Interposed Control Design Conditions for Linear Discrete-time Systems

D. Krokavec and A. Filasová

***Abstract*—**The paper establishes the design procedure for the state feedback control of linear discrete-time systems, considerable as an interposed design criterium in the form of linear matrix inequalities. The goal is to design the feedback control which guarantees bounded H_2 performance index for the system transfer function matrix and H_∞ norm attenuation for the disturbance transfer function matrix, both combined with D-stable circle region parameters. Analyzing the criteria observance, the task is formulated as a feasible problem subject to integral quadratic constraints included in the Lyapunov discrete-time stability condition.

Keywords — H_2/H_∞ control strategy, D-stability region, state control, quadratic Lyapunov function, linear matrix inequalities, interposed design criteria.

I. Introduction

Tasks relating H_2 and H_∞ control design have been studied by many authors (see, e.g., [6], [19], [24] and the references therein), where H_∞ control design is referred mainly with the system frequency performances while H_2 control synthesis sets more suitable achievement on the system transient behavior [10], [21], [27]. Combining H_2 and H_∞ performance analysis, a mixed H_2/H_∞ control problem was formulated in [13] with the goal to optimize H_2 norm of the system transfer function matrix subject the constraint on H_∞ norm of the disturbance transfer function matrix. To derive the state feedback synthesis conditions, the benefit was substantiated by applying the continuous-time mixed H_2/H_∞ performance criterion [2], [4], [7], [20], as well as by formulating the appropriate computational linear matrix inequalities (LMI) technique [8], [16], [22].

To apply LMIs in control law parameter alignment, the H_2/H_∞ control design strategies for discrete-time linear systems are analyzed in the paper. In the sense of common practice [11], [25], the approach extends the design method presented in [15] to obtain associated interposed design criteria. Accordingly, exploiting an extended quadratic Lyapunov function, the obtained parameters of the state controller are designed relying on H_2, H_∞ constraints and combined with D-stability circle region parameters in the set of LMIs.

The outline of this paper is as follows. Section II and Section III. introduce the basic preliminaries in control law parameter design, while in Section IV and Section V. new results in design conditions are established and proven. Section VI. illustrates the properties of the proposed design conditions by a numerical example and in Section VII. some conclusions are established.

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Throughout the paper, the notations is narrowly standard in such way that $diag[\,\cdot\,]$ denotes a block diagonal matrix, $\boldsymbol{x}^T$, $\boldsymbol{X}^T$ denotes the transpose of the vector $\boldsymbol{x}$ and matrix $\boldsymbol{X}$, respectively, for a square matrix $\boldsymbol{X} < 0$ means that $\boldsymbol{X}$ is a symmetric negative definite matrix, $\boldsymbol{I}_n$ marks the n-th order unit matrix, $I\!R$ denotes the set of real numbers and $I\!R^n$, $I\!R^{n\times r}$ refer to the set of all n-dimensional real vectors and $n \times r$ real matrices, respectively.

II. Basic Preliminaries

In this paper, the discrete-time linear MIMO systems are considered, described in the state-space form by the set of equations

$$\boldsymbol{q}(i{+}1) = \boldsymbol{F}\boldsymbol{q}(i) + \boldsymbol{G}\boldsymbol{u}(i) + \boldsymbol{E}\boldsymbol{d}(i)\,, \tag{1}$$

$$\boldsymbol{y}(i) = \boldsymbol{C}\boldsymbol{q}(i)\,, \tag{2}$$

where $\boldsymbol{q}(i) \in I\!R^n$, $\boldsymbol{u}(i) \in I\!R^r$, and $\boldsymbol{y}(i) \in I\!R^m$ are vectors of the system, input and output variables, respectively, $\boldsymbol{d}(i) \in I\!R^p$ is a bounded unknown disturbance and $\boldsymbol{F} \in I\!R^{n\times n}$, $\boldsymbol{G} \in I\!R^{n\times r}$, $\boldsymbol{C} \in I\!R^{m\times n}$, $\boldsymbol{E} \in I\!R^{n\times p}$.

The transfer function matrices to (1), (2) are

$$\boldsymbol{H}(z) = \boldsymbol{C}(z\boldsymbol{I}_n - \boldsymbol{F})^{-1}\boldsymbol{G}\,, \tag{3}$$

$$\boldsymbol{H}_d(z) = \boldsymbol{C}(z\boldsymbol{I}_n - \boldsymbol{F})^{-1}\boldsymbol{E}\,, \tag{4}$$

where a complex z is the transform variable of the transform $\mathcal{Z}$ [17].

Quantifications of the effect of the input onto the output of the system are the so-called H_2 and H_∞ norms of the transfer function matrix $\boldsymbol{H}(z)$ and $\boldsymbol{H}_d(z)$, respectively.

Definition 1: [6] The H_2-norm of the transfer functions matrix (3) is defined as

$$\|\boldsymbol{H}(z)\|_2^2 = \frac{1}{2\pi} tr \int_{-\pi}^{\pi} \boldsymbol{H}(e^{j\omega})\boldsymbol{H}^*(e^{j\omega})\mathrm{d}\omega\,, \tag{5}$$

where $z = e^{j\omega}$, ω is the frequency variable, $j := \sqrt{-1}$ and $\boldsymbol{H}^*(e^{j\omega})$ is the adjoint of $\boldsymbol{H}(e^{j\omega})$.

Definition 2: [23] The H_∞-norm of the transfer function matrix (4) is defined as

$$\begin{aligned}\|\boldsymbol{H}_d(z)\|_\infty &= \sup_{\omega\in\langle-\pi,\pi\rangle} \sigma_o(\boldsymbol{H}_d(e^{j\omega})) = \\ &= \sup_{\omega\in\langle-\pi,\pi\rangle} \sigma_o(\mathrm{eig}(\boldsymbol{H}_d(e^{j\omega})\boldsymbol{H}_d^*(e^{j\omega}))\,,\end{aligned} \tag{6}$$

where σ_o means the largest singular value of the matrix $\boldsymbol{H}_d(e^{j\omega})$.

Definition 3: [12] A square matrix $\boldsymbol{F}$ is stable if every eigenvalue of $\boldsymbol{F}$ lies in the unit circle in the plain of the complex variable z. If $\boldsymbol{F}$ is stable, then the dynamical system (1), (2) has the stable transfer function matrix (3), i.e., the poles of all elements of $\boldsymbol{H}(z)$ lies in the unit circle in the plain of the complex variable z.

Proposition 1: [18] (Lyapunov inequality (LI)) The linear discrete-time system (1), (2) with a bounded disturbance is stable if and only if there exist a symmetric positive definite matrix $\boldsymbol{X} \in I\!R^{n\times n}$ such that

$$\boldsymbol{X} = \boldsymbol{X}^T > 0\,, \tag{7}$$

$$\begin{bmatrix} -\boldsymbol{X} & * \\ \boldsymbol{F}\boldsymbol{X} & -\boldsymbol{X} \end{bmatrix} < 0\,. \tag{8}$$

Hereafter, $*$ labels the symmetric item in a symmetric matrix.

