Dual Iterative Methods for Nonlinear Total Resource Allocation Problems in Telecommunication Networks

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***Abstract*—We consider problems of optimal resource allocation in zonal telecommunication networks with many users. In the simplest formulation the network manager aims to distribute some homogeneous resource (say bandwidth) among users within one region with possible utilization of external resources. We suggest to apply the bi-section method for the nonlinear dual problem with a family of independent nonlinear problems. Next, we consider a more general resource allocation problem where the region a wireless communication network is divided into zones (clusters) and develop an extension of the above dual decomposition method for this problem. We present results of computational experiments which confirm the efficiency of the new method in comparison of the previous ones.**

***Index Terms*—Resource allocation, wireless networks, multi-zonal networks, nonlinear functions, convex optimization, Lagrangian duality methods, decomposition.**

I. Introduction

Despite the existence of powerful processing and transmission devices, increasing demand of different telecommunication services and its variability lead to serious congestion effects and inefficient utilization of network resources; e.g., bandwidth and batteries capacity. This situation forces one to replace the fixed allocation rules with more flexible mechanisms, which are based on proper mathematical models; see e.g. [1]–[3]. In particular, spectrum sharing is now one of the most critical issues in this field and various adaptive mechanisms have been suggested. Most papers are devoted either to game-theoretic or various optimization based models whose implementation is based on decentralized iterative methods; see e.g. [4], [5], [6], [3], [7], [8].

In [9], [10], [11], several optimal resource allocation problems in telecommunication networks and proper decomposition based methods were suggested. They assumed that the network manager can satisfy all the varying users requirements. However, the resource of one network may be not sufficient in some time periods due to instable behavior of many users, hence the network manager can buy additional volumes of the resource. We note that such a strategy is rather typical for telecommunication networks, where WiFi or femtocell communication services are utilized in addition to the usual network resources; see e.g. [12].

In this paper, we consider just this extended formulation of the resource allocation problem with possible external resources, which was proposed in [13]. That is, the network manager also aims to maximize the network profit from allocation of a homogeneous resource (bandwidth) among users of a telecommunication network. As usual, the network income is received from users payments but the suitable network service level requires proper expenses. Besides, the network manager can utilize the resource of the other (external) networks, which implies the proper charge, and the problem involves the balance and capacity constraints. The first convex optimization formulation of this problem was proposed in [13]. In [13], it was suggested to solve the problem with a hierarchical method that reduces this problem to a sequence of multi-level one-dimensional problems. In [14], a dual decomposition Lagrangian method with solution of zonal optimization problem with the conditional gradient method was proposed. Due to the linearization we obtain a family of simple two-side allocation problems with fixed prices, each of them can be solved in a finite number of iterations by a simple arrangement type procedure. The performance of the method from [14] appeared better essentially in comparison with that from [13].

In this paper, we consider the nonlinear convex optimization problem of optimal allocation of resources in a telecommunication network where the network manager can take the resource of external networks. We first consider this optimization problem for the case where all the users are situated in one region. This means that conditions (data) for all the parts of

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this region are almost the same. We now suggest to apply the dual Lagrangian method with respect to the balance constraint without utilization of the conditional gradient method. This approach enables us to apply the bisection method for the nonlinear dual problem with a family of independent non-linear problems. Next, we consider a more general resource allocation problem where the region a wireless communication network is divided into zones (clusters) and develop suitable extensions of the above dual decomposition methods for this problem. We present results of computational experiments which confirm the efficiency of the new method in comparison of the previous ones.

II. Single-zonal resource allocation model

The single-zonal problem of a telecommunication network manager is to find an optimal allocation of a resource among the users in order to maximize the total payment received from the users and to minimize the total network maintenance expenses. That is, x is an unknown quantity of the resource offered by the network, within the capacity bounds $x \in [0, b]$, which yields the network expense (cost of implementation) $u(x)$. Similarly, y_i is the unknown resource offered to user $i \in I$ and $\varphi_i(y_i)$ is the fee (incentive) value paid by user i within the capacity bounds $y_i \in [0, a_i]$, where I is the index set of users. The network manager can take some external resource value $z_j \in [0, c_j], j \in J$ (J denotes the index set of external network providers). Clearly, these values require proper payments $h_j(z_j), j \in J$. The problem is formulated as follows:

$$\max_{(x,y,z)\in D} \to \sum_{i\in I} \varphi_i(y_i) - u(x) - \sum_{j\in J} h_j(z_j), \tag{1}$$

where $z = (z_j)_{j\in J}$, $y = (y_i)_{i\in I}$,

$$D = \left\{ (x, y, z) \;\middle|\; \begin{array}{l} \sum_{i\in I} y_i = x + \sum_{j\in J} z_j, \\ 0 \le y_i \le a_i,\ i \in I,\ 0 \le x \le b, \\ 0 \le z_j \le c_j,\ j \in J \end{array} \right\}.$$

