Processing of Correlated Images through Double PCA-based Transform in Color and Time Domains

Roumen Kountchev and Roumiana Kountcheva

***Abstract*—**In this paper is presented one new approach for decorrelation of groups of color images (for example, multi-view, computer tomography, video sequences, etc.) in two directions: the color and the time domain. The correlation strength depends on the image kind (obtained by a video sensor with fixed or time-changing spatial position), and on their contents. To achieve correlation reduction, here is proposed new approach based on the sequential execution of two PCA-based algorithms: the Adaptive Color PCA for each color image in the group, and the Hierarchical Adaptive PCA - for each group or time-sequence, obtained after the execution of the first color space transform. Both algorithms were already presented in preceding publications of the authors, but their combined impact had not been analyzed yet. In the paper is given also the analysis of the information redundancy reduction as a result of the decorrelation got after the execution of both algorithms, and are indicated the future trends for their development.

***Keywords*—** Adaptive color PCA, Double PCA-based transform, Hierarchical adaptive PCA, Image decorrelation, Principal Component Analysis.

I. INTRODUCTION

As it is known, the non-compressed color image is represented through its primary color components - the matrices [R], [G] and [B], whose size is equal to that of the original image. In such case, a group of correlated images (for example, multi-view images, computer tomography, video sequences, etc.) could be represented by three groups which comprise the corresponding matrices $[R_p]$, $[G_p]$, $[B_p]$, for p=1,2,3,... The source, which could be with fixed, or time-changing spatial position, defines the mutual correlation between the images in the groups and is much higher in the first case.

The objective of this work is to propose new approach for image groups decorrelation both in time- and color domains. The main idea here is to apply sequentially two PCA-based transforms: 1) the Adaptive Color PCA (AC-PCA) for the RGB components of each image in the processed group, and 2) the Hierarchical APCA (HA-PCA) for each group of principal color components (eigen images), got after the execution of AC-PCA. Algorithms AC-PCA and HA-PCA are presented in detail in earlier publications of the authors and here is investigated their combined impact. The choice of these two algorithms for decorrelation of the three groups of RGB components in the color and time domains is determined by their lower computational complexity compared to iterative PCA algorithms, retaining their most important properties (minimum mean square error of the restored images got after the reduction of their information redundancy, and maximum energy concentration in the first decomposition components).

Prof. D. Sc. PhD. R. K. Kountchev is with the Technical University of Sofia, Department of Radio Communications and Video Technologies, Bul. Kl. Ohridsky 8, Sofia 1000, Bulgaria (corresponding author, phone: +899924284) e-mail: rkountch@ tu-sofia.bg).

Dr. R. A. Kountcheva is with TK Engineering, Sofia, Bulgaria (e-mail: kountcheva_r@yahoo.com).

II. RELATED WORKS

Various orthogonal transforms for image decorrelation in the pixel space are already investigated and used in practice: the Principal Component Analysis (PCA) or Karhunen-Loeve Transform (KLT) [1-6], and the related Independent Component Analysis (ICA) [7,8] and the Singular Value Decomposition (SVD) [9,10]. The image representation through ICA in whitened space is obtained by left multiplication with the matrix square root of the inverse covariance matrix [7]. There are also transforms used for processing of multi-channel images, based on the quaternion DFT [11], the KLT in the time domain [1], and the Hierarchical KLT (HKLT) [12] - in the spectrum domain. In [13] the algorithm ICA is used for feature extraction from color and stereo images.

The approach for processing of groups of correlated images in their color and spatial domains, presented here, corresponds to some degree to the method offered in [13], but instead of processing one stereo couple only, it is developed for processing a sequence (group) of 2^n color images. Besides, for the processing are used new PCA-based algorithms with low computational complexity.

The paper comprises the following sections: Section 3: Adaptive Color PCA algorithm for decorrelation of the color image components; Section 4: Algorithm APCA for decorrelation of a couple of images; Section 5: Hierarchical Adaptive PCA (HA-PCA) for decorrelation of a group of 2^n images; Section 6: Evaluation of the decorrelation got after the HA-PCA execution; Section 7: Decorrelation of a group of color images through double transform: AC-PCA and HA-PCA; Section 8: Evaluation of the decorrelation efficiency; Section 9: Conclusions.