Proposition 2: [3] (LMI region) A subset $\mathcal{D}$ of the complex plane $\mathcal{Z}$ is called a stable circle LMI region if

$$\mathcal{D} = \{z \in \mathbb{C}\colon \boldsymbol{f}_{\mathcal{D}}(z) < 0\}\,, \tag{9}$$

where $a, \varrho \in I\!R$, $\varrho \leq a$, $0 < a < 1$ and

$$\boldsymbol{f}_{\mathcal{D}}(z) = \begin{bmatrix} -\varrho & z^* - a \\ z - a & -\varrho \end{bmatrix} \tag{10}$$

is the LMI region characteristic function.

Related by the substitution

$$(\boldsymbol{X}, \boldsymbol{F}\boldsymbol{X}, \boldsymbol{X}\boldsymbol{F}^T) \leftrightarrow (1, z, z^*) \tag{11}$$

it yields

$$\boldsymbol{M}_{\mathcal{D}}(\boldsymbol{F},\boldsymbol{X}) = \begin{bmatrix} -\varrho\boldsymbol{X} & \boldsymbol{X}\boldsymbol{F}^T - a\boldsymbol{X} \\ \boldsymbol{F}\boldsymbol{X} - a\boldsymbol{X} & -\varrho\boldsymbol{X} \end{bmatrix} < 0 \tag{12}$$

and the matrix $\boldsymbol{F}$ of the discrete-time linear system (1), (2) is $\mathcal{D}$-stable if and only if there exists a symmetric positive definite matrix $\boldsymbol{X} \in I\!R^{n\times n}$ such that

$$\boldsymbol{X} = \boldsymbol{X}^T > 0\,, \tag{13}$$

$$\boldsymbol{M}_{\mathcal{D}}(\boldsymbol{F},\boldsymbol{X}) < 0\,. \tag{14}$$

Proposition 3: [5] (quadratic performance) If the matrix $\boldsymbol{F}$ of system (1), (2) is stable and $\boldsymbol{d}(i)$ is bounded then

$$\sum_{i=0}^{\infty}(\boldsymbol{y}^T(i)\boldsymbol{y}(i) - \gamma_\infty^2\boldsymbol{d}^T(i)\boldsymbol{d}(i)) > 0\,, \tag{15}$$

where $\gamma_\infty \in I\!R$ is the H_∞ norm of the discrete-time disturbance transfer function matrix (4).

Proposition 4: [6], [14] (bounded real lemma (BRL)) The discrete-time linear system (1), (2) with a bounded disturbance is stable if there exist a symmetric positive definite matrix $\boldsymbol{X} \in I\!R^{n\times n}$ and a positive scalar $\gamma_\infty \in I\!R$ such that

$$\boldsymbol{X} = \boldsymbol{X}^T > 0\,, \qquad \gamma_\infty > 0\,, \tag{16}$$

$$\begin{bmatrix} -\boldsymbol{X} & * & * & * \\ \boldsymbol{F}\boldsymbol{X} & -\boldsymbol{X} & * & * \\ \boldsymbol{C}\boldsymbol{X} & \boldsymbol{0} & -\gamma_\infty\boldsymbol{I}_m & * \\ \boldsymbol{0} & \boldsymbol{E}^T & \boldsymbol{0} & -\gamma_\infty\boldsymbol{I}_p \end{bmatrix} < 0\,. \tag{17}$$

Lemma 1: If the matrix $\boldsymbol{F}$ of the system (1), (2) is stable, then

$$\gamma_2^2 = \mathrm{tr}(\boldsymbol{C}\boldsymbol{W}_o\boldsymbol{C}^T)\,, \tag{18}$$

where

$$\boldsymbol{F}\boldsymbol{W}_o\boldsymbol{F}^T - \boldsymbol{W}_o + \boldsymbol{G}\boldsymbol{G}^T = \boldsymbol{0}\,, \tag{19}$$

while $\boldsymbol{W}_o \in I\!R^{n\times n}$ is a positive definite symmetric matrix and $\gamma_2 \in I\!R$ is H_2 norm of the discrete-time system transfer function matrix $\boldsymbol{H}(z)$.

Proof: Since a solution of (1), (2) is

$$\boldsymbol{q}(n) = \boldsymbol{F}^n\boldsymbol{q}(0) + \sum_{l=0}^{n-1}\boldsymbol{A}(l)\boldsymbol{u}(n-1-l)\,, \tag{20}$$

where

$$\boldsymbol{A}(l) = \boldsymbol{F}^l\boldsymbol{G}\,, \tag{21}$$

then as an explicit test for linear independence of $\boldsymbol{A}(l)$ can be used its Gramian [1]

$$\boldsymbol{W}(n) = \sum_{l=0}^{n-1}\boldsymbol{F}^l\boldsymbol{G}\boldsymbol{G}^T\boldsymbol{F}^{Tl}. \tag{22}$$

Pre-multiplying the left side of (22) by $\boldsymbol{F}$ and post-multiplying the right side by $\boldsymbol{F}^T$ results in

$$\begin{gathered}\boldsymbol{F}\boldsymbol{W}(n)\boldsymbol{F}^T = \\ = \sum_{l=0}^{n-1}\boldsymbol{F}^{l+1}\boldsymbol{G}\boldsymbol{G}^T(\boldsymbol{F}^T)^{l+1} = \sum_{l=1}^{n}\boldsymbol{F}^l\boldsymbol{G}\boldsymbol{G}^T\boldsymbol{F}^{Tl}\end{gathered} \tag{23}$$

and, subtracting (22) from (23), it yields

$$\boldsymbol{F}\boldsymbol{W}(n)\boldsymbol{F}^T - \boldsymbol{W}(n) = \boldsymbol{F}^n\boldsymbol{G}\boldsymbol{G}^T(\boldsymbol{F}^T)^n - \boldsymbol{G}\boldsymbol{G}^T. \tag{24}$$

Thus, considering that (20) for $l = n$ insert the input variable value $\boldsymbol{u}(-1)$ which is identically equal zero, and defining a stationary solution $\boldsymbol{W}(n) = \boldsymbol{W}_o$, then (24) implies

$$\boldsymbol{F}\boldsymbol{W}_o\boldsymbol{F}^T - \boldsymbol{W}_o + \boldsymbol{G}\boldsymbol{G}^T = \boldsymbol{0}\,. \tag{25}$$

This concludes the proof. ■

Lemma 2: The matrix $\boldsymbol{F}$ of the system (1), (2) is stable and $\|\boldsymbol{H}(z)\|_2 < \gamma_2$ if there exists a symmetric positive definite matrix $\boldsymbol{X} \in I\!R^{n\times n}$ such that

$$\boldsymbol{X} = \boldsymbol{X}^T > 0\,, \tag{26}$$

$$\boldsymbol{F}\boldsymbol{X}\boldsymbol{F}^T - \boldsymbol{X} + \boldsymbol{G}\boldsymbol{G}^T < 0\,, \tag{27}$$

$$\mathrm{tr}(\boldsymbol{C}\boldsymbol{X}\boldsymbol{C}^T) > \gamma_2^2. \tag{28}$$

