Two Term Controller Design for Multivariable Linear Time Delay System

Sonal Singh, Shubhi Purwar

Abstract**—**The proposed work develops a methodology to design a state feedback- two term (2T) controller via composite nonlinear feedback (CNF) controller for reference tracking of multivariable linear time delay system in presence of input saturation. Basic composite nonlinear feedback (CNF) controller only deals with present state whereas 2T controller deals with present and past both the states. Comparative study of controllers (CNF & 2T) proves the superiority of 2T in terms of damping characteristics and fast transient response. The efficiency of the proposed technique is validated through simulation results. Stability is proven by delay independent analysis via Lyapunov-Krasovskii functional. Unknown parameters are found with the help of Linear Matrix Inequalities (LMI) toolbox.

Keywords**—** Composite Nonlinear Feedback Technique, Lyapunov-Krasovskii functional, Time Delay, State Delay.

I. Introduction

DYNAMICAL systems, such as networked control systems, electrical networks, aircrafts, etc, have the feature of time varying or time constant delays in system states [1-4], control inputs [9] or measurements [6] which can be the major cause of instability and performance degradation. This reason captivates researchers in this area from many years and so the analysis of stability and controllers in this area are done to unfold the complications due to time delay. In past decades, many outcomes on delay-independent or delay-dependent stability analysis and controller design for time delay systems have been achieved by performance evaluation with the help of Lyapunov–Krasovskii functional method [3,14] and the Razumikhin lemma [16]. These methods provide solutions in the form of LMIs and Riccati equations which can be solved by LMI toolbox [18] or YALMIP toolbox in MATLAB [8]. But the choices of these functions are critical for deriving stability criteria. There are some special forms that reduce complexity in these functions for delay dependent and independent both cases. In neural systems, delay appears in the state and state derivative both and so complexity increases but proper choice of Lyapunov–Krasovskii function makes the stability analysis simpler [3].

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The output regulation for linear delayed systems subject to input saturation and state delays are major area in literature [1, 12, 15] and the references therein. The topic of analysis and synthesis of controllers for these systems has attracted considerable attention. When the actuator is saturated, the performance of the control system is severely destroyed. To resolve this problem, suitable design procedures are required since control system is subjected to saturation part.

During the past years, many methods have been reported to deal with saturation and time delay problems. Many controllers like H-infinity controller [3], backstepping controller [5], sliding mode controller [2], etc had proven the robustness in the performance of time delayed systems. But most of these controllers are unable to provide small overshoot and fast response simultaneously in tracking problems. They have to compromise between these two characteristics. A CNF controller resolves the issue of compromising either transient characteristics [7, 10]. CNF control consists of two feedback laws: linear and nonlinear which does not contain any switching element. The linear feedback law improves damping ratio to attain a fast response and the nonlinear feedback law tunes the damping ratio to minimize the overshoot which is due to linear part [10].

Moreover, this controller can be applied to the systems having external disturbances or with no disturbances and time delayed systems [1].

In this paper, a 2T controller is presented which claims that adding a simple delay in conventional CNF control, performance of closed loop system becomes robust. Previously, a memory-less state feedback control effort ($u = kx(t)$) has been designed for solution to delay problems [1]. Two term control is created using present and delayed states which improve the performance of time delayed system because delayed part in control law deals with information of past states [3].

State delayed linear system is chosen for performance analysis which is shown by a comparison through numerical example which proves that the 2T control law improves damping characteristics and the transient response becomes faster in comparison to one term controller. Later sections of the paper are categorized as follows:

Section II gives problem formulation followed by controller design in Section III. Stability analysis is given in Section IV and comparison by simulation results is shown in Section V. Section VI gives the concluding remarks.

II. Problem Formulation

A linear system with time delay can be expressed as:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \sum_{i=1}^{N} A_{di}x(t-\tau_i) + Bsat(u) \\ y(t) &= Cx(t) \\ x(t) &= \phi(t), \qquad t \in (-\tau, 0) \end{aligned} \tag{1}$$

where, A, B, A_{di} and C are system parameters, $x(t) \in \Re^n$ is state vector, control input is $u(t) \in \Re^m$, τ_i is time delay and $y(t) \in \Re^P$ is output of the system. N is the number of delay introduced in state vector. $\phi(t)$ is the initial condition that ensures the uniqueness and smoothness of (1).