III. Adaptive Color PCA Algorithm for Decorrelation of the Color Image Components

The Adaptive Color PCA (AC-PCA) transform for decorrelation of the three primary color components [R],[G],[B] of the image of size H×V, is illustrated on Fig. 1. Each component is a matrix of S=H×V elements (pixels). After the AC-PCA calculation are obtained the principal color components - the matrices of the eigen images $[L_1]$,$[L_2]$,$[L_3]$, also of size H×V. To execute the transform, the matrices [R],[G],[B] are represented as 3-component color vectors – one vector for each pixel. As an example, on Fig. 1 are shown the first four color vectors $\vec{C}_s$ with elements R_s, G_s, B_s (s=1,2,3,4), and their corresponding transformed vectors $\vec{L}_s$ with elements L_{1s}, L_{2s}, L_{3s}.

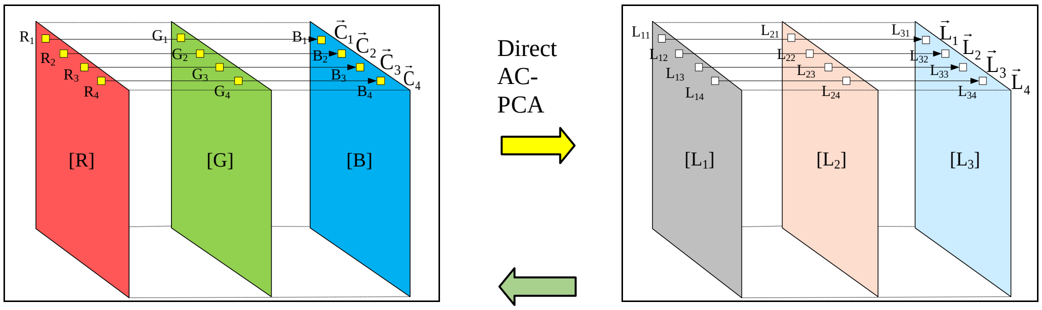

Fig. 1. Transformation of the image RGB components through AC-PCA into corresponding image, which comprises the principal color components L_1, L_2, L_3.

The forward/inverse AC-PCA of the vectors $\vec{C}_s=[R_s,G_s,B_s]^T$ and $\vec{L}_s=[L_{1s},L_{2s},L_{3s}]^T$ for s=1,2,..,S are defined by the relations [14]:

$$\vec{L}_s=[\Phi](\vec{C}_s-\vec{m}_c),\ \ \vec{C}_s=[\Phi]^T\vec{L}_s+\vec{m}_c, \tag{1}$$

where $[\Phi]=\begin{bmatrix} A_1/P_1 & B_1/P_1 & D_1/P_1 \\ A_2/P_2 & B_2/P_2 & D_2/P_2 \\ A_3/P_3 & B_3/P_3 & D_3/P_3 \end{bmatrix}$,

$P_i=\sqrt{A_i^2+B_i^2+D_i^2}$ for i=1,2,3; $\vec{m}_C=[\bar{R},\bar{G},\bar{B}]^T$;

$\bar{B}=(B_s)$; $\bar{R}=(R_s)$; $\bar{G}=E(G_s)$,

($\bar{x}=E(x_s)=(1/S)\sum_{s=1}^{S}x_s$ - averaging operator);

$A_i=(k_3-\lambda_i)[k_5(k_2-\lambda_i)-k_4k_6]$;

$B_i=(k_3-\lambda_i)[k_6(k_1-\lambda_i)-k_4k_5]$;

$D_i=k_6[2k_4k_5-k_6(k_1-\lambda_i)]-k_5^2(k_2-\lambda_i)$;

$k_1=E(R_s^2)-\bar{R}^2$; $k_2=E(G_s^2)-\bar{G}^2$;

$k_3=E(B_s^2)-\bar{B}^2$; $k_4=E(R_sG_s)-(\bar{R})(\bar{G})$;

$k_5=E(R_sB_s)-(\bar{R})(\bar{B})$; $k_6=E(G_sB_s)-(\bar{G})(\bar{B})$;

$\lambda_1=2\sqrt{|p|/3}\cos(\varphi/3)-(a/3)$;

$\lambda_{2,3}=-2\sqrt{|p|/3}\cos[(\varphi\mp\pi)/3]-(a/3)$;

$a=-(k_1+k_2+k_3)$; $b=k_1k_2+k_1k_3+k_2k_3-(k_4^2+k_5^2+k_6^2)$;

$c=k_1k_6^2+k_2k_5^2+k_3k_4^2-(k_1k_2k_3+2k_4k_5k_6)$;

$\varphi=\arccos[-q/2\big/\sqrt{(|p|/3)^3}]$; $q=2(a/3)^3-(ab)/3+c$;

$p=-(a^2/3)+b<0$.