Proof: Let (27) yields for a symmetric positive definite matrix $\boldsymbol{X}$. Then subtracting (19) from (27) leads to the strict inequality

$$\boldsymbol{F}(\boldsymbol{X} - \boldsymbol{W}_o)\boldsymbol{F}^T - (\boldsymbol{X} - \boldsymbol{W}_o) < 0 \tag{29}$$

and with $\boldsymbol{X} > \boldsymbol{W}_o$ the Lyapunov property implies that (29) is negative definite if and only if $\boldsymbol{F}$ is stable. Moreover, the relation $\boldsymbol{X} > \boldsymbol{W}_o$ gives

$$\mathrm{tr}(\boldsymbol{C}\boldsymbol{X}\boldsymbol{C}^T) > \mathrm{tr}(\boldsymbol{C}\boldsymbol{W}_o\boldsymbol{C}^T) = \gamma_2^2 \tag{30}$$

and so (30) implies (28). This concludes the proof. ■

III. State Feedback Design

By applying the controllable system (1), (2) and the control law

$$\boldsymbol{u}(i) = -\boldsymbol{K}\boldsymbol{q}(i)\,, \tag{31}$$

where $\boldsymbol{K} \in I\!R^{r\times n}$, then the closed-loop system description takes the form

$$\boldsymbol{q}(i{+}1) = (\boldsymbol{F}-\boldsymbol{G}\boldsymbol{K})\boldsymbol{q}(i) + \boldsymbol{E}\boldsymbol{d}(i) = \boldsymbol{F}_c\boldsymbol{q}(i) + \boldsymbol{E}\boldsymbol{d}(i)\,, \tag{32}$$

$$\boldsymbol{y}(i) = \boldsymbol{C}\boldsymbol{q}(i)\,, \tag{33}$$

where

$$\boldsymbol{F}_c = \boldsymbol{F} - \boldsymbol{G}\boldsymbol{K}\,. \tag{34}$$

Rewriting that

$$\boldsymbol{F}_c\boldsymbol{X} = \boldsymbol{F}\boldsymbol{X} - \boldsymbol{G}\boldsymbol{Y}\,, \quad \boldsymbol{Y} = \boldsymbol{K}\boldsymbol{X}\,, \tag{35}$$

the standard LMI design conditions are given by the following theorems (some of the proofs are omitted since evidently imply from the formulas stated in Section II).

Theorem 1: (LI synthesis) The control (31) to the system (1), (2) exists if there exist a symmetric positive definite matrix $\boldsymbol{X} \in I\!R^{n\times n}$ and a matrix $\boldsymbol{Y} \in I\!R^{r\times n}$ such that

$$\boldsymbol{X} = \boldsymbol{X}^T > 0\,, \tag{36}$$

$$\begin{bmatrix} -\boldsymbol{X} & * \\ \boldsymbol{F}\boldsymbol{X} - \boldsymbol{G}\boldsymbol{Y} & -\boldsymbol{X} \end{bmatrix} < 0\,. \tag{37}$$

When the above conditions hold, the control law gain is given as

$$\boldsymbol{K} = \boldsymbol{Y}\boldsymbol{X}^{-1}. \tag{38}$$

Theorem 2: (BRL synthesis) The control (31) to the system (1), (2) exists and $\|\boldsymbol{H}_d(z)\|_\infty < \gamma_\infty$ if there exist a symmetric positive definite matrix $\boldsymbol{X} \in I\!R^{n\times n}$, a matrix $\boldsymbol{Y} \in I\!R^{r\times m}$ and a positive scalar $\gamma_\infty \in I\!R$ such that

$$\boldsymbol{X} = \boldsymbol{X}^T > 0\,, \quad \gamma_\infty > 0\,, \tag{39}$$

$$\begin{bmatrix} -\boldsymbol{X} & * & * & * \\ \boldsymbol{F}\boldsymbol{X} - \boldsymbol{G}\boldsymbol{Y} & -\boldsymbol{X} & * & * \\ \boldsymbol{C}\boldsymbol{X} & \boldsymbol{0} & -\gamma_\infty \boldsymbol{I}_m & * \\ \boldsymbol{0} & \boldsymbol{E}^T & \boldsymbol{0} & -\gamma_\infty \boldsymbol{I}_p \end{bmatrix} < 0\,. \tag{40}$$

When the above conditions hold, the control law gain is given by (38).

Theorem 3: (D-stable LI synthesis) The control (31) to the system (1), (2) exists and the closed-loop eigenvalues are clustered in the D-stable circle region if for given $a, \varrho \in I\!R$, $\varrho \le a$, $0 < a < 1$ there exist a symmetric positive definite matrix $\boldsymbol{X} \in I\!R^{n\times n}$ and a matrix $\boldsymbol{Y} \in I\!R^{r\times n}$ such that

$$\boldsymbol{X} = \boldsymbol{X}^T > 0\,, \tag{41}$$

$$\begin{bmatrix} -\varrho\boldsymbol{X} & * \\ \boldsymbol{F}\boldsymbol{X} - \boldsymbol{G}\boldsymbol{Y} - a\boldsymbol{X} & -\varrho\boldsymbol{X} \end{bmatrix} < 0\,. \tag{42}$$

When the above conditions hold, the control law gain is given by (38).

Corollary 1: Considering the extended Lyapunov function

$$v(\boldsymbol{q}(i)) = \boldsymbol{q}^T(i)\boldsymbol{P}\boldsymbol{q}(i) + \frac{(1-\varrho)^2 - a^2}{1-\varrho}\sum_{l=0}^{i-1}\boldsymbol{q}^T(l)\boldsymbol{P}\boldsymbol{q}(l)\,, \tag{43}$$

where $a, \varrho \in I\!R$, $\varrho \le a$, $0 < a < 1$ and a positive definite matrix $\boldsymbol{P} \in I\!R^{n\times n}$, then it yields for the first forward difference of the Lyapunov function (43)

$$\begin{aligned} \Delta v(\boldsymbol{q}(i)) &= v(\boldsymbol{q}(i+1)) - v(\boldsymbol{q}(i)) = \\ &= \boldsymbol{q}^T(i+1)\boldsymbol{P}\boldsymbol{q}(i+1) - \boldsymbol{q}^T(i)\boldsymbol{P}\boldsymbol{q}(i) + \\ &\quad + \tfrac{(1-\varrho)^2 - a^2}{1-\varrho}\boldsymbol{q}^T(i)\boldsymbol{P}\boldsymbol{q}(i) < 0\,. \end{aligned} \tag{44}$$