The saturation function is defined as:

$$sat(u_i) = sign(u_i)\min(|u_i|, \bar{u}_i) \tag{2}$$

where $i = 1, 2, ..., m$ and $\bar{u}_i$ is the maximum control effort of i^{th} channel.

To track a reference model output $y_m(t)$ precisely and quickly, a feedback control law is designed in this paper. This reference model can be described as:

$$\begin{aligned} \dot{x}_m(t) &= A_m x_m(t) \\ y_m(t) &= C_m x_m(t) \end{aligned} \tag{3}$$

where, $x_m(t) \in \Re^{n_m}$ is state vector and $y_m(t)$ is output of the reference model which has same dimension as $y(t)$. Also there is a requirement to bound $x_m(t)$ [15]. Choice of bound is $\|Gx_m(t)\|^2 \le M^*$, where M^* is a positive bounded value (this choice will be clear in later section). To force the output of the system to track reference proximately, two matrices $G \in \Re^{n \times n_m}$ and $H \in \Re^{m \times n_m}$ are introduced [1] as:

$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} G \\ H \end{bmatrix} = \begin{bmatrix} GA_m \\ C_m \end{bmatrix} \tag{4}$$

The solution of (4) is assumed to be controllable and number of inputs *(m)* should always be greater than or equal to number of output *(p)* such that:

$$rank \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = n + p \tag{5}$$

Rewriting (4) as:

$$\begin{bmatrix} G \\ H \end{bmatrix} = \begin{bmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} GA_m \\ C_m \end{bmatrix} \tag{6}$$

(Calculation of (6) is taken from [1])

In later sections following notations are used:

$x(t) = x$, $x(t-\tau_i) = x_{\tau_i}$, $x_m(t) = x_m$, $x(t-d) = x_d$, $\rho(y_m, y) = \rho$ and $\tilde{x} = x(t) - Gx_m(t)$

III. 2T Controller Design

In this section, CNF control law for state feedback is shown which is further unified with delayed state to form 2T controller followed by calculation of unknown parameters by Lyapunov–Krasovskii stability analysis and LMIs. At the end of this section error dynamics is shown.

A. CNF Control Law

For linear time delay system (1), before designing a CNF control law [10] some important assumptions are required i.e. (A, B) is controllable, (A, C) is observable and (A, B, C) is invertible without having any zero at $s = 0$. If above assumptions are true, then CNF control law is designed as:

$$u_B = u_L + u_N \tag{7}$$

where, u_L is a linear feedback law which improves the damping ratio and u_N is a non-linear feedback law which reduces the overshoot as soon as output of the system reaches the reference value. These two laws are:

$$\begin{aligned} u_L &= Kx + (H - KG)x_m \\ u_N &= \rho B^T P \tilde{x} \end{aligned} \tag{8}$$

where, gain matrix $K \in \Re^{m \times n}$ and real symmetric matrix $P \in \Re^{n \times n}$ are chosen in order to make closed loop system asymptotically stable and ρ is a non-positive function (defined in next section) which is locally Lipschitz in y which plays a major role in changing the location of closed loop poles.

B. Choosing a nonlinear function (ρ)

The nonlinear function ρ, can be chosen in order to tune the control law for improving the performance of closed loop system. The settling time is quick due to this nonlinear function and so tracking error will be small by proper choice of ρ. To adapt the changes in tracking, ρ can be selected as:

$$\rho = -\beta \exp(\alpha_0 \alpha)\|y - y_m\| \tag{9}$$

where, $\alpha > 0$, $\beta > 0$ are chosen for tuning the parameters and scaling parameter α_0 changes with different tracking values created by y_m as:

$$\alpha_0 = \begin{cases} \dfrac{1}{\|y_0 - y_m\|} & y_0 \ne y_m \\ 1 & y_0 = y_m \end{cases} \tag{10}$$

C. 2T Control law

Two term control law is basically an addition of a delay state into a present state feedback (CNF) controller u_B which can be expressed as:

$$u = u_B + K_d x_d \tag{11}$$

where, $K_d \in \Re^{m\times n}$ is the controller gain of delayed term. Here u_B is function of present state, $K_d x_d$ is function of delayed state and d is a finite amount of feedback delay introduced with a purpose to improve dynamic response of the time delayed system [3].