As a result of the transform, full decorrelation of the components of the processed vectors $\vec{L}_s=[L_{1s},L_{2s},L_{3s}]^T$ is got. Their covariance matrix $[K_L]$ is diagonal:

$$[K_L]=[\Phi][K_C][\Phi]^T=\begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \tag{2}$$

Its elements λ_1, λ_2, λ_3 are the eigen values of the covariance matrix $[K_C]$ of the color vectors $\vec{C}_s=[R_s,G_s,B_s]^T$. Hence, the corresponding transformed matrices of the eigen images $[L_1]$,$[L_2]$,$[L_3]$ are mutually decorrelated as well.

The consecutive steps of the Direct AC-KLT algorithm in correspondence with (1), are shown on Fig. 2. The experiments with significant number of test images as given in [2] show, that as a result of the direct AC-KLT the power of the matrix $[L_1]$, which comprises the first components L_{1s} of the transformed vectors $\vec{L}_s$, is maximum. The power of the next matrices $[L_2]$ and $[L_3]$, which comprise the elements L_{2s} and L_{3s} correspondingly, decreases quickly and that of the last matrix $[L_3]$ is close to zero.

IV. Algorithm APCA for Decorrelation of a Couple of Images

This transform is basic for the Hierarchical Adaptive PCA (HAPCA), aimed at the processing of a sequence of 2^n images. The APCA transform for a couple of images with corresponding matrices $[C_1]$, $[C_2]$ is noted here as $APCA_{2\times2}$ and is executed by using a transform matrix of size 2×2. Its elements are defined by one parameter only - the value of the rotation angle θ.

The forward/inverse APCA for a couple of matrix images $[C_1]$, $[C_2]$ each of size H×V and comprising S=H×V pixels, are calculated in accordance with the relations below [15]:

$$\begin{bmatrix}L_{1s}\\ L_{2s}\end{bmatrix}=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}C_{1s}-\bar{C}_1\\ C_{2s}-\bar{C}_2\end{bmatrix},$$

$$\begin{bmatrix}C_{1s}\\ C_{2s}\end{bmatrix}=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}L_{1s}\\ L_{2s}\end{bmatrix}+\begin{bmatrix}\bar{C}_1\\ \bar{C}_2\end{bmatrix}\quad \text{for } s=1,2,..,S, \tag{3}$$

where $[\Phi(\theta)]=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$ is the transform rotation matrix; $\vec{C}_s=[C_{1s},C_{2s}]^T$ and $\vec{L}_s=[L_{1s},L_{2s}]^T$ are the input and the transformed vectors.