Using the disturbance free part of (32) then (44) implies

$$\boldsymbol{q}^T(i)\big(\boldsymbol{F}_c^T\boldsymbol{P}\boldsymbol{F}_c - \boldsymbol{P} + \frac{(1-\varrho)^2 - a^2}{1-\varrho}\boldsymbol{P}\big)\boldsymbol{q}(i) < 0\,. \tag{45}$$

The negativeness of (45) demands to be valid

$$\boldsymbol{F}_c^T\boldsymbol{P}\boldsymbol{F}_c - \boldsymbol{P} + \frac{(1-\varrho)^2 - a^2}{1-\varrho}\boldsymbol{P} < 0 \tag{46}$$

and premultiplying the left side of (46) and postmultiplying the right side by the matrix $\boldsymbol{X} = \boldsymbol{P}^{-1}$ leads to

$$\boldsymbol{X}\boldsymbol{F}_c^T\boldsymbol{P}\boldsymbol{F}_c\boldsymbol{X} - \boldsymbol{X} + \frac{(1-\varrho)^2 - a^2}{1-\varrho}\boldsymbol{X} < 0\,. \tag{47}$$

Subsequently, the Schur complement property implies a reformulated form of (47) as

$$\begin{bmatrix} -\boldsymbol{X} + \frac{(1-\varrho)^2 - a^2}{1-\varrho}\boldsymbol{X} & \boldsymbol{X}\boldsymbol{F}_c^T \\ \boldsymbol{F}_c\boldsymbol{X} & -\boldsymbol{X} \end{bmatrix} < 0\,, \tag{48}$$

while, moreover,

$$\begin{aligned} &\tfrac{(1-\varrho)^2 - a^2}{1-\varrho}\boldsymbol{X} = \\ &= (1-\varrho)\boldsymbol{X} - (-a)\boldsymbol{X}(1-\varrho)^{-1}\boldsymbol{X}^{-1}(-a)\boldsymbol{X}\,. \end{aligned} \tag{49}$$

Thus, using the Schur complement property, either it yields

$$\begin{bmatrix} -\boldsymbol{X} & \boldsymbol{X}\boldsymbol{F}_c^T \\ \boldsymbol{F}_c\boldsymbol{X} & -\boldsymbol{X} \end{bmatrix} + \begin{bmatrix} (1-\varrho)\boldsymbol{X} & -a\boldsymbol{X} \\ -a\boldsymbol{X} & (1-\varrho)\boldsymbol{X} \end{bmatrix} < 0 \tag{50}$$

Thus, exploiting (35), it is evident that (50) is identical with (42) which implies that D-circle stability region in the design condition means a quadratic constraint on the Lyapunov function, acceptable in the sense of the Lyapunov-Krasovskii theorem [9].

Theorem 4: (H_2 control synthesis) The control (31) to the system (1), (2) exists and $\|\boldsymbol{H}(z)\|_2 < \gamma_2$ if there exist symmetric positive definite matrices $\boldsymbol{X} \in I\!R^{n\times n}$, $\boldsymbol{Z} \in I\!R^{m\times m}$ and a matrix $\boldsymbol{Y} \in I\!R^{r\times n}$ such that

$$\boldsymbol{X} = \boldsymbol{X}^T > 0\,, \quad \boldsymbol{Z} = \boldsymbol{Z}^T > 0\,, \tag{51}$$

$$\begin{bmatrix} -\boldsymbol{X} & \boldsymbol{F}\boldsymbol{X} - \boldsymbol{G}\boldsymbol{Y} & \boldsymbol{G} \\ * & -\boldsymbol{X} & \boldsymbol{0} \\ * & * & -\boldsymbol{I}_r \end{bmatrix} < 0\,, \tag{52}$$

$$\begin{bmatrix} \boldsymbol{X} & \boldsymbol{X}\boldsymbol{C}^T \\ * & \boldsymbol{Z} \end{bmatrix} > 0\,. \tag{53}$$

When the above conditions hold, the control law gain is given by (38).

Proof: Rearranging the inequality (27) by using the Schur complement property it yields

$$\begin{bmatrix} -\boldsymbol{X} & \boldsymbol{F}\boldsymbol{X} & \boldsymbol{G} \\ \boldsymbol{X}\boldsymbol{F}^T & -\boldsymbol{X} & \boldsymbol{0} \\ \boldsymbol{G}^T & \boldsymbol{0} & -\boldsymbol{I}_r \end{bmatrix} < 0\,. \tag{54}$$

Supplanting $\boldsymbol{F}$ in (54) by (35) modifies the LMI (54) as

$$\begin{bmatrix} -\boldsymbol{X} & \boldsymbol{F}\boldsymbol{X} - \boldsymbol{G}\boldsymbol{Y} & \boldsymbol{G} \\ \boldsymbol{X}\boldsymbol{F}^T - \boldsymbol{Y}^T\boldsymbol{G}^T & -\boldsymbol{X} & \boldsymbol{0} \\ \boldsymbol{G}^T & \boldsymbol{0} & -\boldsymbol{I}_r \end{bmatrix} < 0\,. \tag{55}$$

Prom this it follows easily (52).

By H_2 control nomination the inequality (28) could be minimized, but this form cannot be directly included into the set of LMIs. Introducing the inequality

$$\boldsymbol{Z} > \boldsymbol{C}\boldsymbol{X}\boldsymbol{C}^T = \boldsymbol{C}\boldsymbol{X}\boldsymbol{X}^{-1}\boldsymbol{X}\boldsymbol{C}^T, \tag{56}$$

with $\boldsymbol{Z} \in I\!R^{m\times m}$ being symmetric and positive definite, and applying appropriate the Schur complement property, then (56) implies (53). This concludes the proof. ■

Corollary 2: It is evident that

$$\eta = \mathrm{tr}(\boldsymbol{Z}) > \mathrm{tr}(\boldsymbol{C}\boldsymbol{X}\boldsymbol{C}^T) > \gamma_2^2. \tag{57}$$

IV. Multi-objective Design

An integration of the above presented approaches can be formulated by the multi-objective principle.

Theorem 5: (D-stable BRL synthesis) The control (31) to the system (1), (2) exists, $\|\boldsymbol{H}_d(z)\|_\infty < \gamma_\infty$ and the closed-loop system matrix eigenvalues are clustered in the D-stable circle region if for given $a, \varrho \in I\!R$, $\varrho \le a$, $0 < a < 1$ there exist a symmetric positive definite matrix $\boldsymbol{X} \in I\!R^{n\times n}$ a matrix $\boldsymbol{Y} \in I\!R^{r\times n}$ and a positive scalar $\gamma_\infty \in I\!R$ such that

$$\boldsymbol{X} = \boldsymbol{X}^T > 0\,, \quad \gamma_\infty > 0\,, \tag{58}$$

$$\begin{bmatrix} -\varrho\boldsymbol{X} & * & * & * \\ \boldsymbol{FX}-\boldsymbol{GY}-a\boldsymbol{X} & -\varrho\boldsymbol{X} & * & * \\ \boldsymbol{CX} & \boldsymbol{0} & -\gamma_\infty\boldsymbol{I}_m & * \\ \boldsymbol{0} & \boldsymbol{E}^T & \boldsymbol{0} & -\gamma_\infty\boldsymbol{I}_p \end{bmatrix} < 0\,. \tag{59}$$

When the above conditions hold, the control law gain is given by (38)

Proof: The proof of this theorem is a modification of the argument given in the Corollary 1.

