Inner feedback robust control of a laboratory heat exchanger

Libor Pekar and Roman Prokop

***Abstract*—**Controller design for a laboratory circuit heating plant with model feedback delays by means of a branched control system in a robust sense is the aim of this contribution. The reader is concisely acquainted with a mathematical model of then plant that includes all significant delays and latencies first; hence, the model is infinite-dimensional. Relevant algebraic tools, namely, a special ring quasipolynomial functions, Bézout identity and particular controller parameterizations for a feedback control system with additional inner loop follow. Although the controller structure is fixed, its parameters are eventually tuned by introduced robustness principles in order to meet robust stability and performance. For the practical implementation, rather complex controllers' laws, which are of a delayed nature, are simplified by a rationalization and their discrete-time formulation is suggested as well. The results are verified by simulations in MATLAB® and Simulink®. All the obtained results are promising for a future real-life implementation of the presented approach.

***Keywords*—**Algebraic control design, Discretization, Heat exchanger, MATLAB®, Rationalization, Robustness, Time-delay system.

I. Introduction

By a simple anisochronic modelling principle [1]-[3] and also many others ([4], [5]) it has been proved that circuit heating and thermal plants and processes are typical representatives of nonlinear systems with feedback internal delays mainly due to transmission latencies in pipelines. A heat exchanger is a device that exchanges heat between two streams, heating one and cooling the other. A subset of recuperating (through-flow) heater exchangers consists in the hot and cold fluids that are separated by a wall and heat is transferred by conduction through the wall. An exchanger of this type was recently developed and assembled at the Faculty of Applied Informatics of Tomas Bata University and this appliance has become a rewarding object for many scientific experiments giving some interesting and promising results [3], [6]-[8] where, however, only input-output delays have been considered.

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The family of delayed (or time-delay) systems is not trivial to be controlled by the use of many traditional or conventional control design approaches – or, these methods are even impossible to be utilized directly [9]-[12], for instance, without a rationalization. Some proposed ad-hoc approaches, e.g. Lyapunov-Krasovskii methods [13] or H∞ optimal controllers [14], [15], are too complex and troublesome to be practically implemented by engineers. However, algebraic control design means proved to be very effective and in many cases engineeringly acceptable as well [16]-[18]. For instance, the ring of proper and stable quasipolynomial meromorphic functions (RMS) [19], [20], as a point of authors' interest, provides one with a simple model-based design yielding a delayed controller structure which, however, can be implemented e.g. via programmable logic controllers [21]. The RMS ring can also be used to design controllers meeting some robustness conditions in some sense [22], [23]. Note that robust stabilization, control, controller parameterization, delay values determination etc. has widely been studied in the literature, and many algorithms, approaches and results has been derived and published [10], [11], [15], [24]-[26].

As up to the amount of 95 percent of control loops in industry are equipped by proportional-integral-derivative (PID) controllers [27], the implementation of which is well solved and managed, it is desirable for the practice to approximate the final delayed (anisochronic) controller law by the PID one (or its linear generalization). Methods based on a rational approximation of exponential (delay) terms ([28], [29]) do not effectively enable to control the order of the obtained approximation model. Moreover, they are mostly used to plant model rationalization prior to controller design, rather than to controller simplification. A way how to cope with this problem is to find the closest finite-dimensional model with a prescribed fixed structure to the infinite-dimensional original e.g. by the matching of spectral properties [30] or via some advanced interpolation/extrapolation methods [31], [32].

Another practically-oriented problem is connected to discrete-time control law implementation. Most of control design approaches utilize a controlled plant discretized model prior to a controller structure determination either in the input-output [8], [33], [34] or state-space formulation [35]. There is, however, a lack of well-applicable digital implementation procedures for continuous-time controllers – so called delta models [36] represent a family of possible approaches solving this task.

This contribution provides a reader with a complex robust control design for the above introduced circuit laboratory heat exchanger evincing internal feedback delays. It follows up with and extends our previous works [22], [23], [37] – details appear in the state-of-the-art section – in the sense that the branched Two-Feedkack-Controllers (TFC) control structure is used and, in particular, the eventual controller rationalization and discretization are proposed. The obtained theoretical results are verified by means of MATLAB® / Simulink® environment tools.

A novel combination of anisochronic modelling, algebraic control design for unconventional control system and controller parameterization in a robust sense for a complex family of delayed systems stand for the most contributive idea and result of this paper.

The paper is organized as follows. Section 2 provides the reader with a concise state of the art on robust control of the considered circuit heat exchanger and our motivation. The appearance of the appliance and its mathematical model is summarized in Section 3. In Section 4, principles of algebraic controller design in the RMS ring and basic results on robust stability and performance for the TFC control system are given. The complete controller structure design and parameterization in the robust sense for the particular model are derived in Section 5. Section 6, preceding the conclusions, brings about controller rationalization and discretization proposal that are proved by simulations.

II. Heat exchanger robust control – State of the art

As mentioned above, circuit heating plants and exchangers of the type considered in this paper, evincing internal feedback delays, were modeled and served for control design verification many times. Regarding frequency-domain-based (robust) control of our plant considering internal delays (not only input-output ones), works of Zítek and his colleagues have to be referenced [2], [19], [21], [38], [39] first, where so-called anisochronic (i.e. delayed) controllers were tuned mostly by direct pole placement methodology. Model predictive control (MPC) and discrete-time approaches, in which a kind of robustness is also included, were referred above; see e.g. [7], [8], [40], [41].

