular sampled fields with jitter, *WSEAS Transactions on Systems*, Vol. 5, Issue 8, 2006, pp. 1771-1776.

[10] V. Kazakov, Non-uniform sampling and reconstruction of stochastic processes with the limited number of samples, WSEAS *Transactions on Systems*, Vol. 5, No. 8, 2006, pp. 1777-1784.

[11] V. Kazakov, Y. Olvera, Sampling-reconstruction procedures of non-Gaussian processes by two algorithms, *International Journal of Circuits, Systems and Signal Processing*, Issue 5, Vol. 5, 2011, pp. 557–564.

[12] V. A. Kazakov, Sampling-reconstruction procedures of non-Gaussian process realizations - Chapter 10 in book: *Probability: Interpretation, Theory and applications*, Edited by Y. Shmaliy. Nova Science Publishers Inc., USA, 2012, pp. 299 – 326.

[13] Y. Goritskiy, V.Kazakov, Sampling and Reconstruction of piece-constant random processes with Erlang's time staying in states, *Journal of Computer and Systems Sciences International*, Vol. 50, No 6, 2011, pp. 870 – 883.

[14] Y. I. Khurgin, V. P. Yakovlev, *Finite functions in physic and technology*, Naukova, Moscow, 1971. (In Russian)

[15] V. L. Rvachyov, *Theory of R-functions and some applications*, Naukova Dumka, 1982. (In Russian)
[16] M. A. Basarab, E. G. Zelkin, V. F. Kravchenko, V. P. Iakovlev, *Digital signal processing on the basis of Whittaker-Kotel'nikov-Shannon´s theorem*, Radiotecnika, 2004. (In Russian)
[17] R. L. Stratonovich, *Topics in the theory of random noise*, Gordon and Breach, 1963.
[18] V. F. Kravchenko, *Course of the atomic functions theory*, Radiotecnika, 2003. (In Russian)
[19] V. F. Kravchenco, V. L. Rvachyov, *Logic algebra, atomic functions and wavelets*, Fizmatlit, 2006. (In Russian)
[20] V. F. Kravchenko, H. M. Pérez, V. I. Ponomaryov, *Adaptive digital processing of multidimensional signals with applications*, Fizmatlit, 2009.

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