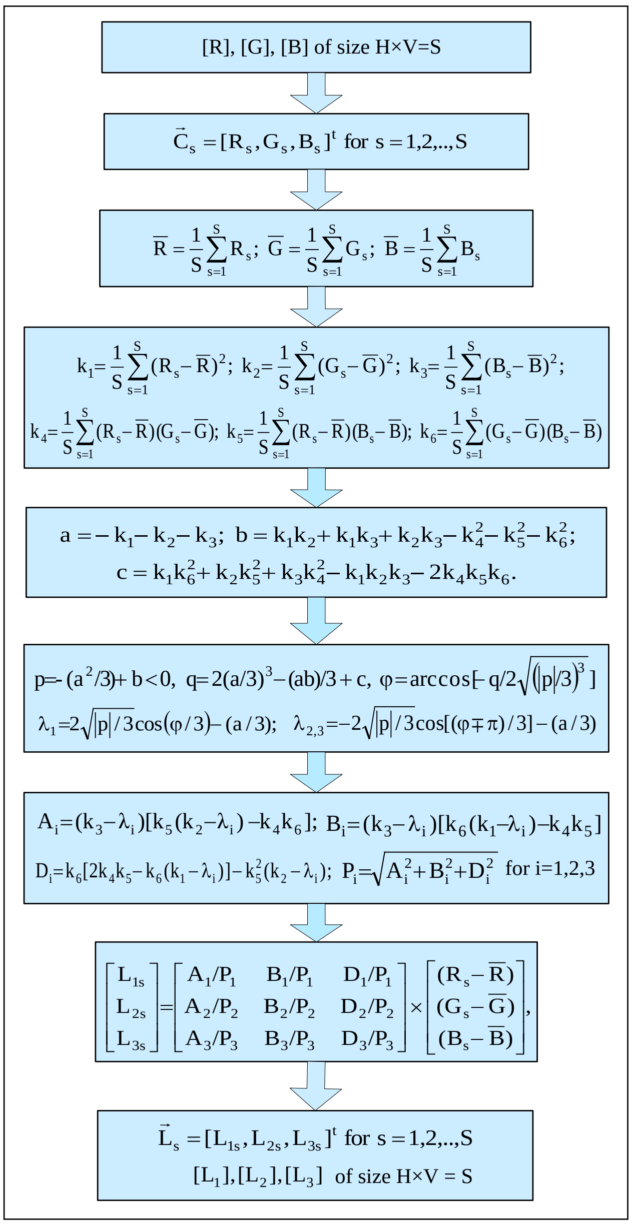

Fig. 2. Block diagram of Algorithm AC-PCA for processing of RGB components

The components of vectors $\vec{C}_s$ are defined by the couples of pixels of same spatial position in the matrices $[C_1]$ and $[C_2]$ (Fig. 3a), or with spatial displacement (Fig. 3b). In the first case, the couple of pixels is in the stationary area, and in the second - in the movement area. The displacement of the pixels in $[C_2]$ regarding their positions in $[C_1]$ is defined by the corresponding movement vectors, colored in red on Fig. 3b. These vectors are usually calculated by using the well-known methods for movement block compensation [4].

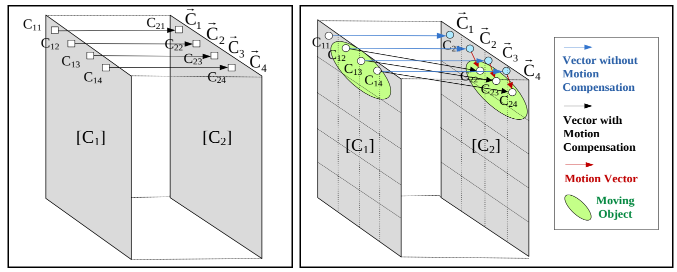

Fig. 3. Definition of vectors $\vec{C}_s$ for the calculation of the matrix APCA on the basis of $[C_1]$ and $[C_2]$: a - components of vectors $\vec{C}_s$ without movement compensation; b - components of vectors $\vec{C}_s$ with block movement compensation.

The algorithm for Forward APCA of the matrices $[C_1]$, $[C_2]$ transform into matrices $[L_1]$, $[L_2]$ is shown on Fig. 4.

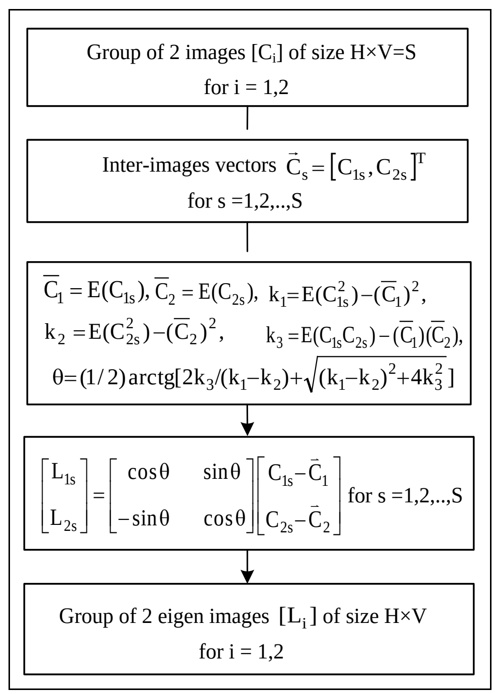

Fig. 4. APCA Algorithm for processing a couple of images

In (2) the transform matrix $[\Phi(\theta)]$ depends on the rotation angle θ, calculated in accordance with the relation below [3]:

$$\theta=\mathrm{arctg}\left(\frac{2k_3}{k_1-k_2+\sqrt{(k_1-k_2)^2+4k_3^2}}\right), \tag{4}$$

where:

$k_1 = E(C_{1s}^2) - \overline{C}_1^2$; $k_2 = E(C_{2s}^2) - \overline{C}_2^2$; $k_3 = E(C_{1s}C_{2s}) - \overline{C}_1\,\overline{C}_2$; $\overline{C}_1 = (C_{1s})$; $\overline{C}_2 = (C_{2s})$.

V. Hierarchical Adaptive PCA (HA-PCA) for Decorrelation of a Group of 2N Images

The Hierarchical Adaptive PCA is an n-level PCA-based transform aimed at the decorrelation of a group of 2^n images. The computational complexity of HA-PCA is lower than that of the well-known iterative algorithms (for example, for n=4 it is two times lower) [16].