The work presented herein the paper, in fact, follows up with and extends our some previous works. Namely, the One-Degree-of-Freedom (1DoF) control system and robust control design were implemented to the heating exchanger in [22], [37] by simulations as well as real laboratory measurements. The latter paper, moreover, suggests a way how to rationalize and discretize the final infinite-dimensional controller. A more advanced branched TFC structure [42], see Fig. 1, with a delayed controller satisfying robust stability and performance is designed in [23]. In this paper, the whole procedure is summarized and, moreover, a rationalization of the controller together with its sampled-data algorithm is suggested.

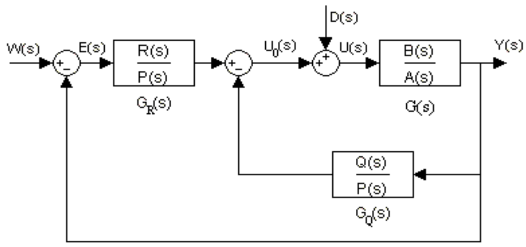

Fig. 1 TFC control system

III. Heat exchanger appearance and model

A. Laboratory Model Appearance

A rough scheme of the laboratory heat exchanger to be controlled is provided to the reader in Fig. 2 [3]. The model works as follows: Distilled water inside the piping is driven by a pump {6} - continuously controllable via the voltage $u_P(t)$ - through a flow heater {1} with maximum power $P_H(t)$ of 750 W. The heater output temperature, $\vartheta_{HO}(t)$, is measured by a platinum thermometer. Warmed liquid then goes through a 15 meters long insulated coiled pipeline {2} which causes the significant delay in the system. A heat-consuming appliance is represented by the air-water heat exchanger (cooler) {3} equipped with a continuously adjustable (by means of the voltage $u_C(t)$) and an on/off cooling fans {4, 5}. Input and output temperatures of the cooler, $\vartheta_{CI}(t)$ and $\vartheta_{CO}(t)$, respectively, are measured by platinum thermometers as well. The expansion tank {7} compensates for the expansion effect of the heat fluid. Let the ambient temperature be $\vartheta_A(t)$.

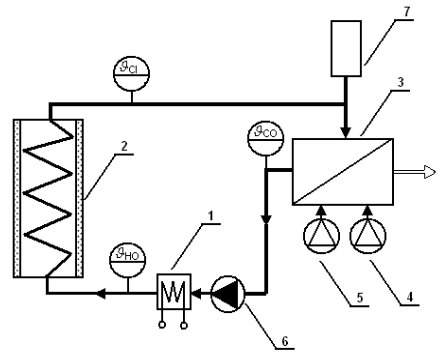

Fig. 2 Circuit heating exchanger

The laboratory appliance can be considered as a small-scaled model of a real-word system, e.g. as the cooling system in cars or a house central heating system.

B. Mathematical Model

The anisochronic modeling approach utilized here, [1], [2], is based on the comprehension of all significant delays and latencies in the model. Two step are performed when modeling. Models of separate functional parts of the plant are found first and, as second, the obtained sub-models are combined by means of their common physical quantities taking delays between them into consideration.

The sub-models based on heat balance equations are the following:

The heater

$$cM_H \frac{\mathrm{d}\vartheta_{HO}(t)}{\mathrm{d}t} = P_H(t-0.5\tau_H) + c\dot{m}(t)\left(\vartheta_{HI}(t-\tau_H) - \vartheta_{HO}(t)\right) - K_H(t)\left(\frac{\vartheta_{HO}(t) + \vartheta_{HI}(t-\tau_H)}{2} - \vartheta_A\right)$$

The long pipeline

$$cM_P \frac{\mathrm{d}\vartheta_{CI}(t)}{\mathrm{d}t} = c\dot{m}(t)\left(\vartheta_{HO}(t-\tau_{HC}) - \vartheta_{CI}(t)\right) - K_P\left(\frac{\vartheta_{CI}(t) + \vartheta_{HO}(t-\tau_{HC})}{2} - \vartheta_A\right)$$

The cooler (water-air exchanger)

$$cM_C \frac{\mathrm{d}\vartheta_{CO}(t)}{\mathrm{d}t} = c\dot{m}(t)\left(\vartheta_{CI}(t-\tau_C) - \vartheta_{CO}(t)\right) - K_C(t)\left(\frac{\vartheta_{CO}(t) + \vartheta_{CI}(t-\tau_C)}{2} - \vartheta_A\right)$$

where

c [J·kg^{-1}·K^{-1}] is the specific water heat capacity,
$\dot{m}(t)$ [kg·s^{-1}] is the water mass flow rate,
M_H [kg] is the overall water mass in the heater,
M_C [kg] is the overall water mass in the cooler,
M_P [kg] is the overall water mass in the pipeline,
$\vartheta_{HI}(t)$ [°C] means heater input temperature,
ϑ_A [°C] stands for ambient temperature,
τ_H [s] expresses the delay of a water flow through the heater,
τ_{HC} [s] is the delay of a water flow between the heater and the cooler,
τ_C [s] means the delay of a water flow through the cooler,
[s] is the delay between a control signal to the cooling fan and the output temperature of the cooler,
τ_{KC} [s] stands for the delay of a water flow between the cooler and the heater,
$u_P(t)$ [V] is pump input voltage
$u_C(t)$ [V] is cooling fan input voltage
$K_H(t)$ [W·K^{-1}] expresses the overall heater wastage energy heat transmission coefficient
$K_C(t)$ [W·K^{-1}] is the overall cooler heat transmission coefficient, and
K_P [W·K^{-1}] is the overall long pipeline heat transmission coefficient.