Suppose that the set D is non-empty, the functions $u(x)$ and $h_j(z_j)$, $j \in J$ are strictly convex, whereas all the functions $\varphi_i(y_i)$, $i \in I$ are strictly concave. Then (1) is a convex optimization problem with a unique solution. Set $g(x) = u'(x)$, $v_j(z_j) = h'_j(z_j)$ and $w_i(y_i) = \varphi'_i(y_i)$. The necessary and sufficient optimality condition for problem (1) is written in the form of the variational inequality: find $(\bar{x}, \bar{y}, \bar{z}) \in D$ such that

$$\begin{array}{l} g(\bar{x})(x - \bar{x}) + \sum_{j\in J} v_j(\bar{z}_j)(z_j - \bar{z}_j) \\ - \sum_{i\in I} w_i(\bar{y}_i)(y_i - \bar{y}_i) \ge 0, \quad \forall (x, y, z) \in D. \end{array} \tag{2}$$

Moreover, they are equivalent to the following conditions:

$$(\bar{x}, \bar{y}, \bar{z}) \in D, \exists \bar{p},$$

$$g(\bar{x}) \begin{cases} \ge \bar{p} & \text{if } \bar{x} = 0, \\ = \bar{p} & \text{if } \bar{x} \in (0, b), \\ \le \bar{p} & \text{if } \bar{x} = b; \end{cases} \tag{3}$$

$$v_j(\bar{v}_j) \begin{cases} \ge \bar{p} & \text{if } \bar{z}_j = 0, \\ = \bar{p} & \text{if } \bar{z}_j \in (0, c_j), \\ \le \bar{p} & \text{if } \bar{z}_j = c_j, \end{cases} \quad \text{for } j \in J; \tag{4}$$

$$w_i(\bar{y}_i) \begin{cases} \le \bar{p} & \text{if } \bar{y}_i = 0, \\ = \bar{p} & \text{if } \bar{y}_i \in (0, a_i), \\ \ge \bar{p} & \text{if } \bar{y}_i = a_i, \end{cases} \quad \text{for } i \in I. \tag{5}$$

This problem is treated as above two-side market equilibrium model; see [15]–[17] for more details. In the case where all the functions $u(x)$, $h_j(z_j)$, and $\varphi_i(y_i)$ are affine, (3)–(5) is a market equilibrium problem with fixed prices, which can be solved by a simple ordering algorithm; see [17]. The well-known conditional gradient method (see [18]–[20]) just reduces (1) to a sequence of problems of form (2) with fixed prices. For the sake of simplicity, we describe this method for the general format:

$$\min_{\tilde{w}\in D} \to \quad \eta(\tilde{w}),\ \tilde{w} = (x, y, z).$$

(CGM) Take an arbitrary initial point $\tilde{w}^0 \in D$ and a number $\delta > 0$. At the s-th iteration, $s = 0, 1, \ldots$, we have a point $\tilde{w}^s \in D$ and calculate $\tilde{w}^s \in D$ as a solution of the linear programming problem

$$\min_{\tilde{u}\in D} \to \langle \eta'(\tilde{w}^s), \tilde{u} \rangle. \tag{6}$$

Then we set $p^s = \tilde{u}^s - \tilde{w}^s$. If $\langle \eta'(\tilde{w}^s), p^s \rangle \ge -\delta$, stop, we have an approximate solution. Otherwise we find the next iterate $\tilde{w}^{s+1} = \tilde{w}^s + \theta_s p^s$, where $\theta_s \in (0, 1)$ is a stepsize parameter.

The stepsize θ_s can be chosen with the inexact line search procedure: Find m as the minimal non-negative integer such that

$$\eta(\tilde{w}^s + \gamma^m p^s) \le \eta(\tilde{w}^s) + \alpha\gamma^m \langle \eta'(\tilde{w}^s), p^s \rangle,$$

for some $\alpha \in (0, 1)$ and $\gamma \in (0, 1)$, and set $\theta_s = \gamma^m$; see [20]. Observe that (6) coincides (2) where the derivatives (prices) are fixed at $\tilde{w}^s$.

Let us now describe the bi-section algorithm **(BS)**. In [11], it was proposed for problems with affine prices and without external providers. First we write the Lagrange function of problem (1) with the negative sign:

$$\begin{aligned} M(x,y,z,p) = \quad & u(x)+\sum_{j\in J}h_j(z_j)-\sum_{i\in I}\varphi_i(y_i) \\ & -p\left(x+\sum_{j\in J}z_j-\sum_{i\in I}y_i\right) \\ = \quad & (u(x)-px)+\sum_{j\in J}(h_j(z_j)-pz_j) \\ & -\sum_{i\in I}(\varphi_i(y_i)-py_i). \end{aligned}$$

In order to find a value of the dual cost function

$$\theta(p)=\min_{x\in[0,b],\ y\in[0,a],\ z\in[0,c]} M(x,y,z,p),$$

where $a=(a_i)_{i\in I}$ and $c=(c_j)_{j\in J}$, we have to solve one-dimensional problems:

$$\min_{0\le x_k\le b_k} \to (u(x)-px), \tag{7}$$

$$\min_{0\le z_j\le c_j} \to (h_j(z_j)-pz_j),\ \text{for } j\in J, \tag{8}$$

$$\min_{0\le y_i\le a_i} \to (-\varphi_i(y_i)+py_i),\ \text{for } i\in I. \tag{9}$$

Solutions of these problems denoted by $x(p)$, $z_j(p)$, $j\in J$, and $y_i(p)$, $i\in I$, respectively, are defined uniquely.