In the first level of HA-PCA the processed group of 2^n images is divided into 2^{n-1} sub-groups (each comprising two images). For both images in each sub-group is calculated $APCA_{2\times2}$. The principal components obtained from all sub-groups are rearranged in accordance with the value of their power, P (for one component with elements $x_{i,j}$ it is defined by the relation $P=\sum_i \sum_j x_{i,j}^2$). For each sub-group are calculated two principal components. The power of the first component is higher than that of the second and they follow as shown on Fig. 5. In the next row the components are rearranged in accordance with their values - the first group comprises the components with higher power, the second - these with lower power, etc. The components with minimum power are in the last group. In the second hierarchical level the rearranged couples of principal components are processed with $APCA_{2\times2}$, and the so calculated new principal components are rearranged again in the way, used for the first level. Then the $APCA_{2\times2}$ is calculated again for all couples, until the last HA-PCA level is processed.

One example for the full structure of the HA-PCA algorithm for a group of T=8 images is shown on Fig. 5. The principal components calculated through $APCA_{2\times2}$ for each sub-group in the consecutive hierarchical levels are tinted in two colors - yellow for the components with high power, and blue - for these with the low-power. The output components in the last (third) level are tinted in eight colors in correspondence with their power decrease - from orange (for the maximum component), to dark blue (for the component with minimum power).

VI. Evaluation of the Decorrelation After the HA-PCA Execution

For the Level 1 of HA-PCA, the corresponding covariance sub-matrices $[K^1_{L_p,L_p}]$ of size 2×2 for each sub-group of transformed vectors $\vec{L}^1_{p,s}$ for p=1,..,4 and s=1,2,...,S are:

$$[K^1_{L_p,L_p}] = [\Phi^1_p][K_{C_{p,1}}][\Phi^1_p]^T = \begin{bmatrix} \lambda_1^{p,1}(1) & 0 \\ 0 & \lambda_2^{p,1}(1) \end{bmatrix}. \qquad (5)$$

Here, $[K_{C_{p,1}}] = \frac{1}{S}\sum_{s=1}^{S} \vec{C}^1_{p,s}\vec{C}^{1T}_{p,s} - \vec{m}^1_{c,p}\vec{m}^{1T}_{c,p}$ for p=1,..,4 is the covariance matrix of the vectors $\vec{C}^1_{p,s}$ in the sub-group p from level; $\vec{m}^1_{c,p}$ - the mean vector for each sub-group p from level 1; $\lambda_1^{p,1}(1), \lambda_2^{p,1}(1)$ - the corresponding eigen values of the covariance sub-matrices $[K_{C_{p,1}}]$; $[\Phi^1_p]$ - the transform matrix for the vectors $\vec{C}^1_{p,s}$ for the sub-group p for level 1.

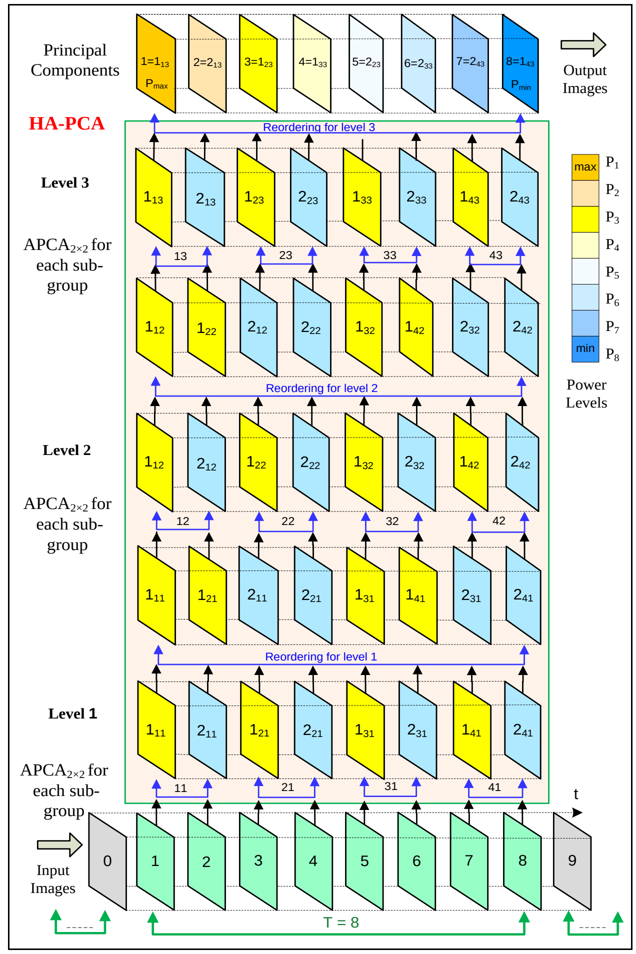

Fig. 5. Hierarchical APCA algorithm for a group of T=8 images, based on the use of $APCA_{2\times2}$ for each image couple in all three decomposition levels