The complete linearized model describing the dependence of three measured outputs, $\vartheta_{HO}(t)$, $\vartheta_{CI}(t)$, $\vartheta_{CO}(t)$, (or, their deviations from an operating point, more precisely) on three manipulated inputs, $P_H(t)$, $u_P(t)$, $u_C(t)$ can be obtained by introducing additional static relations and linearization [3]. For the sake of this paper it is, however, sufficient to consider the relationship between $\Delta u_P(t)$ and $\Delta\vartheta_{CO}(t)$ governed by the transfer function

$$G(s) = \frac{\Delta\vartheta_{CO}(s)}{\Delta P_H(s)} = \frac{(b_{0D}\exp(-\tau_0 s) + b_0)\exp(-\tau s)}{s^3 + a_2 s^2 + a_1 s + a_0 + a_{0D}\exp(-\vartheta s)} \tag{1}$$

where the prefix Δ means the difference from an operating point, the particular setting of which is given by

$$\begin{aligned}&\lfloor u_p, u_c, P_H, \vartheta_{HO}, \vartheta_{CI}, \vartheta_{CO}, \vartheta_A \rfloor_0 \\ &= [5\,\mathrm{V}, 3\,\mathrm{V}, 300\,\mathrm{W}, 44.1°\mathrm{C}, 43.8°\mathrm{C}, 36°\mathrm{C}, 24°\mathrm{C}]\end{aligned} \tag{2}$$

Then, eventual model parameters values read

$$\begin{aligned}&b_{0D} = 2.334\cdot 10^{-6}, b_0 = -2.146\cdot 10^{-7}, a_2 = 0.1767, \\ &a_1 = 0.009, a_0 = 1.413\cdot 10^{-4}, a_{0D} = -7.624\cdot 10^{-5}, \\ &\tau_0 = 1.5, \tau = 131, \vartheta = 143\end{aligned} \tag{3}$$

IV. Algebraic Control Design and Robustness Issues

A. R_{MS} Ring and Control Design for TFC

In this subsection, basic algebraic notions and conditions for controller structure derivation by means of the TFC system are summarized. For further details, the reader is kindly referred to [23].

1) RMS ring

The design starts with the formulation of the plant model and all external signals in the Laplace transform (the reference - $W(s)$, load disturbance - $D(s)$, control error - $E(s)$, controller output - $U_0(s)$, manipulated input affected by $D(s)$ - $U(s)$, and output - $Y(s)$, see Fig. 1) in R_{MS}. This ring was defined in [20] as follows: $T(s) = n(s)/d(s) \in R_{MS}$ if $n(s)$, $d(s)$ are quasipolynomials with $n(s) = \tilde{n}(s)\exp(-\tau s)$, $\tau \ge 0$, $T(s) \in H_\infty$ ($\mathbb{C}^+$), and the term is formally stable and proper, i.e.

$$\exists R > 0 : \sup_{\mathrm{Re}\, s>0, |s|\ge R} |T(s)| < \infty$$

2) Feedback system stability

Prior to the introduction of the stability condition formulation, dynamics' relations of the TFC system in terms of transfer functions ought to be given to the reader. Transfer functions that can be derived from the TFC structure are given in (4) where the corresponding characteristic meromorphic function is provided to the reader in (5).

$$G_{WY}(s)=\frac{Y(s)}{W(s)}=\frac{B(s)R(s)}{M(s)},\ G_{DY}(s)=\frac{Y(s)}{D(s)}=\frac{B(s)P(s)}{M(s)},\quad G_{WE}(s)=\frac{E(s)}{W(s)}=\frac{A(s)P(s)+B(s)Q(s)}{M(s)} \tag{4}$$

$$M(s)=A(s)P(s)+B(s)(Q(s)+R(s)) \tag{5}$$

From (4), (5), the following theorem can be proofed [16], [17], [23]: Given a Bézout coprime pair $A(s),B(s)\in R_{MS}$ the closed-loop TFC system is stable (in R_{MS} sense) if and only if there exist coprime pairs $P(s),Q(s)\in R_{MS}$ and $P(s),R(s)\in R_{MS}$ satisfying the Bézout identity (5).

Moreover, if we set $O(s):=Q(s)+R(s)$, all stabilizing solutions of (5) can be parameterized as

$$P(s)=P_0(s)\pm B(s)Z(s)\neq 0,\quad O(s)=O_0(s)\mp A(s)Z(s) \tag{6}$$

for an arbitrary $Z(s)\in R_{MS}$ where the subscript $\cdot_0$ means a particular solution.

3) *Load disturbance rejection*

The load disturbance $d(t)=L^{-1}\{D(s)\}$ (where L^{-1} means the inverse Laplace transform) is asymptotically rejected if it holds that $\lim_{t\to\infty} y_D(t)=\lim_{s\to 0} sG_{DY}(s)D(s)=0$. This can be algebraically fulfilled if and only if $F_D(s)$ divides $B(s)P(s)$, where $D(s)=\cdot/F_D(s)$, i.e.

$$B(s)P(s)/F_D(s)\in R_{MS} \tag{7}$$

in R_{MS}, which can be deduced from (4). Condition (7) can be satisfied by a suitable choice of $Z(s)$ in (6).