It follows that the function $\theta(p)$ is concave and differentiable with

$$\theta'(p)=\sum_{i\in I}y_i(p)-x(p)-\sum_{j\in J}z_j(p).$$

Besides, the one- dimensional dual problem

$$\max_p \to \theta(p)$$

coincides with the simple equation

$$\theta'(p)=0, \tag{10}$$

where $\theta'(p)$ is non-increasing. If p^* is a solution of (10), then we can find the solution of the initial problem (1) from (7)–(9) at $p=p^*$; cf. (3)–(5).

Set $p'=\min\{g(0),\min_{j\in J}v_j(0)\}$. Then, due to the monotonicity $p'<g(b)$ and $p'<v_j(c_j)$ for all $j\in J$. Next, set $p''=\max_{i\in I}w_i(0)$. Then, due to the anti-monotonicity $p''>w_i(a_i)$ for all $i\in I$.

Suppose that $p''\le p'$. Then taking $\bar p\in[p'',p']$ and using (3)–(5) or (7)–(9) gives immediately the zero solutions. So, we can consider only the non-trivial case where $p'<p''$. Take $p=p'$. Then $x(p)=0$ and $z_j(p)=0$, $j\in J$, are the unique solutions of problems (7)–(8). Besides, there exists at least one index $i\in I$ such that $y_i(p)>0$. It follows that $\theta'(p')>0$. Now take $p=p''$. Then $y_i(p)=0$, $i\in I$, are the unique solutions of problems (9). Besides, either $x(p)>0$ or there exists at least one index $j\in J$ such that $z_j(p)>0$. It follows that $\theta'(p'')<0$. These properties enable us to find a solution of (3) by the simple bisection procedure, denoted as Algorithm **(BS)**.

Given an accuracy $\varepsilon>0$ and the initial segment $[p',p'']$, we take $\tilde p=0.5(p'+p'')$, calculate $\theta'(\tilde p)$. Then we set $p'=\tilde p$ if $\theta'(\tilde p)>0$ and $p'=\tilde p$ otherwise, until $(p''-p')<\varepsilon$.

III. Multi-zonal network problem

Let us consider a more general model where a telecommunication network is divided into several zones (clusters). This means that conditions (data) for different zones vary for both providers and users. The problem of a manager of the network is to find the optimal allocation of all the resources, including those received from external providers. Besides, the own network resource volume is limited. The problem hence again consists in maximizing the total profit containing the total income from consumers' fees minus maintenance expenses and payments to the external providers.

Let us use the following notation:

- n is the number of zones;
- I_k is the index set of users (currently) located in zone k $(k=1,\dots,n)$;
- J_k is the index set of external providers (currently) located in zone k $(k=1,\dots,n)$;
- B is the total resource supply (the total bandwidth) for the system (network);
- x_k is an unknown quantity of the resource allotted to zone k with the upper bound b_k and $f_k(x_k)$ is the maintenance expenses for this quantity of the resource for zone k $(k=1,\dots,n)$;
- z_j is an unknown quantity of the additional resource bought from j-th external provider in zone k with the upper bound c_j and $h_j(z_j)$ is the payment to the j-th external provider for utilization of this quantity z_j the additional resource in zone k $(j\in J_k,\ k=1,\dots,n)$;
- y_i is the resource amount received by user i with the upper bound a_i and $\varphi_i(y_i)$ is the charge value paid by user i for the resource value y_i.

The multi-zonal network manager problem is formulated as follows:

$$\max \to \sum_{k=1}^{n}\left[\sum_{i\in I_k}\varphi_i(y_i)-f_k(x_k)-\sum_{j\in J_k}h_j(z_j)\right], \tag{11}$$

subject to

$$\sum_{k=1}^{n}x_k\le B; \tag{12}$$

$$\sum_{i\in I_k}y_i=x_k+\sum_{j\in J_k}z_j,\ k=1,\dots,n; \tag{13}$$

$$\begin{aligned} &0\le y_i\le a_i,\ i\in I_k,\ 0\le x_k\le b_k,\\ &0\le z_j\le c_j,\ j\in J_k,\ k=1,\dots,n. \end{aligned} \tag{14}$$

That is, (13) is the balance equation for demand and supply in each zone, (14) contains capacity constraints for users and network supply values in each zone, and (12) gives the upper bound for the total resource supply.