Here present state controller is CNF controller which is presented in (7). Substituting (7) in (11), the 2T controller will finally forms:

$$u = K\tilde{x} + Hx_m + u_N + K_d x_d \tag{12}$$

The controller parameters K, K_d and P are calculated by solving LMIs which are developed in later section. Feedback delay d is chosen as:

$$d = \max(\tau_1, \tau_2, \ldots, \tau_N) \tag{13}$$

D. Lyapunov–Krasovskii Functional

In terms of transformed state $\tilde{x}$, the behavior of (1) driven by controller (12), can be described as:

$$\dot{\tilde{x}}(t) = A_c\tilde{x} + B_d\tilde{x}_d + \sum_{i=1}^{N} A_{di}\tilde{x}_{\tau_i} + B\omega + B_d G x_{m_d} + \sum_{i=1}^{N} A_{di} G x_{m_{\tau_i}} \tag{14}$$

where, $A_c = A + BK$, $B_d = BK_d$ and

$$\omega = sat(u) - (K\tilde{x} + Hx_m + K_d x_d) \tag{15}$$

Choosing Lyapunov–Krasovskii function to investigate stability and obtaining controller parameters for closed loop time delayed system as:

$$V = \tilde{x}^T P\tilde{x} + \int_{t-d}^{t} \tilde{x}^T(s) Q\tilde{x}(s)ds + \sum_{i=1}^{N}\int_{t-\tau_i}^{t} \tilde{x}^T(s) R_i\tilde{x}(s)ds + \int_{t-\tau_i}^{t} x_m{}^T(s)x_m(s)ds + \int_{t-d}^{t} x_m{}^T(s)x_m(s)ds \tag{16}$$

where, $R_i > 0$ and $Q > 0$ with dimensions similar to P and $\tilde{x}$ is initially assumed at rest.

Taking derivative of V:

$$\dot{V} = 2\tilde{x}^T P\dot{\tilde{x}} + \tilde{x}^T(Q + \sum_{i=1}^{N} R_i)\tilde{x} + 2x^T{}_m x_m - \tilde{x}_d^T Q\tilde{x}_d - x_{m_d}{}^T x_{m_d} - \sum_{i=1}^{N}(x_{m_{\tau_i}}{}^T x_{m_{\tau_i}} + \tilde{x}_{\tau_i}{}^T R_i\,\tilde{x}_{\tau_i}) \tag{17}$$

Substituting values from (14), (17) becomes:

$$\dot{V} = 2\tilde{x}^T PA_c\tilde{x} + 2\tilde{x}^T PB_d\tilde{x}_d + 2\sum_{i=1}^{N}\tilde{x}^T PA_{di}\tilde{x}_{\tau_i} + 2\tilde{x}^T PB\omega + 2\tilde{x}^T PB_d x_{m_d} + 2\sum_{i=1}^{N}\tilde{x}^T PA_{di}x_{m_{\tau_i}} + \tilde{x}^T(Q + \sum_{i=1}^{N} R_i)\tilde{x} + 2x_m{}^T x_m - \tilde{x}_d^T Q\tilde{x}_d - x_{m_d}{}^T x_{m_d} - \sum_{i=1}^{N}(x_{m_{\tau_i}}{}^T x_{m_{\tau_i}} + \tilde{x}_{\tau_i}^T R_i\,\tilde{x}_{\tau_i}) \tag{18}$$

One may rewrite (18) as:

$$\dot{V} = Z^T\Delta Z + 2\tilde{x}^T PB\omega + 2(Gx_m)^T Gx_m \tag{19}$$

where,

$Z = \begin{bmatrix}\tilde{x} & \tilde{x}_d & \tilde{x}_{\tau_1} & \tilde{x}_{\tau_2} & \ldots & \tilde{x}_{\tau_N} & Gx_{m_d} & Gx_{m_{\tau_1}} & Gx_{m_{\tau_2}} & \ldots & Gx_{m_{\tau_N}}\end{bmatrix}^T$ and Δ is shown in (22).