4) *Reference tracking*

The reference signal $w(t)=L^{-1}\{W(s)\}$ is tracked if $\lim_{t\to\infty} e_W(t)=\lim_{s\to 0} sG_{WE}(s)W(s)=0$. It holds in R_{MS} if and only if

$$(A(s)P(s)+B(s)Q(s))/F_W(s)\in R_{MS} \tag{8}$$

where $W(s)=H_W(s)/F_W(s)$, i.e. $F_W(s)$ divides the product $A(s)P(s)$ in R_{MS} and, simultaneously, $F_W(s)$ divides $B(s)Q(s)$. The natural question is how to meet both the conditions, (7) and (8). This task can be solved by so called decomposition of $O(s)$ that is described in the following subsection.

5) *Decomposition of $O(s)$*

Let

$$O(s)=\frac{o_N(s)}{o_D(s)}=\frac{s^n+\sum_{i=0}^{n}\sum_{j=1}^{k_i} o_{ij}s^i\exp(-\vartheta_{ij}s)}{o_D(s)}$$

Introduce a set of real selectable parameters $\gamma_{ij}\in[0,1], i=0,1,...n, j=1...k_i$ where n is the degree of $o_N(s)$, k_i expresses the number of non-zero delay terms for s^i, $n_O=1+\sum_{i=1}^{n}k_i$ is the number of all non-zero terms in $o_N(s)$, and set

$$R(s)=\frac{\gamma_{n0}s^n+\sum_{i=0}^{n}\sum_{j=1}^{k_i}\gamma_{ij}o_{ij}s^i\exp(-\vartheta_{ij}s)}{o_D(s)},\quad Q(s)=\frac{(1-\gamma_{n0})s^n+\sum_{i=0}^{n}\sum_{j=1}^{k_i}(1-\gamma_{ij})o_{ij}s^i\exp(-\vartheta_{ij}s)}{o_D(s)} \tag{9}$$

If the number n_O of free parameters is not enough to solve (8), $O(s)$ has to be expanded as

$$O_1(s)=O(s)\frac{s^{n_\beta}+\beta_{n_\beta-1}s^{n_\beta-1}+...+\beta_1 s+\beta_0}{s^{n_\beta}+\beta_{n_\beta-1}s^{n_\beta-1}+...+\beta_1 s+\beta_0}$$

where $\beta_i, i=0...n_\beta-1$ stand for additional selectable parameters.

B. Robust Stability and Performance

A triple motivation for robust analysis of the heat exchanger can be found. First, some physical quantities of the laboratory model and the room environment can vary in time. Second, measurement and identification uncertainties can naturally appear when modeling and measurements. Third, we intend to determine a possible range for the free controller parameters such that the control system is robust against these perturbations.

It is well known that robust stability and performance analysis is based on the Nyquist criterion [43]; hence, the first step is devoted to its validity for time-delay systems and the selected TFC control system. Then, the main results on robust stability and performance published by the authors earlier are reviewed.

1) *Nyquist criterion for TFC with delayed plant*

Let $G_0(s)$ express the nominal plant transfer function and $G(s)=(1+\Delta(s)W_M(s))G_0(s)$ be a family of perturbed transfer functions where $W_M(s)$ is a fixed stable weight function expressing the uncertainty frequency distribution and for a perturbation stable transfer function holds $\|\Delta(s)\|_\infty\le 1$. Moreover, $G(s)$ and $G_0(s)$ have the same number of unstable poles. Hence, it holds that

$$\left|\frac{G(j\omega)}{G_0(j\omega)}-1\right|\le|W_M(j\omega)|,\forall\omega$$

Function $W_M(s)$ ought to be selected so that it covers all systems from the family over some suitable frequency range.

Regarding the TFC control structure, there are more possibilities how to define the criterion since it depends on how the feedback transfer function is constructed. Let, for instance, the function be of the form

$$
\begin{aligned}
G_{WY}(s) &= \frac{Y(s)}{W(s)} = \frac{G_R(s)G(s)}{1+G(s)\left(G_R(s)+G_Q(s)\right)} \\
L_0(s) &= G(s)\left(G_R(s)+G_Q(s)\right)
\end{aligned}
\tag{10}
$$

If retarded delayed systems are considered [9] (since system (1) is of this type), the following theorem can be proved [23], [44]:

Theorem 1 (Nyquist criterion for retarded time-delay systems with TFC). Let the plant and the controller in the TFC structure have transfer functions with distributed or lumped delays and $G_R(s)+G_Q(s)=o(s)/p(s)$, $G(s)=b(s)/a(s)$, where $a(s), b(s), o(s), p(s)$ are retarded quasipolynomials, $G(s)$ is strictly proper and $G_R(s)+G_Q(s)$ is proper. Moreover, let $a(s)$ and $p(s)$ have no root on the imaginary axis, i.e. $a(s)\neq 0, p(s)\neq 0$ for any $s=\mathrm{j}\omega$, $\omega\in\mathbb{R}^+$, and define $m_{ap}(s)$ as the denominator of $L_0(s)$.