The covariance matrix $[K^1_L] = \frac{1}{S}\sum_{s=1}^{S}\vec{L}^1_s\vec{L}^{1T}_s - \vec{m}_L\vec{m}_L^T$ of size 8×8 for the 8-component vectors $\vec{L}^1_s$ calculated for the group of 8 input images in Level 1 of HA-PCA after rearrangement of the calculated eigen images, is represented as:

$$[K^1_L] = \begin{bmatrix} [K^1_{L_1,L_1}] & [K^1_{L_1,L_2}] & [K^1_{L_1,L_3}] & [K^1_{L_1,L_4}] \\ [K^1_{L_1,L_2}] & [K^1_{L_2,L_2}] & [K^1_{L_2,L_3}] & [K^1_{L_2,L_4}] \\ [K^1_{L_1,L_3}] & [K^1_{L_2,L_3}] & [K^1_{L_3,L_3}] & [K^1_{L_3,L_4}] \\ [K^1_{L_1,L_4}] & [K^1_{L_2,L_4}] & [K^1_{L_3,L_4}] & [K^1_{L_4,L_4}] \end{bmatrix} =$$

$$= \begin{bmatrix} \begin{array}{c|c} \lambda_1^{1,1}(1) & 0 \\ \hline 0 & \lambda_2^{1,1}(1) \end{array} & 0 & [K^1_{L_1,L_3}] & 0 \\ 0 & \begin{array}{c|c} \lambda_1^{2,1}(1) & 0 \\ \hline 0 & \lambda_2^{2,1}(1) \end{array} & 0 & [K^1_{L_2,L_4}] \\ [K^1_{L_1,L_3}] & 0 & \begin{array}{c|c} \lambda_1^{3,1}(1) & 0 \\ \hline 0 & \lambda_2^{3,1}(1) \end{array} & 0 \\ 0 & [K^1_{L_2,L_4}] & 0 & \begin{array}{c|c} \lambda_1^{4,1}(1) & 0 \\ \hline 0 & \lambda_2^{4,1}(1) \end{array} \end{bmatrix}, \qquad (6)$$

where $[K^1_{L_k,L_p}] = E\{\vec{L}^1_{k,s}.\vec{L}^{1T}_{p,s}\} - E\{\vec{L}^1_{k,s}\}E\{\vec{L}^1_{p,s}\}^T$ for k, p =1,..,4 and k≠p is the cross-covariance sub-matrix of size 2×2 for the 2-component vectors $\vec{L}^1_{k,s}$ and $\vec{L}^1_{p,s}$ from groups k and p in the Level 1 of HA-PCA. In this case only the sub-matrices $[K^1_{L_1,L_3}]$ and $[K^1_{L_2,L_4}]$ are not equal to zero. For the remaining covariance sub-matrices is satisfied the condition

$$[K^1_{L_1,L_2}]=[K^1_{L_1,L_4}]=[K^1_{L_2,L_3}]=[K^1_{L_3,L_4}]=0 \,, \tag{7}$$

because their corresponding couples of vectors $\vec{L}^1_{k,s}$ and $\vec{L}^1_{p,s}$ are in sub-groups with different power values (tinted in yellow and blue, respectively).

By analogy with the first level, in the decomposition Level 2 the covariance matrix $[K^2_L]$ of size 8×8 for the 8-component vectors $\vec{L}^2_s$ in the group of the rearranged 8 eigen images, is:

$$[K^2_L]=\begin{bmatrix} [K^2_{L_1,L_1}] & [K^2_{L_1,L_2}] & [K^2_{L_1,L_3}] & [K^2_{L_1,L_4}] \\ [K^2_{L_1,L_2}] & [K^2_{L_2,L_2}] & [K^2_{L_2,L_3}] & [K^2_{L_2,L_4}] \\ [K^2_{L_1,L_3}] & [K^2_{L_2,L_3}] & [K^2_{L_3,L_3}] & [K^2_{L_3,L_4}] \\ [K^2_{L_1,L_4}] & [K^2_{L_2,L_4}] & [K^2_{L_3,L_4}] & [K^2_{L_4,L_4}] \end{bmatrix}=$$

$$=\begin{bmatrix} \begin{array}{c|c} \lambda_1^{1,1}(2) & 0 \\ \hline 0 & \lambda_2^{1,1}(2) \end{array} & 0 & [K^2_{L_1,L_3}] & 0 \\ 0 & \begin{array}{c|c} \lambda_1^{2,1}(2) & 0 \\ \hline 0 & \lambda_2^{2,1}(2) \end{array} & 0 & [K^2_{L_2,L_4}] \\ [K^2_{L_1,L_3}] & 0 & \begin{array}{c|c} \lambda_1^{3,1}(2) & 0 \\ \hline 0 & \lambda_2^{3,1}(2) \end{array} & 0 \\ 0 & [K^2_{L_2,L_4}] & 0 & \begin{array}{c|c} \lambda_1^{4,1}(2) & 0 \\ \hline 0 & \lambda_2^{4,1}(2) \end{array} \end{bmatrix}, \tag{8}$$