In what follows we assume that there exists at least one feasible point satisfying conditions (12)–(14), all the functions $f_k(x_k)$ and $h_j(z_j)$ are strictly convex, whereas all the functions $\varphi_i(y_i)$, $i \in I$ are strictly concave. This means that (11)–(14) is a convex optimization problem. However, due to large dimensionality and inexact data one can meet serious drawbacks in solving this problem with custom iterative solution methods. In order to create an efficient method we have to take into account its separability and apply certain decomposition approach. However, the standard duality approach using the Lagrangian function with respect to all the functional constraints leads to the multi-dimensional dual optimization problem. We will apply another approach, which was suggested in [21]. Let us define the Lagrange function of problem (11)–(14) as follows:

$$L(x,y,z,\lambda) = \sum_{k=1}^{n}\left[\sum_{i\in I_k}\varphi_i(y_i) - f_k(x_k) - \sum_{j\in J_k} h_j(z_j)\right] -\lambda\left(\sum_{k=1}^{n} x_k - B\right).$$

We utilize the Lagrangian multiplier λ only for the upper bound constraint of the total resource supply (13). We can now write the dual for problem (11)–(14):

$$\min_{\lambda\geq 0} \to \psi(\lambda), \tag{15}$$

where

$$\psi(\lambda) = \max_{(x,y,z)\ \in\ W} L(x,y,z,\lambda)$$
$$= \lambda B + \max_{(x,y,z)\ \in\ W} \sum_{k=1}^{n}\left[\sum_{i\in I_k}\varphi_i(y_i) - f_k(x_k) -\lambda x_k - \sum_{j\in J_k} h_j(z_j)\right],$$

and

$$W = \left\{ (x,y,z) \;\middle|\; \begin{array}{l} \sum_{i\in I_k} y_i = x_k + \sum_{j\in J_k} z_j, \\ 0 \leq y_i \leq a_i,\ i\in I_k, \\ 0\leq z_j \leq c_j,\ j \in J_k, \\ 0 \leq x_k \leq b_k,\ k=1,\dots,n \end{array} \right\}.$$

By duality (see e.g. [22], [23]), problems (11)–(14) and (15) have the same optimal value. But solution of (15) can be found by one of well-known single-dimensional optimization algorithms; see e.g. [23]. In order to calculate the value of $\psi(\lambda)$ we have to solve the inner problem:

$$\max \to \sum_{k=1}^{n}\left[\sum_{i\in I_k}\varphi_i(y_i) - f_k(x_k) - \lambda x_k - \sum_{j\in J_k} h_j(z_j)\right],$$

subject to

$$\sum_{i\in I_k} y_i = x_k + \sum_{j\in J_k} z_j,$$
$$0 \leq y_i \leq a_i,\ i\in I_k,$$
$$0\leq z_j \leq c_j,\ j \in J_k,$$
$$0 \leq x_k \leq b_k,\ k=1,\dots,n.$$

Obviously, this problem decomposes into n independent zonal optimization problems

$$\max \to \left[\sum_{i\in I_k}\varphi_i(y_i) - f_k(x_k) - \lambda x_k - \sum_{j\in J_k} h_j(z_j)\right], \tag{16}$$

subject to

$$\sum_{i\in I_k} y_i = x_k + \sum_{j\in J_k} z_j,$$
$$0 \leq y_i \leq a_i,\ i\in I_k,$$
$$0\leq z_j \leq c_j,\ j \in J_k,$$
$$0 \leq x_k \leq b_k,$$

for $k = 1,\dots,n$. Each k-th independent zonal problem (16) clearly coincides with problem (1) where $u(x) = f_k(x_k) + \lambda x_k$, $I = I_k$, $J = J_k$. Therefore, we can find its solution by the algorithms described in Section II.

IV. Numerical experiments

In order to evaluate the performance of the methods we made a number of computational experiments. We denote by (DBS) the above dual method with bi-section, and by (CGDM) the above dual method with the conditional gradient method. The methods were implemented in C++ with a PC with the following facilities: Intel(R) Core(TM) i7-4500, CPU 1.80 GHz, RAM 6 Gb.

The initial intervals for choosing the dual variable λ were taken as [0,1000]. Values of b_k were chosen by trigonometric functions in $[1, 51]$, values of a_i were chosen by trigonometric functions in $[1, 2]$, values of c_j were chosen by trigonometric functions in $[1, 10]$. Value B were taken equal 1000. Values γ and α in (CGDM) was chosen to be 0.7 and 0.4, respectively. The number of external providers in all the zones was equal and varied from 0 to 10. The number of zones was varied from 5 to 105, the number of users was varied from 210 to 10010. Users were distributed in zones either uniformly or according to the normal distribution.

For all the methods of finding solution of problem (11)–(14) the accuracy of upper dual problem solution were varied from 10^{-1} to 10^{-4}. The accuracy of lower level problem solution was fixed and equal to 10^{-2}. For each set of the parameters made 50 tests. Let I denote the total number of users, J denote the number of external providers in one zone (let us remind that the number of external providers in each zone were taken equal), T_ε the total processor time in seconds. Method (CGDM) with the initial point $w^0 = 0$ is marked as (CGDM0), whereas the same method with the initial point w^0 taken in the upper boundary of D is marked as (CGDMB).