Then if $\underset{s=\mathrm{j}\omega,\omega\in[0,\infty)}{\Delta\arg} m_{ap}(s)=l\pi/2$, the closed-loop system is asymptotically stable if

$$
\underset{s=\mathrm{j}\omega,\omega\in[0,\infty)}{\Delta\arg}\left(1+L_0(s)\right)=\left(n-l-2\overline{N}_u\right)\frac{\pi}{2}=\overline{N}_{u,ap}\pi
\tag{11}
$$

where n is the highest s-power in $m_{ap}(s)$, $\overline{N}_u$ means the number of common zeros of the numerator and denominator of $L_0(s)$ in $\mathbb{C}^+$ and $\overline{N}_{u,ap}$ stands for the number of unstable zeros of $m_{ap}(s)$ which are not included in the $L_0(s)$ numerator. ∎

2) Robust stability

Due to Theorem 1, another theorem for the whole family of perturbed plants can be derived [23]:

Theorem 2 (Robust stability of time-delay systems with TFC). If $L_0(s)$ is stable, the TFC control system is robustly stable (i.e. remains stable for all perturbed plants) if and only if

$$
\left\| W_M(\mathrm{j}\omega)T_0(\mathrm{j}\omega)\left(1+\frac{G_Q(\mathrm{j}\omega)}{G_R(\mathrm{j}\omega)}\right)\right\|_\infty < 1
\tag{12}
$$

where $T_0(s)=G_{WY,0}(s)$ is the so called nominal complementary sensitivity function. ∎

3) Robust performance

Prior to the presentation of robust performance condition, the notion of nominal performance must be introduced [43].

The feedback system meets nominal performance if

$$
\left\| W_P(\mathrm{j}\omega)S_0(\mathrm{j}\omega)\right\|_\infty < 1
\tag{13}
$$

where $\left|W_P(\mathrm{j}\omega)\right|$ stands for the sensitivity weighting function and $S_0(s)=1-T_0(s)=G_{WE,0}(s)$ means the sensitivity function for the nominal plant. Then we can state [23]:

Theorem 3 (Robust performance of time-delay systems with TFC). If $L_0(s)$ is stable, the TFC control system satisfies robust performance (i.e. it is robustly stable according to Theorem 2 and meets performance (13) for the whole family of perturbed plants) if and only if

$$
\left\| \begin{aligned} &\left| W_M(\mathrm{j}\omega)T_0(\mathrm{j}\omega)\left(1+\frac{G_Q(\mathrm{j}\omega)}{G_R(\mathrm{j}\omega)}\right)\right| \\ &+\left| W_P(\mathrm{j}\omega)\left(S_0(\mathrm{j}\omega)+W_M(\mathrm{j}\omega)T_0(\mathrm{j}\omega)\frac{G_Q(\mathrm{j}\omega)}{G_R(\mathrm{j}\omega)}\right)\right| \end{aligned} \right\|_\infty < 1
\tag{14}
$$

V. Controllers Structures Design and Robust Parameterization for the Heating Exchanger

Let us recall basic results already introduced in [23]. Considering a linearwise reference function and a stepwise disturbance one, the eventual controllers via the procedure (5)-(9) can be derived

$$
\begin{aligned}
G_Q(s) &= m_0^3\frac{\left(s^3+\sum_{i=0}^2 a_i s^i + a_{0D}\exp(-\vartheta s)\right)(1-\gamma)o_1 s^2}{\left(\sum_{i=2}^4 p_i s^i + p_1(s)s+p_0(s)\right)(s+m_1)} \\
G_R(s) &= m_0^3\left(\gamma o_1 s^2+(o_1 m_1+o_0)s+o_0 m_1\right) \\
&\quad \frac{\left(s^3+\sum_{i=0}^2 a_i s^i + a_{0D}\exp(-\vartheta s)\right)}{\left(\sum_{i=2}^4 p_i s^i + p_1(s)s+p_0(s)\right)(s+m_1)}
\end{aligned}
\tag{15}
$$

where

$$
\begin{aligned}
&p_4=(b_0+b_{0D})^2, p_3=4m_0(b_0+b_{0D})^2, p_2=6m_0^2(b_0+b_{0D})^2, \\
&p_1(s)=m_0^3 \\
&\quad\begin{pmatrix} 4(b_0+b_{0D})^2-(b_0+b_{0D}\exp(-\tau_0 s)) \\ (b_0(m_0\tau+4)+b_{0D}(m_0(\tau+\tau_0)+4))\exp(-\tau s) \end{pmatrix} \\
&p_0(s)=m_0^4(b_0+b_{0D})(b_0(1-\exp(-\tau s))+b_{0D}(1-\exp(-(\tau+\tau_0)s))) \\
&o_1=b_0(m_0\tau+4)+b_{0D}(m_0(\tau+\tau_0)+4), o_0=m_0(b_0+b_{0D})
\end{aligned}
$$

Real valued parameters $m_0, m_1>0$ and $0\leq\gamma\leq 1$ (defined in (9)) are selectable controller parameters.

By taking year-long variations in $\vartheta_A(t)\in[16,30]$ inside the laboratory room and identification uncertainties of transmission coefficients of the heater (boiler), $K_P\in[0.1,0.5]$, and heating exchanger (cooler), $K_C\in[15,22]$, into consideration, the following estimation of $W_M(s)$ can be written as

$$
\begin{aligned}
W_M(s) &= 0.36\frac{(200s+1)(10s+1)}{(340s+1)(15s+1)} \\
&= \frac{720s^2+75.6s+0.36}{5100s^2+355s+1}
\end{aligned}
\tag{16}
$$

see Fig. 3 – there is still a gap in finding a suitable, not so conservative, envelop curve. Note that parameters of (16) have been obtained by the application of the standard knowledge of frequency (filtering) properties of particular factors in $W_M(s)$, see e.g. [45].