In this case, only the cross-covariance matrices $[K^2_{L_1,L_3}]$ and $[K^2_{L_2,L_4}]$ of size 2×2 are not equal to zero, while for the remaining matrices is satisfied the condition

$$[K^2_{L_1,L_2}]=[K^2_{L_1,L_4}]=[K^2_{L_2,L_3}]=[K^2_{L_3,L_4}]=0 \,. \tag{9}$$

For the Level 3 of HA-PCA are calculated the covariance sub-matrices of size 2×2 for each sub-group of the rearranged vectors, obtained after performing the HA-PCA Level 2. The covariance matrix of size 8×8 for the 8-component vectors $\vec{L}^2_s$ in the group of the processed 8 input images for Level 2 is calculated by analogy with (7), but the upper index for all matrices and sub-matrices is 3. In this level, as a result of the principal components rearrangement, all cross-covariance sub-matrices are equal to zero, including also the sub-matrices $[K^3_{L_1,L_3}]$ and $[K^3_{L_2,L_4}]$. Then, the covariance matrix of size 8×8 for the 8-component vectors $\vec{L}^3_s$ is:

$$[K^3_L] = \\ = \mathrm{diag}[\lambda_1^{1,1}(3),\lambda_2^{1,1}(3),\lambda_1^{2,1}(3),\lambda_2^{2,1}(3),\lambda_1^{3,1}(3),\lambda_2^{3,1}(3),\lambda_1^{4,1}(3),\lambda_2^{4,1}(3)] \tag{10}$$

Using the already calculated matrix $[K^r_L]$ for r=1,2,3 could be evaluated the decorrelation of the corresponding 8 eigen images in the processed group. When full decorrelation is obtained, the matrix $[K^r_L]$ is diagonal, and the following condition should be satisfied:

$$[K^r_{L_k,L_p}] = E\{\vec{L}^r_{k,s}.\vec{L}^{rT}_{p,s}\} - E\{\vec{L}^r_{k,s}\}E\{\vec{L}^r_{p,s}\}^T = 0 \tag{11}$$

for k,p=1,..,4, r=1,2,3 and $k \neq p$.

In the general case this requirement is satisfied after the rearrangement of the vectors in the last level (r=3) of the algorithm. The calculation of HA-PCA could be stopped even without full decorrelation, if the condition below is satisfied:

$$|\, E\{\vec{L}^r_k.\vec{L}^{rT}_p\} - E\{\vec{L}^r_k\}E\{\vec{L}^r_p\}^T| \leq \delta, \tag{12}$$

where δ is a pre-defined threshold. In this case the full structure of the algorithm shown on Fig. 5 is simplified through cutting-off the sub-groups containing the vectors $\vec{L}^r_p$, which have sub-threshold cross-correlation in respect of the sub-groups of vectors, $\vec{L}^r_k$.

VII. Decorrelation of a Group of Color Images Through Double Transform: AC-PCA and HA-PCA

The principle of the double transform for a group of 2^n color images is based on the consecutive execution of algorithms AC-PCA and HA-PCA. The first algorithm (AC-PCA) is used to transform the matrices of the primary colors $[R_p]$,$[G_p]$,$[B_p]$ for p=1,2,..,2^n into corresponding groups of principal color components, represented by the matrices $[L_{1p}]$,$[L_{2p}]$,$[L_{3p}]$. On Fig. 6 a,b is shown an example for the transformation of the group of 8 color images represented by the sequences $[R_p]$,$[G_p]$,$[B_p]$ for p=1,2,..,8 (n=3), through AC-PCA. The colors of the three groups of principal components' matrices from Fig. 6b is different, and is changing depending on their power. On Fig. 6c is shown the second transform, based on the execution of the algorithm HA-PCA for each group of principal color components, which comprise the matrices $[L_{1p}]$,$[L_{2p}]$,$[L_{3p}]$, respectively. The matrices $[F_{1p}]$,$[F_{2p}]$,$[F_{3p}]$, obtained after the second transform, represent the three groups of second principal components, colored in correspondence with their power distribution. On the basis of their power, it is possible to select and retain for further processing the components with over-threshold power. The PCA-based transforms AC-PCA and HA-PCA are reversible and this permits to restore the original group of RGB images with minimum mean-square error, when the retained principal components only were used.