Considering the extended Lyapunov function

$$\begin{aligned} v(\boldsymbol{q}(i)) = {} & \boldsymbol{q}^T(i)\boldsymbol{P}\boldsymbol{q}(i) + \tfrac{(1-\varrho)^2-a^2}{1-\varrho}\sum_{l=0}^{i-1}\boldsymbol{q}^T(l)\boldsymbol{P}\boldsymbol{q}(l) + \\ & +\gamma_\infty^{-1}\sum_{l=0}^{i-1}(\boldsymbol{y}^T(l)\boldsymbol{y}(l) - \gamma_\infty^2\boldsymbol{d}^T(l)\boldsymbol{d}(l))\,, \end{aligned} \tag{60}$$

where $\boldsymbol{P} \in I\!R^{n\times n}$ is a positive definite symmetric matrix and $\gamma_\infty \in I\!R$ is the H_∞ norm of the disturbance transfer function matrix, then using (2) it yields

$$\begin{aligned} \Delta v(\boldsymbol{q}(i)) = {} & \boldsymbol{q}^T(i{+}1)\boldsymbol{P}\boldsymbol{q}(i{+}1) - \boldsymbol{q}^T(i)\boldsymbol{P}\boldsymbol{q}(i) + \\ & +\gamma_\infty^{-1}\boldsymbol{q}^T(i)\boldsymbol{C}^T\boldsymbol{C}\boldsymbol{q}(i) - \gamma_\infty\boldsymbol{d}^T(i)\boldsymbol{d}(i) + \\ & +\tfrac{(1-\varrho)^2-a^2}{1-\varrho}\boldsymbol{q}^T(i)\boldsymbol{P}\boldsymbol{q}(i) < 0\,. \end{aligned} \tag{61}$$

Now let, with (32) and the notation

$$\boldsymbol{q}_c^T(i) = \begin{bmatrix} \boldsymbol{q}^T(i) & \boldsymbol{u}^T(i) \end{bmatrix} \tag{62}$$

the inequality (61) is written as

$$\Delta v(\boldsymbol{q}_c(i)) = \boldsymbol{q}_c^T(i)\boldsymbol{P}_c\boldsymbol{q}_c(i) < 0\,, \tag{63}$$

$$\begin{aligned} & \boldsymbol{P}_c = \\ & = \begin{bmatrix} \boldsymbol{F}_c^T\boldsymbol{P}\boldsymbol{F}_c - \boldsymbol{P} + \gamma_\infty^{-1}\boldsymbol{C}^T\boldsymbol{C} + \frac{(1-\varrho)^2-a^2}{1-\varrho}\boldsymbol{P} & \boldsymbol{PE} \\ \boldsymbol{E}^T\boldsymbol{P} & -\gamma_\infty\boldsymbol{I}_p \end{bmatrix}, \end{aligned} \tag{64}$$

while $\boldsymbol{P}_c < 0$, $\boldsymbol{P}_c \in I\!R^{(n+p)\times(n+p)}$. Defining the transform matrix

$$\boldsymbol{T} = \text{diag}\begin{bmatrix} \boldsymbol{X} & \boldsymbol{I}_p \end{bmatrix}, \qquad \boldsymbol{X} = \boldsymbol{P}^{-1} \tag{65}$$

and premultiplying the left side and postmultiplying the right side of (64) by the matrix $\boldsymbol{T}$ then

$$\begin{bmatrix} \boldsymbol{X}\boldsymbol{F}_c^T\boldsymbol{P}\boldsymbol{F}_c\boldsymbol{X} - \boldsymbol{X} + \gamma_\infty^{-1}\boldsymbol{X}\boldsymbol{C}^T\boldsymbol{C}\boldsymbol{X} + \frac{(1-\varrho)^2-a^2}{1-\varrho}\boldsymbol{X} & \boldsymbol{E} \\ \boldsymbol{E}^T & -\gamma_\infty\boldsymbol{I}_p \end{bmatrix} < 0\,. \tag{66}$$

As explained above, an equivalent form of (66) is

$$\begin{aligned} & \begin{bmatrix} -\varrho\boldsymbol{X} & \boldsymbol{X}\boldsymbol{F}_c^T - a\boldsymbol{X} \\ \boldsymbol{F}_c\boldsymbol{X} - a\boldsymbol{X} & -\varrho\boldsymbol{X} \end{bmatrix} + \\ & + \begin{bmatrix} \gamma_\infty^{-1}\boldsymbol{X}\boldsymbol{C}^T\boldsymbol{C}\boldsymbol{X} & \boldsymbol{0} \\ \boldsymbol{0} & \gamma_\infty^{-1}\boldsymbol{E}^T\boldsymbol{E} \end{bmatrix} < 0 \end{aligned} \tag{67}$$

Thus, using (35) and applying the Schur complement property then (67) implies (59). This concludes the proof. ■

V. Interposed Design Criteria

Combining the algorithms for H_2 and H_∞ control design, as well as the D-stable circle constraints, the following theorems can be introduced.

Theorem 6: (mixed H_2/H_∞ synthesis) The state feedback control (31) to the system (1), (2) exists and $\|\boldsymbol{H}(z)\|_2 < \gamma_2$ as well as $\|\boldsymbol{H}_d(z)\|_\infty < \gamma_\infty$ if there exist symmetric positive definite matrices $\boldsymbol{X} \in I\!R^{n\times n}$, $\boldsymbol{Z} \in I\!R^{m\times m}$, a matrix $\boldsymbol{Y} \in I\!R^{r\times n}$ and a positive scalar $\gamma_\infty \in I\!R$ such that

$$\boldsymbol{X} = \boldsymbol{X}^T > 0\,, \quad \boldsymbol{Z} = \boldsymbol{Z}^T > 0\,, \quad \gamma_\infty > 0\,, \tag{68}$$

$$\begin{bmatrix} -\boldsymbol{X} & * & * & * \\ \boldsymbol{FX}-\boldsymbol{GY} & -\boldsymbol{X} & * & * \\ \boldsymbol{CX} & \boldsymbol{0} & -\gamma_\infty\boldsymbol{I}_m & * \\ \boldsymbol{0} & \boldsymbol{E}^T & \boldsymbol{0} & -\gamma_\infty\boldsymbol{I}_p \end{bmatrix} < 0\,, \tag{69}$$

$$\begin{bmatrix} -\boldsymbol{X} & \boldsymbol{FX}-\boldsymbol{GY} & \boldsymbol{G} \\ * & -\boldsymbol{X} & \boldsymbol{0} \\ * & * & -\boldsymbol{I}_r \end{bmatrix} < 0\,, \tag{70}$$

$$\begin{bmatrix} \boldsymbol{X} & \boldsymbol{X}\boldsymbol{C}^T \\ * & \boldsymbol{Z} \end{bmatrix} > 0\,. \tag{71}$$

When the above conditions hold, the control law gain is given by (38).