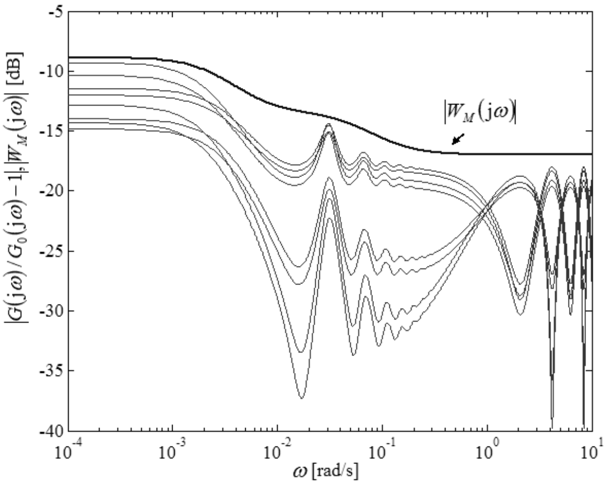

Fig. 3 $|W_M(j\omega)|$ estimation

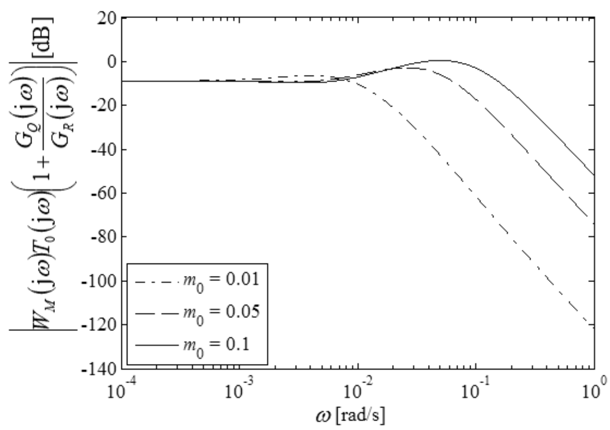

Fig. 4 Robust stability test

Whereas, surprisingly, neither m_1 nor γ affects (12), let us select $m_0 = \{0.01, 0.05, 0.1\}$ from a sufficient range test the robust stability condition. Results are presented in Fig. 4. Thus, as a robustly stable range, we can asses $m_0 \in [0, 0.09]$.

The weighting function $W_P(s)$ is selected in order to keep the condition (13) for all selected combinations of controller parameters values, namely, $\gamma = \{0.3, 0.5, 0.7\}$, $m_1 = \{0.01, 0.05, 0.1\}$, $m_0 = \{0.01, 0.05\}$, see Fig. 5.

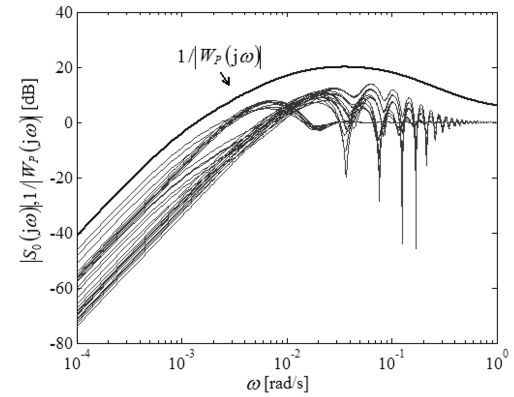

Fig. 5 $|W_P(j\omega)|$ determination

Its possible eventual Laplace form might be

$$
\begin{aligned}
1/W_P(s) &= 9\cdot 10^5 \frac{s^2(1.5s+1)}{(1000s+1)(80s+1)(10s+1)} \\
&= 9\cdot 10^5 \frac{1.5s^3+s^2}{8\cdot 10^5 s^3 + 9.08\cdot 10^4 s^2 + 1090s + 1}
\end{aligned}
\tag{17}
$$

where the level of conservativeness is a rather low on low frequencies. Once functions $W_M(s)$ and $W_P(s)$ are fixed, possible values of the free controller parameter can be benchmarked by simulations of robust performance condition (14), see a bunch of Bode plots in Fig. 6. The result of the test can be summarized as follows: Possible ranges of controller parameter values can be:

- $m_0 = 0.01$, $m_1 \in [0.001, 0.01]$, $\gamma = [0.5, 0.8]$, or
- $m_0 = 0.05$, $m_1 \in [0.001, 0.005]$, $\gamma = [0.3, 0.5]$, or
- $m_0 = 0.05$, $m_1 = 0.01$, $\gamma = 0.3$

It can be shown that the endeavor to reduce the maximum overshoot yields the maximization of m_0 and the minimization of γ [23]. Thus, the triplet $m_0 = 0.02, m_1 = 0.005, \gamma = 0.4$ was chosen as eventual controller parameters.

VI. Controllers Structures Rationalization and Discretization

Linear delayed controllers (15) are characterized by an infinite spectrum, which is not suitable for practical reasons since the most of industrial feedback loops are equipped with conventional PID controllers. Hence, it is highly desirable to approximate them by a finite-dimensional linear control law. Moreover, the control law to be implemented via a machine working with digital signals and values should be formulated also in a discrete-time form. Therefore, propositions of controllers' rationalization and discretization follow.