TABLE I
RESULTS OF TESTING EXP WITH $J = 0$, $I = 510$, $n = 70$

ε_λ	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (DBS)
10^{-1}	0.0128	0.3854	0.0010
10^{-2}	0.0130	0.4556	0.0013
10^{-3}	0.0163	0.5553	0.0009
10^{-4}	0.0194	0.6378	0.0019

TABLE II
RESULTS OF TESTING EXP WITH $J = 5$, $I = 510$, $n = 70$

ε_λ	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (DBS)
10^{-1}	0.0188	0.3452	0.0027
10^{-2}	0.0174	0.3953	0.0025
10^{-3}	0.0237	0.4938	0.0025
10^{-4}	0.0296	0.5799	0.0034

We took the quadratic concave fee functions

$$\varphi_i(y_i) = 0.5\alpha_i y_i^2 + \beta_i y_i,\ i \in I_k,\ k = 1, \dots, n;$$

with the coefficients

$$\alpha_i = -3|\cos(2i + 1)| - 3,\ \beta_i = |\sin(i + 2)| + 1.$$

We first took the exponential convex functions

$$f_k(x_k) = \gamma_k e^{\delta_k x_k}$$

and

$$h_j(z_j) = \tilde{\gamma}_j e^{\tilde{\delta}_j z_j}$$

with the coefficients

$$\gamma_k = |\cos(2k + 2)| + 1,\ \delta_k = |\cos(k + 1)| + 3,$$
$$\tilde{\gamma}_j = |\sin(2j + 2)| + 1,\ \tilde{\delta}_j = |\sin(j + 1)| + 3.$$

We mark as EXP this series of experiments. The results of computations are given in Tables I–X.

We also took the non-convex logarithmic functions

$$f_k(x_k) = \ln(\gamma_k + \delta_k x_k)$$

and

$$h_j(z_j) = \ln(1 + \tilde{\gamma}_j + \tilde{\delta}_j z_j)$$

with the coefficients

$$\gamma_k = |2 * \cos(2k + 2)| + 1,\ \delta_k = |\cos(k + 1)| + 1,$$
$$\tilde{\gamma}_j = |2 * \sin(2j + 2)|,\ \tilde{\delta}_j = |\sin(j + 1)| + 1.$$

Observe that we then in principle have a global optimization problem. We mark as LOG this series of experiments. The results of computations are given in Tables XI–X.

TABLE III
RESULTS OF TESTING EXP WITH $J = 10$, $I = 510$, $n = 70$

ε_λ	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (DBS)
10^{-1}	0.0278	0.4518	0.0015
10^{-2}	0.0266	0.5312	0.0031
10^{-3}	0.0310	0.6516	0.0032
10^{-4}	0.0390	0.7157	0.0032

TABLE IV
RESULTS OF TESTING EXP WITH $J = 0$, $n = 70$, $\varepsilon = 10^{-2}$

I	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (DBS)
210	0.0094	0.1099	0.0003
310	0.0093	0.2012	0.0009
410	0.0116	0.3181	0.0013
510	0.0130	0.4556	0.0013
610	0.0152	0.6057	0.0009
710	0.0177	0.7820	0.0012
810	0.0197	0.9729	0.0022
910	0.0219	1.1644	0.0012
1010	0.0231	1.3875	0.0019
2010	0.0428	3.3751	0.0028
3010	0.0615	5.2503	0.0024
4010	0.0797	7.1370	0.0044
5010	0.0969	9.0495	0.0056
6010	0.1188	11.0049	0.0050
7010	0.1372	12.9454	0.0075
8010	0.1565	14.9661	0.0056
9010	0.1774	16.9421	0.0087
10010	0.1955	18.9424	0.0097

TABLE V
RESULTS OF TESTING EXP WITH $J = 0$, $I = 510$, $\varepsilon = 10^{-2}$

n	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε:(DBS)
5	0.0090	0.9197	0.0003
15	0.0109	0.8677	0.0003
25	0.0106	0.8047	0.0015
35	0.0112	0.7032	0.0006
45	0.0135	0.6128	0.0004
55	0.0134	0.5281	0.0012
65	0.0137	0.4752	0.0010
75	0.0136	0.4392	0.0019
85	0.0141	0.4075	0.0010
95	0.0150	0.3832	0.0022
105	0.0157	0.3608	0.0009

TABLE VI
RESULTS OF TESTING EXP WITH $J = 5$, $n = 70$, $\varepsilon = 10^{-2}$

I	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (DBS)
210	0.0157	0.1843	0.0016
310	0.0171	0.2655	0.0016
410	0.0172	0.3435	0.0021
510	0.0174	0.3953	0.0025
610	0.0219	0.4577	0.0016
710	0.0266	0.5641	0.0014
810	0.0251	0.6453	0.0019
910	0.0297	0.7202	0.0010
1010	0.0312	0.8031	0.0017
2010	0.0484	2.0846	0.0015
3010	0.0671	5.5736	0.0032
4010	0.0874	8.4238	0.0015
5010	0.1061	10.3438	0.0016
6010	0.1203	12.1534	0.0047
7010	0.1483	14.2568	0.0048
8010	0.1635	16.2254	0.0048
9010	0.1827	18.2004	0.0062
10010	0.2031	20.1129	0.0060