Proof: Setting down a unique solution of $\boldsymbol{K}$ within the above conditions then (39), (40), (51)-(53) imply (68)-(71). This concludes the proof. ■

Theorem 7: (interposed H_2/H_∞ synthesis) The control (31) to the system (1), (2) exists, $\|\boldsymbol{H}_d(z)\|_\infty < \gamma_\infty$, $\|\boldsymbol{H}_d(z)\|_\infty < \gamma_\infty$ and the closed-loop system matrix eigenvalues are clustered in the D-stable circle region if for given $a, \varrho \in I\!R$, $\varrho \le a$, $0 < a < 1$ there exist symmetric positive definite matrices $\boldsymbol{X} \in I\!R^{n\times n}$, $\boldsymbol{Z} \in I\!R^{m\times m}$, a matrix $\boldsymbol{Y} \in I\!R^{r\times n}$ and a positive scalar $\gamma_\infty \in I\!R$ such that

$$\boldsymbol{X} = \boldsymbol{X}^T > 0\,, \quad \boldsymbol{Z} = \boldsymbol{Z}^T > 0\,, \quad \gamma_\infty > 0\,, \tag{72}$$

$$\begin{bmatrix} -\varrho\boldsymbol{X} & * & * & * \\ \boldsymbol{FX}-\boldsymbol{GY}-a\boldsymbol{X} & -\varrho\boldsymbol{X} & * & * \\ \boldsymbol{CX} & \boldsymbol{0} & -\gamma_\infty\boldsymbol{I}_m & * \\ \boldsymbol{0} & \boldsymbol{E}^T & \boldsymbol{0} & -\gamma_\infty\boldsymbol{I}_p \end{bmatrix} < 0\,. \tag{73}$$

$$\begin{bmatrix} -\boldsymbol{X} & \boldsymbol{FX}-\boldsymbol{GY} & \boldsymbol{G} \\ * & -\boldsymbol{X} & \boldsymbol{0} \\ * & * & -\boldsymbol{I}_r \end{bmatrix} < 0\,, \tag{74}$$

$$\begin{bmatrix} \boldsymbol{X} & \boldsymbol{X}\boldsymbol{C}^T \\ * & \boldsymbol{Z} \end{bmatrix} > 0\,. \tag{75}$$

When the above conditions hold, the control law gain is given by (38).

Proof: This is an immediate consequence of the above LMI conditions. ■

VI. Illustrative example

To illustrate the proposed method, a system whose dynamics is described by equations (1), (2) is considered with the sampling period $t_s = 0.01\ s$, the disturbance noise variance $\sigma_d^2 = 0.028$ and the matrix parameters

$$\boldsymbol{F} = \begin{bmatrix} 1.0142 & -0.0018 & 0.0651 & -0.0546 \\ -0.0057 & 0.9582 & -0.0001 & 0.0067 \\ 0.0103 & 0.0417 & 0.9363 & 0.0563 \\ 0.0004 & 0.0417 & 0.0129 & 0.9797 \end{bmatrix},$$

$$\boldsymbol{G}=\begin{bmatrix} 0.0000 & -0.0010 \\ 0.0556 & 0.0000 \\ 0.0125 & -0.0304 \\ 0.0125 & -0.0002 \end{bmatrix},\ \boldsymbol{E}=\begin{bmatrix} 0.0063 \\ 0.0216 \\ 0.0131 \\ 0.0044 \end{bmatrix},$$

$$\boldsymbol{C}=\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

Solving (39)–(40) using Self-Dual-Minimization (SeDuMi) package, the H_∞ control design problem is feasible while

$$\boldsymbol{X}=\begin{bmatrix} 0.0755 & 0.0645 & -0.0733 & -0.0054 \\ 0.0645 & 1.0991 & -0.0586 & -0.1332 \\ -0.0733 & -0.0586 & 0.4652 & 0.0062 \\ -0.0054 & -0.1332 & 0.0062 & 0.2011 \end{bmatrix},$$

$$\boldsymbol{Y}=\begin{bmatrix} -0.0073 & 1.3335 & 0.3439 & 1.0031 \\ -0.4794 & 0.0571 & -2.3094 & 0.3805 \end{bmatrix},$$

which results the control loop structure parameters

$$\boldsymbol{K}_1=\begin{bmatrix} -0.6037 & 2.0568 & 0.8192 & 6.3077 \\ -13.6687 & 0.7487 & -7.0524 & 2.2354 \end{bmatrix},$$

$$\rho(\boldsymbol{F}_c)=\left\{\ 0.8320\pm0.0842\,\mathrm{i}\quad 0.8952\pm0.0599\,\mathrm{i}\ \right\},$$

$$\gamma_\infty<1.7586,\quad \mathrm{tr}\,(\boldsymbol{CXC}^T)=0.7309>\gamma_2^2,$$

$$\text{Frobenius norm of } \boldsymbol{K}_1=16.9464\,.$$

Solving (68)-(71) then

$$\boldsymbol{X}=\begin{bmatrix} 0.1415 & 0.1030 & -0.1383 & -0.0049 \\ 0.1030 & 1.3972 & -0.1322 & -0.1814 \\ -0.1383 & -0.1322 & 0.6909 & 0.0183 \\ -0.0049 & -0.1814 & 0.0183 & 0.3182 \end{bmatrix},$$

$$\boldsymbol{Y}=\begin{bmatrix} -0.0732 & 3.0467 & 0.5694 & 1.5381 \\ -0.7805 & 0.0778 & -3.8454 & 0.6599 \end{bmatrix},$$

$$\boldsymbol{Z}=\begin{bmatrix} 1.3909 & -0.0854 \\ -0.0854 & 1.5655 \end{bmatrix},\quad \gamma_\infty<3.1650,$$

$$\boldsymbol{K}_2=\begin{bmatrix} -1.7621 & 3.2559 & 0.9198 & 6.6108 \\ -14.0272 & 0.6547 & -8.3200 & 2.7112 \end{bmatrix},$$

$$\rho(\boldsymbol{F}_c)=\left\{\ 0.7671\pm0.0499\,\mathrm{i}\quad 0.9058\pm0.0382\,\mathrm{i}\ \right\},$$

$$\mathrm{tr}\,\boldsymbol{Z}=2.9563>\mathrm{tr}\,(\boldsymbol{CXC}^T)=1.1409>\gamma_2^2,$$

$$\text{Frobenius norm of } \boldsymbol{K}_2=44.2207\,.$$

On the other side, applying the D-circle parameters $a=0.5$, $\rho=0.41$ the LMI-based conditions (58), (59) are solvable with