A. Controller Rationalization

The Padé approximation, which is usually performed in such a way that the approximation is applied to separate exponential terms [28], has been eventually used to the finite-dimensional approximation of controllers (15). We, in the contrary, decided to apply it to the whole transfer function.

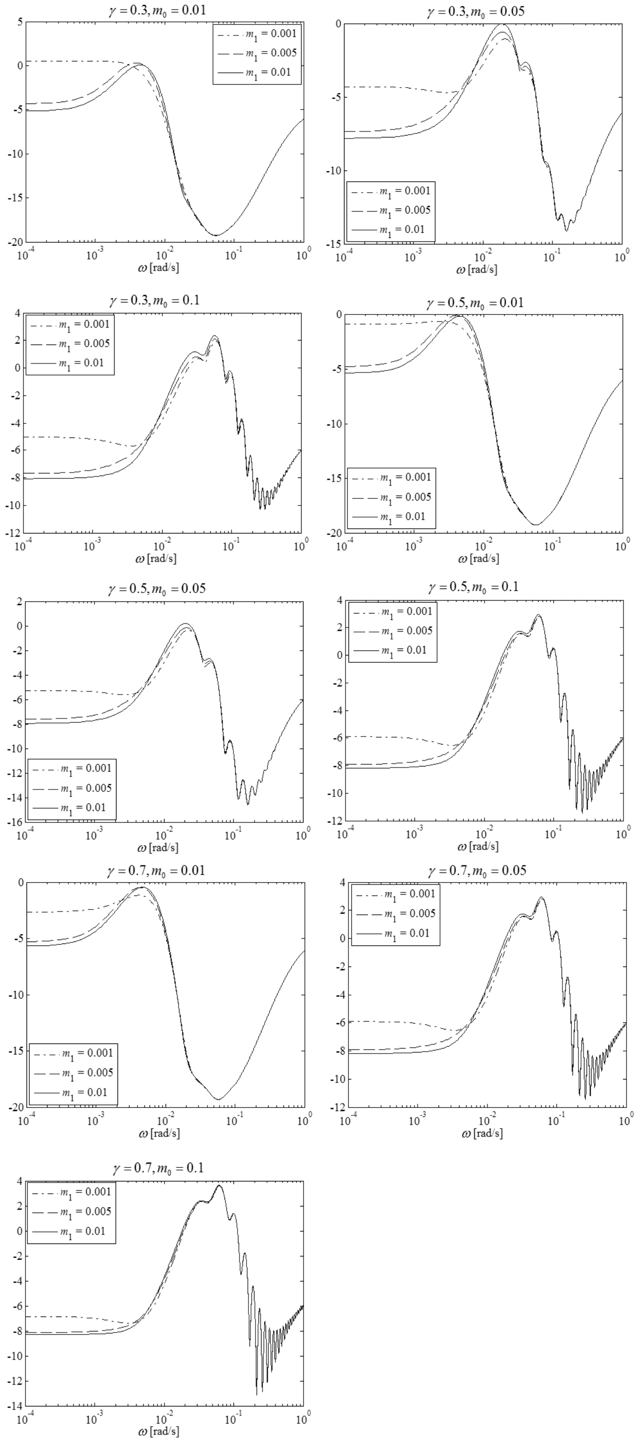

Fig. 6 Robust performance tests

The well known Maclaurin series expansion, $Mc(s)$ (if it converges), of function $f(s)$ is equivalent to the rule

$$\left[\frac{\mathrm{d}^i}{\mathrm{d}s^i} Mc(s)\right]_{s=0} = \left[\frac{\mathrm{d}^i}{\mathrm{d}s^i} f(s)\right]_{s=0}, i = 0,1,\ldots\infty \tag{18}$$

We intend to approximate controllers (15) by proportional (P) and proportional-derivative (PI) laws, respectively, as

$$\overline{G}_R(s) = \frac{\overline{r}_2 s^2 + \overline{r}_1 s + \overline{r}_0}{s^2}, \overline{G}_Q(s) = \overline{q}_0 \tag{19}$$

which respects the asymptotic behaviour of (15) because $G_R(s)$ has a double integral action, while $G_Q(s)$ does not exhibit any. Since $G_R(0) \to \infty$, the inversion of $G_R(s)$ is used in (18); moreover, $\left[G_R^{-1}(s)\right]_{s=0} = \left[\overline{G}_R^{-1}(s)\right]_{s=0} = 0$, hence the zero derivative can be omitted. To sum up, the final particular matching rules read

$$\begin{gathered}\left[\frac{\mathrm{d}^i}{\mathrm{d}s^i} G_R^{-1}(s)\right]_{s=0} = \left[\frac{\mathrm{d}^i}{\mathrm{d}s^i} \overline{G}_R^{-1}(s)\right]_{s=0}, i = 1,2,3 \\ G_Q^{-1}(0) = \overline{G}_Q^{-1}(0)\end{gathered} \tag{20}$$

which yields

$$\begin{gathered}\overline{G}_R(s) = \frac{47.2676 s^2 + 0.416887 s + 6.16545 \cdot 10^{-4}}{s^2} \\ \overline{G}_Q(s) = 24.6114\end{gathered} \tag{21}$$

The simulation comparison of original control responses (where the step load disturbance $d(t) = -50$ W enters at $t = 6000$ s) by means of controllers (15) with those using simplified rational P and PI controllers (21) are displayed in Figs. 7 and 8.