According to bounds on reference model:

$$(Gx_m)^T Gx_m = \|Gx_m\|^2 \le M^* \tag{20}$$

this makes (19) as:

$$\dot{V} \le Z^T\Delta Z + 2\tilde{x}^T PB\omega + M^* \tag{21}$$

As we know, if the Lyapunov function V is positive definite ($V(0) = 0$, $V(x) > 0$) and the time derivative of V is locally negative definite ($\dot{V} \le 0$), then the equilibrium is proven to be local asymptotical stable. Hence, (21) is local asymptotical stable if below conditions are satisfied:

C1. Δ should be negative definite : $\Delta \le -\lambda$. Here λ is a positive constant chosen according to C3.

C2. $\tilde{x}^T PB\omega$ should be negative definite: $\tilde{x}^T PB\omega < 0$.

C3. According to (20), M^* is a positive bounded value [15, 19]. To nullify its effect on stability, we choose $\lambda \ge M^*$.

C1. Substituting values of A_c, B_d, pre and post multiplying P^{-1} in Δ for solving it in the form of LMI by using Schur Compliment formula [4, 17], we will get (23).

where,

$$T = P^{-1}(PA + PBK + A^T P + K^T B^T P + Q + \sum_{i=1}^{N} R_i)P^{-1}$$

Assuming $Y = P^{-1}$, $V = K_d Y$, $S = KY$, $Q^* = P^{-1}QP^{-1}$ $R_i^* = P^{-1}R_i P^{-1}$, $L = P^{-1}IP^{-1}$, $E_i = L$, (23) becomes (24).

C2. In $\tilde{x}^T PB\omega$, ω contains saturated input channels. The following investigations are done to make this term negative definite for the sake of stability:

- If input channels are unsaturated, i.e. $|u| \le u_{\max}$ then from (12) and (15):

$$\begin{aligned}\omega &= K\tilde{x} + Hx_m + u_N + K_d x_d - (K\tilde{x} + Hx_m + K_d x_d) \\ &= u_N\end{aligned} \tag{25}$$

hence,

$$\tilde{x}^T PB\omega = \tilde{x}^T PBu_N = \rho\tilde{x}^T PBB^T P\,\tilde{x}$$

As we know ρ is a non-positive function and $\tilde{x}^T PBB^T P\,\tilde{x}$ is positive function, hence

$$\tilde{x}^T PB\omega < 0$$

$$\Delta = \begin{bmatrix} PA_c + A_c^T P + Q + \sum_{i=1}^{N} R_i & PB_d & PA_{\tau_1} & PA_{\tau_2} & \dots & PA_{\tau_N} & PB_d & PA_{\tau_1} & PA_{\tau_2} & \dots & PA_{\tau_N} \\ * & -Q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -R_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -R_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & * & * & * & \ddots & 0 & 0 & 0 & 0 & \ddots & 0 \\ * & * & * & * & * & -R_N & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -I & 0 & 0 \\ \vdots & * & * & * & \ddots & * & * & * & * & \ddots & 0 \\ * & * & * & * & * & * & * & * & * & * & -I \end{bmatrix} \tag{22}$$

$$\Delta = \begin{bmatrix} T & P^{-1}PBK_dP^{-1} & P^{-1}PA_{\tau_1}P^{-1} & P^{-1}PA_{\tau_2}P^{-1} & \dots & P^{-1}PA_{\tau_N}P^{-1} & P^{-1}PBK_dP^{-1} & P^{-1}PA_{\tau_1}P^{-1} & P^{-1}PA_{\tau_2}P^{-1} & \dots & P^{-1}PA_{\tau_N}P^{-1} \\ * & -P^{-1}QP^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -P^{-1}R_1P^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -P^{-1}R_2P^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & * & * & * & \ddots & 0 & 0 & 0 & 0 & \ddots & 0 \\ * & * & * & * & * & -P^{-1}R_NP^{-1} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -P^{-1}IP^{-1} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -P^{-1}IP^{-1} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -P^{-1}IP^{-1} & 0 & 0 \\ \vdots & * & * & * & \ddots & * & * & * & * & \ddots & 0 \\ * & * & * & * & * & * & * & * & * & * & -P^{-1}IP^{-1} \end{bmatrix} \tag{23}$$

$$\Delta = \begin{bmatrix} AY + BS + YA^T + S^TB^T + Q^* + \sum_{i=1}^{N} R_i^* & BV & A_{\tau_1}Y & A_{\tau_2}Y & \dots & A_{\tau_N}Y & BV & A_{\tau_1}Y & A_{\tau_2}Y & \dots & A_{\tau_N}Y \\ * & Q^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -R_1^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -R_2^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & * & * & * & \ddots & 0 & 0 & 0 & 0 & \ddots & 0 \\ * & * & * & * & * & -R_N^* & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -L & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -E_1 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -E_2 & 0 & 0 \\ \vdots & * & * & * & \ddots & * & * & * & * & \ddots & 0 \\ * & * & * & * & * & * & * & * & * & * & -E_N \end{bmatrix} \le -\lambda \tag{24}$$