TABLE VII
RESULTS OF TESTING EXP WITH $J = 5$, $I = 510$, $\varepsilon = 10^{-2}$

n	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (DBS)
5	0.0109	0.8782	0.0016
15	0.0141	0.7280	0.0010
25	0.0140	0.4625	0.0015
35	0.0154	0.4173	0.0021
45	0.0201	0.4124	0.0012
55	0.0158	0.3873	0.0016
65	0.0158	0.4358	0.0019
75	0.0232	0.4157	0.0016
85	0.0249	0.4375	0.0016
95	0.0235	0.4218	0.0027
105	0.0236	0.4328	0.0031

TABLE VIII
RESULTS OF TESTING EXP WITH $J = 10$, $n = 70$, $\varepsilon = 10^{-2}$

I	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (DBS)
210	0.0233	0.2922	0.0031
310	0.0234	0.3656	0.0021
410	0.0280	0.4892	0.0023
510	0.0266	0.5312	0.0031
610	0.0298	0.6030	0.0016
710	0.0296	0.6784	0.0032
810	0.0298	0.7498	0.0016
910	0.0344	0.8344	0.0016
1010	0.0342	0.9235	0.0022
2010	0.0547	2.0797	0.0031
3010	0.0783	3.8813	0.0015
4010	0.0969	7.1769	0.0016
5010	0.1108	10.2110	0.0064
6010	0.1360	12.6454	0.0061
7010	0.1533	14.8563	0.0031
8010	0.1689	16.9521	0.0046
9010	0.1909	19.0532	0.0077
10010	0.2143	21.1689	0.0047

TABLE IX
RESULTS OF TESTING EXP WITH $J = 10$, $I = 510$, $\varepsilon = 10^{-2}$

n	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (DBS)
5	0.0093	0.9468	0.0016
15	0.0140	0.5046	0.0012
25	0.0157	0.4828	0.0016
35	0.0205	0.4766	0.0024
45	0.0251	0.4746	0.0016
55	0.0252	0.5093	0.0031
65	0.0236	0.5232	0.0028
75	0.0278	0.5361	0.0011
85	0.0311	0.5641	0.0016
95	0.0327	0.5765	0.0017
105	0.0343	0.5907	0.0022

TABLE X
RESULTS OF TESTING EXP WITH $I = 510$, $n = 70$, $\varepsilon = 10^{-2}$

J	T_ε:(CGDM0)	T_ε:(CGDMB)	T_ε:(DBS)
0	0.0130	0.4556	0.0013
5	0.0174	0.3953	0.0025
10	0.0266	0.5312	0.0031

TABLE XI
RESULTS OF TESTING LOG WITH $J = 0$, $I = 510$, $n = 70$

ε_λ	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (DBS)
10^{-1}	0.5376	0.5530	0.0094
10^{-2}	0.8096	0.7876	0.0126
10^{-3}	1.1438	1.0534	0.0156
10^{-4}	1.4748	1.2596	0.0220

TABLE XII
RESULTS OF TESTING LOG WITH $J = 5$, $I = 510$, $n = 70$

ε_λ	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (DBS)
10^{-1}	33.5816	33.6256	0.0532
10^{-2}	34.8609	35.5017	0.0570
10^{-3}	46.2510	47.5979	0.0633
10^{-4}	53.1885	54.2517	0.0781

TABLE XIII
RESULTS OF TESTING LOG WITH $J = 10$, $I = 510$, $n = 70$

ε_λ	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (DBS)
10^{-1}	43.9518	43.4692	0.0631
10^{-2}	48.8135	49.4540	0.0780
10^{-3}	60.6781	62.6795	0.0933
10^{-4}	71.3513	73.0485	0.1097

TABLE XIV
RESULTS OF TESTING LOG WITH $J = 0$, $n = 70$, $\varepsilon = 10^{-2}$

I	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (DBS)
210	0.1096	0.1278	0.0040
310	0.3594	0.2874	0.0064
410	0.4972	0.4626	0.0092
510	0.8096	0.7876	0.0126
610	1.1530	1.1406	0.0128
710	1.4782	1.4126	0.0158
810	1.9750	1.9186	0.0154
910	2.5688	2.4688	0.0188
1010	3.3440	2.9998	0.0250
2010	29.0004	12.2876	0.0436
3010	99.6226	30.9940	0.0624
4010	203.2690	64.3018	0.0780
5010	192.3142	113.7266	0.1002
6010	346.9848	188.6262	0.1250
7010	456.8418	282.0170	0.1530
8010	662.6904	428.4620	0.1688
9010	784.1608	592.4142	0.1936
10010	619.5352	705.0890	0.2184