$$\boldsymbol{X}=\begin{bmatrix} 0.0243 & 0.0265 & -0.0579 & 0.0120 \\ 0.0265 & 1.4924 & -0.0659 & -0.0588 \\ -0.0579 & -0.0659 & 0.3791 & -0.0061 \\ 0.0120 & -0.0588 & -0.0061 & 0.0623 \end{bmatrix},$$

$$\boldsymbol{Y}=\begin{bmatrix} 0.0643 & 2.9413 & -0.0268 & 0.5633 \\ 0.0127 & -0.0821 & -3.0039 & 0.3096 \end{bmatrix},$$

which results the control loop structure parameters

$$\boldsymbol{K}_3=\begin{bmatrix} -8.2100 & 2.5999 & -0.6640 & 13.0181 \\ -37.4652 & 0.4664 & -13.3803 & 11.3358 \end{bmatrix},$$

$$\rho(\boldsymbol{F}_c)=\left\{\ 0.7421\pm0.0785\,\mathrm{i}\quad 0.8306\pm0.0817\,\mathrm{i}\ \right\},$$

$$\gamma_\infty<1.8527,\quad \mathrm{tr}\,(\boldsymbol{CXC}^T)=0.4897>\gamma_2^2,$$

$$\text{Frobenius norm of } \boldsymbol{K}_3=18.2214\,.$$

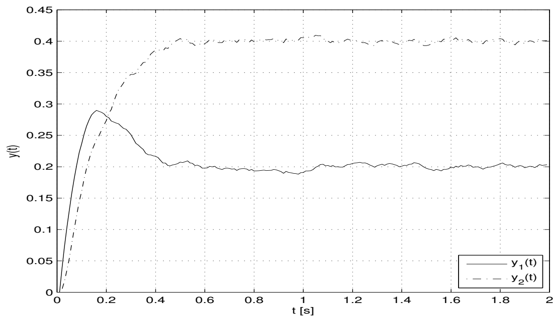

Fig. 1. Closed-loop system output response - H_∞

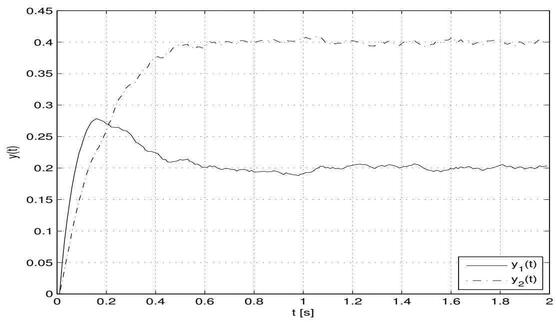

Fig. 2. Closed-loop system output response - H_2/H_∞

while the common solution of (68)–(71) gives the following result

$$\boldsymbol{Q}=\begin{bmatrix} 0.0411 & 0.0437 & -0.0969 & 0.0207 \\ 0.0437 & 2.0370 & -0.1092 & -0.0801 \\ -0.0969 & -0.1092 & 0.6430 & -0.0142 \\ 0.0207 & -0.0801 & -0.0142 & 0.0958 \end{bmatrix},$$

$$\boldsymbol{Y}=\begin{bmatrix} 0.0982 & 4.7108 & -0.0011 & 0.9448 \\ -0.0353 & 0.0164 & -5.2944 & 0.5093 \end{bmatrix},$$

$$\boldsymbol{Z}=\begin{bmatrix} 1.2588 & -0.0725 \\ -0.0725 & 1.5552 \end{bmatrix},\quad \gamma_\infty<3.0524,$$

$$\boldsymbol{K}_4=\begin{bmatrix} -9.4462 & 3.0486 & -0.5891 & 14.3629 \\ -40.7424 & 0.6257 & -13.9865 & 12.5492 \end{bmatrix},$$

$$\rho(\boldsymbol{F}_c)=\left\{\ 0.7210\pm0.0856\,\mathrm{i}\quad 0.8196\pm0.0616\,\mathrm{i}\ \right\},$$

$$\mathrm{tr}\,\boldsymbol{Z}=2.8140>\mathrm{tr}\,(\boldsymbol{CXC}^T)=0.8213>\gamma_2^2.$$

$$\text{Frobenius norm of } \boldsymbol{K}_4=48.1519\,.$$

All simulations are done in the forced mode, where

$$\boldsymbol{u}(i)=-\boldsymbol{K}_j\boldsymbol{q}(i)+\boldsymbol{W}_j\boldsymbol{w}_o\,,\ j\in\langle 1,4\rangle\,,$$

the set of gain matrices is given above and the associated signal gain matrices $\boldsymbol{W}_j$ are computed by using the static decoupling principle [26] as

$$\boldsymbol{W}_j=(\boldsymbol{C}(\boldsymbol{I}_n-(\boldsymbol{F}-\boldsymbol{GK}_j))^{-1}\boldsymbol{G})^{-1}.$$

Therefore, the signal gain matrices are given as

$$\boldsymbol{W}_1=\begin{bmatrix} 0.9608 & 7.6607 \\ -9.7879 & 7.6909 \end{bmatrix},\ \boldsymbol{W}_2=\begin{bmatrix} -3.8384 & 19.8243 \\ -27.2241 & 32.4430 \end{bmatrix},$$

$$\boldsymbol{W}_3=\begin{bmatrix} 0.1562 & 9.3619 \\ -9.9902 & 7.1958 \end{bmatrix},\ \boldsymbol{W}_4=\begin{bmatrix} -4.5478 & 22.4230 \\ -29.6214 & 36.1152 \end{bmatrix}.$$

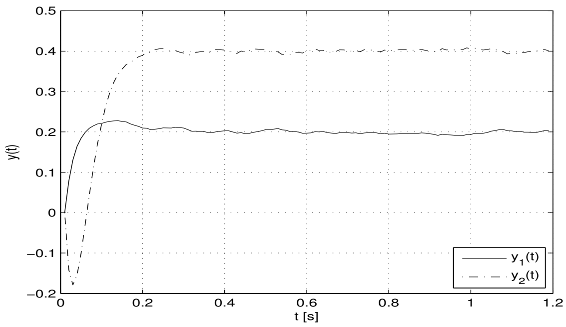

Fig. 3. Closed-loop system output response - D/H_∞

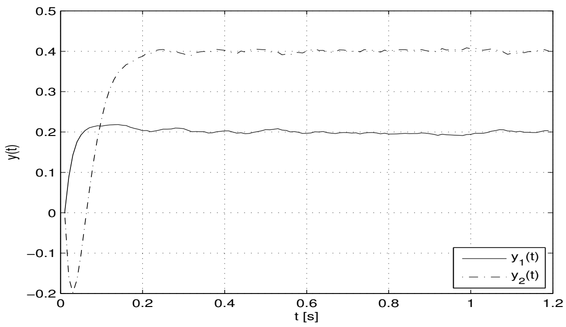

Fig. 4. Closed-loop system output response - $H_2/D/H_\infty$

The trajectories of the output of this system are drawn for the system state initial vector $\boldsymbol{q}(0) = \mathbf{0}$ and the desired steady state vector of the output variables $\boldsymbol{w}_o^T = [\,0.2 \;\; 0.4\,]$. The simulation results for H_∞ and H_2/H_∞ methodology as well as D/H_∞ and $H_2/D/H_\infty$ principle are presented with respect to the closed-loop systems responses in the Fig. 1 - Fig. 4.