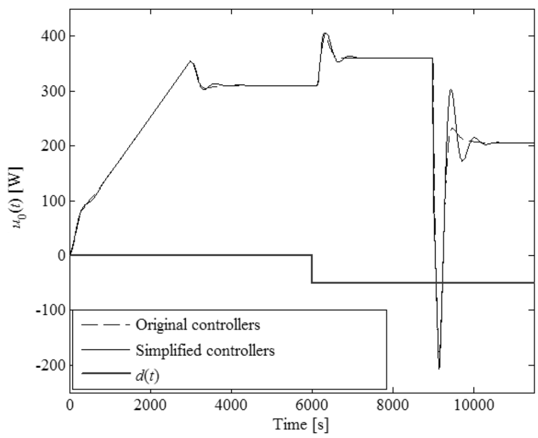

Fig. 7 Simulated control responses of $u_0(t)$ for the TFC system with controllers (15) vs. (21)

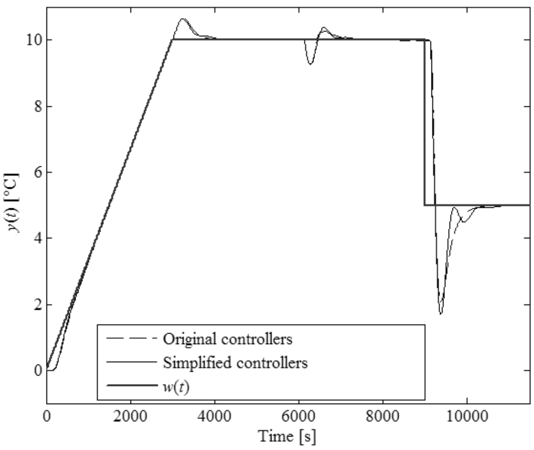

Fig. 8 Simulated control responses of $y(t)$ for the TFC system with controllers (15) vs. (21)

These results prove a very good agreement of the original and approximating responses except for the reaction on the step-down change in $w(t)$. Note that it is possible to calculate identities (20) at a different point from $s = 0$, e.g. in the neighborhood of the leading (most dominant) pole.

B. *Controller Discretization*

In this subsection, the reader is briefly provided with the idea of possible delayed controllers (15) discrete-time formulations based on the transfer function (i.e. input-output) description – unlike some advanced but computationally heavy state-space algorithms [35], for instance. In particular, delta models [36] and linear delay interpolation [46] are implemented herein.

Delay exponential terms are subjected to the transformation $\exp(-\eta s)X(s) \to x(t-\eta)$ and interpolated as $x(t-\eta_i) \approx$ $\approx (1-\alpha_i)x(t-\tau_{d,i}) + \alpha_i x(t-\tau_{d+1,i})$ where $d_i = \lfloor \eta_i / T_s \rfloor$, $\tau_{d,i} \le \eta_i \le \tau_{d+1,i}$, $\tau_{d,i} = d_i T_s$, $\tau_{d+1,i} = (d_i + 1)T_s$, $\alpha_i = (\eta_i - \tau_{d,i})/T_s$ and T_s means the sampling period. The goal is that $\tau_{d,\cdot}$ are commensurate delays as integer multiples of the sampling period. Then $x(t - d_i T_s) \to z^{-d_i} X(z)$ where z is the z-transform variable associated with the shifting operator q.

The approximation of derivatives resides in the introduction of variable γ associated with the delta operator δ defined as

$$\gamma = \frac{z-1}{\beta T_s z + (1-\beta)T_s} \tag{22}$$

where $\beta \in [0,1]$ represents a weighting parameter. In this paper, the Tustin (trapezoidal) approximation governed by the setting $\beta = 0.5$ is used.

The eventual particular discrete-time control laws with $T_s = 1$ s are omitted since the complete ones are too long yet sufficiently simple to be implemented by a computer. The PC connected to the heat exchanger is equipped with the data acquisition card AD622 and Real-Time Toolbox for MATLAB®, which enables to use quasi-continuous algorithms with $T_s \approx 0.01s$ or a higher. Note that orders of corresponding linear difference equations equal 146 and 149, respectively.

Similarly, a graphical comparison of continuous-time and discrete-time simulation control responses is useless since both the courses are almost identical and indistinguishable by a human sight.

VII. Conclusion

In this contribution, we have presented a complex control design for a laboratory circuit heating plant (exchanger) in a robust sense with a branched Two-Feedback-Controllers control system. The plant mathematical model evinces internal delays and thus the process ought to be considered as an infinite-dimensional one. The introduced utilization of the ring of stable and proper meromorphic functions has resulted in delayed (anisochronic) controllers with unknown, tunable, parameters which have been determined by using robust control tools and simulation experiments. Namely, robust stability and robust performance conditions have been met. For practitioners, a rationalization procedure and a possible digital implementation of the control law have been suggested, yielding conventional linear finite-dimensional continuous-time and discrete-time control laws, respectively. Both the results have given a very good agreement with the original control responses.

For the future research, practical real-life verification of all these promising results should be performed. A multi-input, multi-output control design might be a suitable task for the future research on the exchanger as well.

Obtained results are beneficial for both scientists and engineers providing a possible procedure to design an advanced control law for an arduosly controllable family of delayed systems.

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