TABLE XV
RESULTS OF TESTING LOG WITH $J = 0$, $I = 510$, $\varepsilon = 10^{-2}$

n	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (DBS)
5	110.1356	13.5044	0.0158
15	8.7282	3.9936	0.0156
25	1.5020	2.2940	0.0064
35	2.0746	1.5470	0.0094
45	1.2124	1.2188	0.0096
55	1.0406	1.0064	0.0094
65	0.7940	0.8498	0.0092
75	0.7688	0.7502	0.0128
85	0.8130	0.6472	0.0126
95	0.6530	0.5968	0.0126
105	0.5222	0.4938	0.0124

TABLE XVI
RESULTS OF TESTING LOG WITH $J = 5$, $n = 70$, $\varepsilon = 10^{-2}$

I	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (DBS)
210	5.7184	4.9695	0.0324
310	12.6603	12.0622	0.0471
410	23.3911	23.7660	0.0637
510	34.8609	35.5017	0.0570
610	53.7515	54.4546	0.0623
710	71.9853	71.6409	0.0783
810	93.3601	92.7030	0.0639
910	122.5000	119.9533	0.0947
1010	152.6745	147.5653	0.0786
2010	569.8197	566.8493	0.1416
3010	1289.3284	1270.3125	0.1871
4010	2385.9490	2344.7468	0.2345
5010	3640.6492	3593.2404	0.3753
6010	5046.9814	4970.5373	0.3445
7010	6708.4588	6249.4869	0.3916
8010	8311.4033	7437.4960	0.4389
9010	10139.8603	8701.0172	0.5321
10010	11900.9121	10015.6371	0.6111

As we can see from the results in the tables, in all the cases the suggested methods were capable to find a solution. Moreover, for the same accuracy, both the methods gave the same numbers of upper iterations, so that the main difference was in the processor time which showed that utilization of Algorithm (BS) for inner optimization problems gives better performance, especially on large problems. Also in the non-convex case (see Tables XI-XX) Algorithm (DBS) appeared faster significantly than (CGDM).

TABLE XVII
RESULTS OF TESTING LOG WITH $J = 5$, $I = 510$, $\varepsilon = 10^{-2}$

n	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (DBS)
5	474.7975	441.7209	0.0311
15	185.2360	185.7031	0.0313
25	102.5799	97.4371	0.0310
35	79.8297	79.2511	0.0475
45	56.8925	56.7504	0.0463
55	48.2816	47.7207	0.0478
65	42.4691	43.5637	0.0632
75	36.2815	35.5634	0.0635
85	29.2823	29.0943	0.0629
95	25.2664	26.2046	0.0781
105	22.0949	21.5310	0.0791

TABLE XVIII
RESULTS OF TESTING LOG WITH $J = 10$, $n = 70$, $\varepsilon = 10^{-2}$

I	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (DBS)
210	6.6720	6.3446	0.0473
310	15.9533	15.9076	0.0624
410	31.1725	31.8287	0.0787
510	48.8135	49.4540	0.0780
610	70.7205	71.1993	0.0781
710	100.3282	100.6011	0.0944
810	121.1159	123.7584	0.0930
910	155.9370	155.7436	0.1102
1010	186.7681	191.0187	0.1099
2010	681.9694	678.8754	0.1721
3010	1452.2272	1440.2179	0.2181
4010	2535.7178	2514.0659	0.2812
5010	3974.3384	3945.2081	0.3296
6010	5905.8590	5802.3383	0.3605
7010	7812.5409	7858.1012	0.4223
8010	10151.1191	10137.5370	0.4903
9010	12648.5315	12701.1092	0.5784
10010	16040.0027	15915.1477	0.6428

TABLE XIX
RESULTS OF TESTING LOG WITH $J = 10$, $I = 510$, $\varepsilon = 10^{-2}$

n	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (DBS)
5	529.6564	513.4950	0.0474
15	198.3463	199.3771	0.0470
25	122.5231	121.8903	0.0473
35	96.7040	97.0791	0.0630
45	73.7825	74.2585	0.0636
55	63.5329	64.6424	0.0786
65	58.1260	57.7986	0.0787
75	47.9841	48.3767	0.0783
85	38.1881	38.5440	0.0949
95	36.4686	36.7199	0.1102
105	35.2355	33.5791	0.0931

V. CONCLUSIONS

We considered nonlinear convex optimization problems of optimal allocation of resources in a telecommunication network where the network manager can take the resource of external networks. We presented a dual bi-section type solution method for the case where the network manager aims to distribute some homogeneous resource (bandwidth) among users of one region and compare it with the conditional gradient method. Next, we considered a more general resource allocation problem where the region a wireless communication network is divided into zones (clusters) and develop suitable

TABLE XX
RESULTS OF TESTING LOG WITH $I = 510$, $n = 70$, $\varepsilon = 10^{-2}$

J	T_ε:(CGDM0)	T_ε:(CGDMB)	T_ε: (DBS)
0	0.8096	0.7876	0.0126
5	34.8609	35.5017	0.0570
10	48.8135	49.4540	0.0780

extensions of the above dual methods for this problem, where calculation of the cost function value leads to independent solution of zonal problems, which coincide with the single region problem. We present results of computational experiments which confirm the efficiency of the new method in comparison of the previous ones.