From the numerical results above it can see that by adding an additional H_2 constraint, these new conditions can provide comparable H_∞-norm of the closed-loop disturbance transfer function but substantially decrease the value of H_2-norm of the closed-loop system transfer function. Consequently, the feasible solutions can be obtained in the same manner.

It is natural that in terms of closed loop system dynamics, it is essential to define the pole cluster of the closed loop characteristic polynomial by using the D-stability region, also because it is linked to a common matrix of the Ljapunov function verifying the closed-loop stability.

VII. Concluding remarks

This paper modifies the use of a control design approach, destined for MIMO linear systems with disturbance attenuations by using γ_2 and γ_∞ norms of the closed-loop discrete-time transfer function matrices, solving the interposed $H_2/D/H_\infty$ control design task. Using a originally constructed extended BRL with D-stability region parameters, the problems of H_∞ and H_2/H_∞ control are redefined and proven, documenting that the D-circle stability region means a new quadratic constraint on the Lyapunov function in the discrete system stability condition.

The proposed control design method is linear and established as a set of LMIs utilizing quadratic constraints. This design strategy is easy implementable, making it an eligible method to factual applications. The numerical example is given to show the feasibility and advantage of the criteria.

References

[1] P. J. Antsaklis and A. N. Michel, *A Linear Systems Primer*, Boston, MA, USA: Birkhäuser, 2007.

[2] R. Bambang, E. Shimemura and K. Uchjida, "Mixed H_2/H_∞ control with pole placement. State feedback case," *Proc. 2009 American Control Conf.*, San Francisco, CA, USA, pp. 2777-2779, 1993.

[3] M. Chilali and P. Gahinet, "H_∞ design with pole placement constraints. An LMI Approch", *IEEE Tran. Automatic Control*, vol. 41, no. 3, pp. 358-361, 1996.

[4] J. C. Doyle, K. Glover, P. P. Khargonekar, and B. A. Francis, "State-space solutions to standard H_2 and H_∞ control problems", *IEEE Tran. Automatic Control*, vol. 34, no. 8, pp. 831-847, 1989.

[5] A. Filasová and D. Krokavec, "H_∞ control of discrete-time linear systems constrained in state by equality constraints," *Int. J. Applied Mathematics and Computer Science*, vol. 22, no. 3, pp. 551-560, 2012.

[6] B. A. Francis, *A Course in H_∞ Control Theory*, Berlin, Germany: Springer-Verlag, 1987.

[7] J. C. Geromel, P. L. D. Peres and S. R. Souza, "Mixed H_2/H_∞ control for continuous-time linear systems," *Proc. 31st Conf. Decision and Control*, Tuscon, AZ, USA, pp. 3717-3722, 1992.

[8] E. N. Goncalves, R. M. Palhares and R. H. C. Takahashi, "Multiobjective optimization applied to robust H_2/H_∞ state-feedback control synthesis," *Proc. 2004 American Control Conference*, Boston, MA, USA, pp. 4619-4624, 2004.

[9] W. M. Haddad and V. Chellaboina, *Nonlinear Dynamical Systems and Control. A Lyapunov-Based Approach*, Princeton, NJ, USA: Princeton University Press, 2008.

[10] M. Hou and R. J. Patton, "An LMI approach to $H\_/H_\infty$ fault detection observers," *Proc. UKACC Int. Conf. Control CONTROL'96*, Exeter, England, pp. 305-10, 1996.

[11] M. Kchaou, M. Souissi, and A. Toumi, "Robust H_∞ output feedback control with pole placement constraints for uncertain discrete-time fuzzy systems," *Soft Computing*, vol. 16, no. 4, 13p, 2012.

[12] H. K. Khalil, *Nonlinear Systems*, Englewood Cliffs, NJ, USA: Prentice Hall, 2002.

[13] P. P. Khargonekar and M. A. Rotea, "Mixed H_2/H_∞ control. A Convex optimization approach," *IEEE Tran. Automatic Control*, vol. 36, no. 7, pp. 824-831, 1991.

[14] D. Krokavec and A. Filasová, "On pole placement LMI constraints in control design for linear discrete-time systems," *Proc. 19th Int. Conf. Process Control PC'2013*, Štrbské Pleso, Slovakia, pp. 69-74, 2013.

[15] D. Krokavec and A. Filasová, "Discrete-time state feedback control design for linear systems," *Proc. 20th Int. Conf. Process Control PC'2015*, Štrbské Pleso, Slovakia, pp. 7-12, 2015.

[16] M. Meisami-Azad, J. Mohammadpour, and K. M. Grigoriadis, "Upper bound mixed H_2/H_∞ control and integrated design for collocated structural systems," *Proc. 2009 American Control Conf.*, St. Louis, MO, USA, pp. 4563-4568, 2009.

[17] K. Ogata, *Discrete-Time Control Systems*, Upper Saddle River, NJ, USA: Prentice-Hall, 1995.

[18] M. C. Oliveira de, J. Bernussou and J. C. Geromel, "A new discrete-time robust stability condition," *Systems & Control Letters*, vol. 37, no. 4, pp. 261-265, 1999.

[19] Y. V. Orlov and L. T. Aguilar, *Advanced H_∞ Control. Towards Nonsmooth Theory and Applications*, New York, NY, USA: Springer Science, 2014.

[20] M. A. Rotea and P. P. Khargonekar "H_2-optimal control with an H_∞ constraint. The state feedback case," *Automatica*, vol. 27, no. 2, pp. 307-316, 1991.

[21] A. Schaft van der, *L_2-Gain and Passivity Techniques in Nonlinear Control*, London, UK: Springer-Verlag, 2000.

[22] C. Scherer, "Mixed H_2/H_∞ control," *Trends in Control. A European Perspective*, A. Isidori Ed. Berlin, Germany: Springer-Verlag, pp. 173-216, 1995.

[23] R. E. Skelton, T. Iwasaki, and K. Grigoriadis, *A Unified Algebraic Approach to Linear Control Design*, London, UK: Taylor & Francis, 1998.

[24] A. A. Stoorvogel, *The H_∞ Control Problem. State-Space Approach*, Englewood Cliffs, N.J., USA: Prentice-Hall, 1992.

[25] J. G. Van Antwerp and R. D. Braatz, "A tutorial on linear and bilinear matrix inequalities," *J. Process Control*, vol. 10, pp. 363-385, 2000.

[26] Q. G. Wang, *Decoupling Control*, Berlin, Germany: Springer-Verlag, 2003.

[27] A. G. Wu and G. R. Duan, "Enhanced LMI representations for H_2 performance of polytopic uncertain systems. Continuous-time case," *Int. J. Automation and Computing*, vol. 3, no. 3, pp. 304-308, 2006.