- If input channels exceed their upper bound i.e. $u > u_{\max}$:

$$K\tilde{x} + Hx_m + u_N + K_d x_d > u_{\max}$$
$$u_N > u_{\max} - (K\tilde{x} + Hx_m + K_d x_d) \quad (26)$$

This condition exists only when:

$$u_{\max} > K\tilde{x} + Hx_m + K_d x_d$$
$$u_{\max} - (K\tilde{x} + Hx_m + K_d x_d) > 0$$

From (15), above inequality becomes :

$$\omega > 0 \text{ and } u_N = \rho B^T P \tilde{x} > \omega > 0 \quad (27)$$

ρ is non-positive and to satisfy (27)

$B^T P \tilde{x} = \tilde{x}^T PB < 0$, hence

$$\tilde{x}^T PB\omega < 0$$

- If input channels exceed their lower bound i.e. $u < -u_{\max}$:

$$K\tilde{x} + Hx_m + u_N + K_d x_d < -u_{\max}$$
$$u_N < -u_{\max} - (K\tilde{x} + Hx_m + K_d x_d) \quad (28)$$

This condition exists only when:

$$-u_{\max} < (K\tilde{x} + Hx_m + K_d x_d)$$
$$-u_{\max} - (K\tilde{x} + Hx_m + K_d x_d) < 0$$

From (15), above inequality becomes :

$$\omega < 0 \text{ and } u_N = \rho B^T P \tilde{x} < \omega < 0 \quad (29)$$

ρ is non-positive and to satisfy (29)

$B^T P \tilde{x} = \tilde{x}^T PB > 0$, hence

$$\tilde{x}^T PB\omega < 0$$

C3. Value of M^* is calculated by bounded condition of x_m which is already explained in (20).

E. Tracking Error Dynamics

The tracking error of output is defined as:

$$e = y - y_m \quad (30)$$

From (1) and (3) we can obtain:

$$e = C(x - x_m) \quad (31)$$

In order to make error dynamics zero, following assumptions are made :

$$\|e\| \le \|C\| \|x - x_m\| \quad (32)$$

Where $\|C\|$ is a finite value. From (32) we can say:

if $\|x - x_m\| \to 0$ then it yields $\|e\| \to 0$.

IV. NUMERICAL EXAMPLE AND SIMULATION RESULTS

In this section the designed two term CNF controller is implemented on a second order time delay linear system with saturation and time invariant delays.

Consider an example which is taken from [1, 20] described as:

$$A = \begin{bmatrix} 1 & 1.5 \\ 0.3 & -2 \end{bmatrix},\ A_{d1} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},\ A_{d2} = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 10 \\ 1 \end{bmatrix},\ C = \begin{bmatrix} 1 & 1 \end{bmatrix},\ N = 2,\ \bar{u} = 5,$$

$$\tau_1 = \tau_2 = 0.1\text{sec}$$

Tuning parameters chosen as $\alpha = 0.126$ and $\beta = 1.12$. Parameters of reference model (2) are :

$$A_m = \begin{bmatrix} -20 & 0 \\ 0 & -20 \end{bmatrix},\ C_m = \begin{bmatrix} -5.2 & -2.6 \end{bmatrix}$$

Initial conditions are taken as $x(0) = [-4 \quad 2]^T$ and $x_m(0) = [-2 \quad 1.5]^T$. Here $M^* = 400$ and the unknown parameters (P, K, K_d) of controller are found by using LMI (24) and G and H are calculated via (6). The simulation results are:

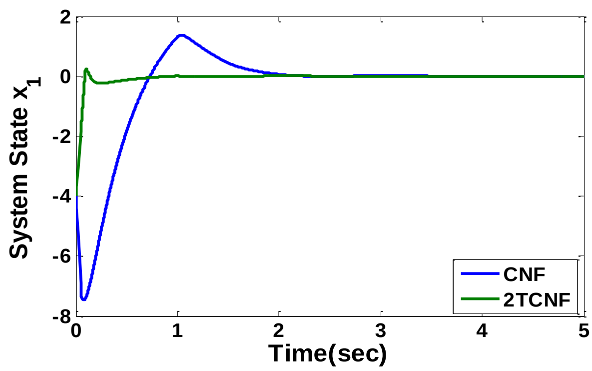

Figure 1. Trajectory of the system state x_1

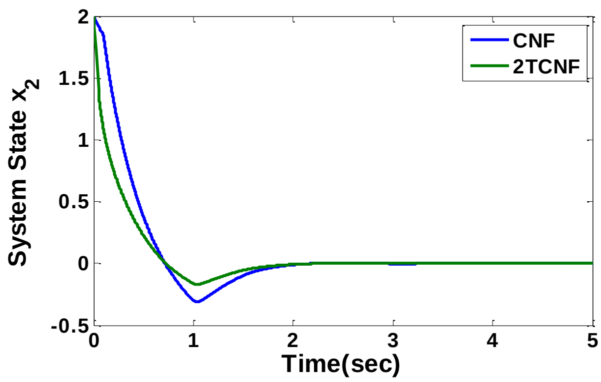

Figure 2. Trajectory of system state x_2

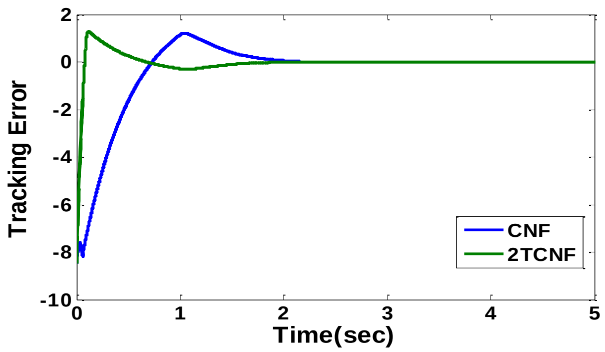

Figure 3. Tracking error of the trajectory.

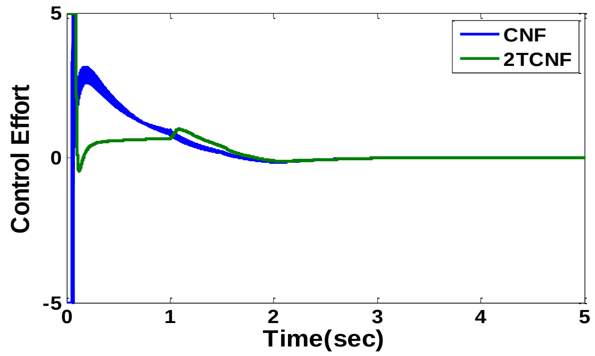

Figure 4. Control effort given to the system.

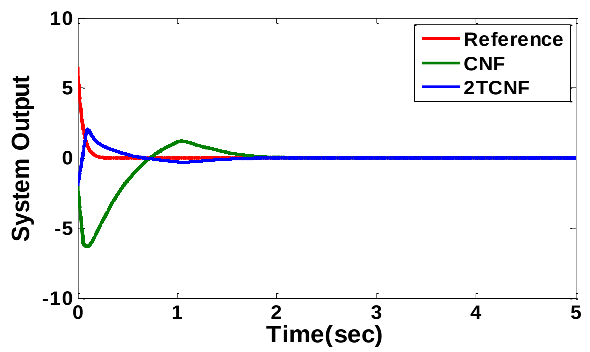

Figure 5. Outputs of reference model and system with CNF controller and two term CNF controller.

Comparing the above simulation results it can be seen that tracking error reduces more quickly in 2T control than CNF control. The damping reduces and transient response improves due to effect of delay. Also figure.5 shows that output tracking is much better in 2T controller. (CNF control parameters are taken from [1]).

V. Conclusion

2T control law deals with the previous and present state information which improves the damping characteristics when compared with single term CNF control law in presence of state delay. The calculated controller parameters by solving inequalities formed by Lyapunov–Krasovskii functional, with the help of LMI Toolbox, ensures the stability of the given closed loop time delay system. From simulation studies we can conclude that by adding information of past state in controller (2T), transient response became fast and tracking error reduces quickly in comparison to present state controller (CNF).

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