REFERENCES

[1] Courcoubetis, C., Weber, R.: Pricing Communication Networks: Economics, Technology and Modelling. John Wiley & Sons, Chichester (2003)

[2] Stańczak, S., Wiczanowski, M., Boche, H.: Resource Allocation in Wireless Networks. Theory and Algorithms. Springer, Berlin (2006)

[3] Wyglinski, A.M., Nekovee, M., Hou, Y.T.: (eds.) Cognitive Radio Communications and Networks: Principles and Practice. Elsevier, Amsterdam (2010)

[4] Huang, J., Berry, R.A., Honig, M.L.: Auction-based spectrum sharing, ACM/Springer Mobile Networks and Appl., vol. 11, pp. 405–418 (2006)

[5] Leshem, A., Zehavi, E.: Game theory and the frequency selective interference channel: A practical and theoretic point of view, IEEE Signal Process., vol. 26, pp. 28–40 (2009)

[6] Koutsopoulos, I., Iosifidis, G.: Auction mechanisms for network resource allocation, in: Proc. of Workshop on Resource Allocation in Wireless Networks (WiOpt 2010), pp. 554–563 (2010)

[7] Nordin, R.: Interference-aware subcarrier allocation in a correlated MIMO downlink transmission, WSEAS Trans. on Comm., vol. 11, pp. 158–169 (2012)

[8] Huang, X., Leng, S., Wu, F., Mao, Y.: Energy-efficient and QoS-ware resource allocation for relay-enhanced OFDMA wireless cellular networks, WSEAS Trans. on Comm., vol. 13, pp. 394–401 (2014)

[9] Konnov, I.V., Kashina, O.A., Laitinen, E.: Optimisation problems for control of distributed resources, Int. J. Model., Ident. and Contr., vol.14, pp.65–72 (2011)

[10] Konnov, I.V., Kashina, O.A., Laitinen, E.: Two-level decomposition method for resource allocation in telecommunication network, Int. J. Dig. Inf. Wirel. Comm., vol.2, pp.150–155 (2012)

[11] Konnov, I.V., Kashuba, A.Yu.: Decomposition method for zonal resource allocation problems in telecommunication networks, in: IOP Conf. Series: Materials Science and Engineering, vol. 158, Proc. of the 11th International Conference on “Mesh Methods for Boundary-value Problems and Applications”, Art. No. 012054. 7 pp. (2016)

[12] Gao, L., Iosifidis, G., Huang, J., Tassiulas, L.: Economics of mobile data offloading, IEEE Conference on Computer Communications, pp.351–356 (2013)

[13] Konnov, I.V., Laitinen, E., Kashuba, A.Yu.: Optimization of zonal allocation of total network resources, in: Proc. of the 11th International Conference “Applied Computing 2014”, Porto, pp.244–248 (2014)

[14] Konnov, I.V., Kashuba, A.Yu., Laitinen, E.: Application of the conditional gradient method to resource allocation in wireless networks. Lobachevskii J. Mathem., vol.37, pp.626–635 (2016)

[15] Konnov, I.V.: On modeling of auction type markets. Issled. Inform., vol.10, pp.73–76 (2006), Available at SSRN: http://ssrn.com/abstract=2482282

[16] Konnov, I.V.: An alternative economic equilibrium model with different implementation mechanisms. Adv. Model. Optim., vol. 17, pp.245–265 (2015)

[17] Konnov, I.V. Modelling of auction type markets. Universita degli Studi di Bergamo, DMSIA Report No. 7. Bergamo, Italy. 28 pp. (2007)

[18] Frank, M., Wolfe, P.: An algorithm for quadratic programming, Nav. Res. Logist. Quart., vol. 3, pp.95–110 (1956)

[19] Levitin, E.S., Polyak, B.T.: Constrained minimization methods, USSR Comput. Maths. Math. Phys., vol. 6, pp.1–50 (1966)

[20] Pshenichnyi, B.N., Danilin, Yu.M.: Numerical Methods in Extremal Problems. MIR, Moscow (1978)

[21] Konnov, I.V., Kashuba, A.Yu., Laitinen, E.: A simple dual method for optimal allocation of total network resources, in: Recent Advances in Mathematics. Proceedings of the International Conference ”PMAMCM 2015”, Zakynthos, Ed. by I.J. Rudas, pp.19–21 (2015)

[22] Polyak, B.T.: Introduction to Optimization. Nauka, Moscow (1983) [Engl. transl. in Optimization Software, New York, 1987]

[23] Konnov, I.V.: Nonlinear Optimization and Variational Inequalities. Kazan Univ. Press, Kazan (2013)