On Fig. 6c are shown the 7 components (framed) selected from the total group of 24 components, i.e. $[F_{11}]$,$[F_{12}]$,$[F_{13}]$,$[F_{14}]$,$[F_{21}]$,$[F_{22}]$,$[F_{31}]$. In these components is concentrated the main part of the energy from the three groups $[F_{1p}]$,$[F_{2p}]$,$[F_{3p}]$, for p=1,2,..,8. The concentration degree depends on the correlation in color- and time domains, existing between images in the original group. Higher concentration permits to reduce the number of the retained second principal components, and to get lower dimensionality of the new space got after the double transform of the original group of images. To achieve this, the number of images in the group (2^n) should be selected so that to ensure maximum mutual correlation in time- and color domains.

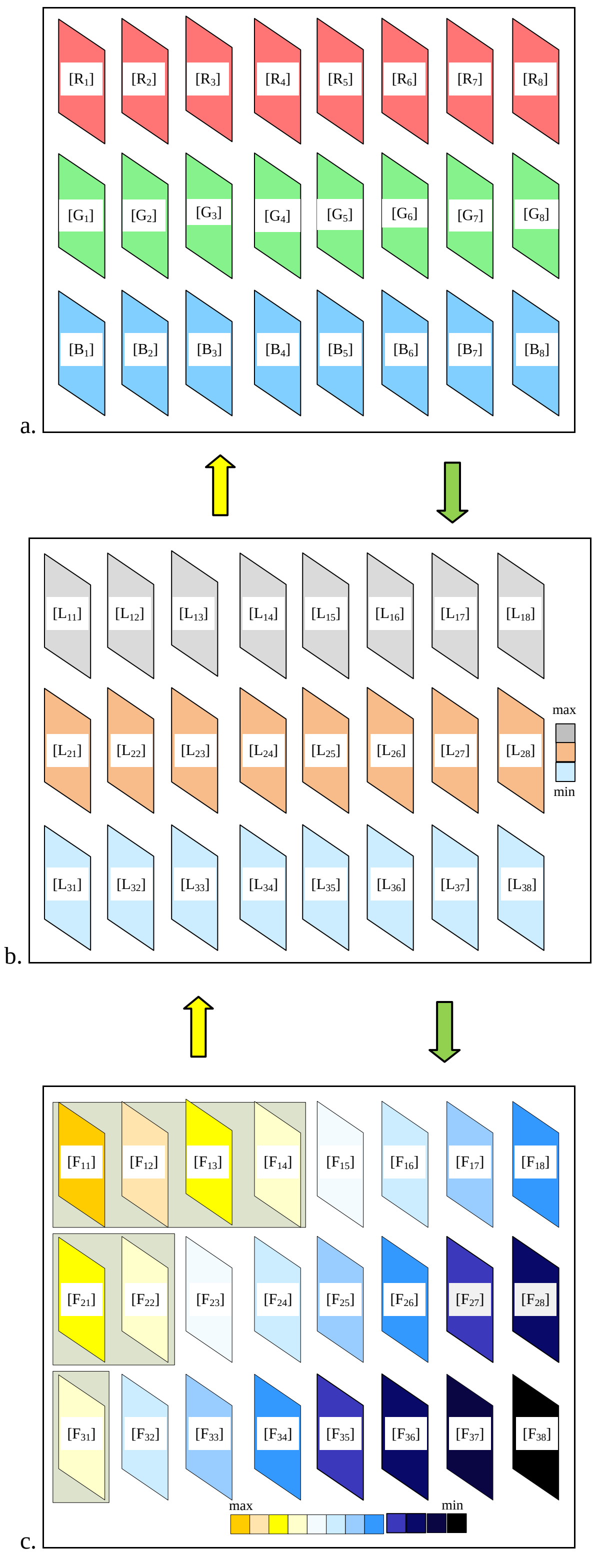

Fig. 6. Double PCA-based transform for a group of 8 RGB images.
a - input group of 8 images, represented by their R,G,B components;
b - first transform: the AC-PCA is applied on the R,G,B components of each image in the group, and as a result are got three groups of principal components;
c - second transform: the HA-PCA is applied on each of the three groups of principal components, got after the AC-PCA transform.

On Fig. 6c are shown the 7 components (framed) selected from the total group of 24 components, i.e. $[F_{11}],[F_{12}],[F_{13}],[F_{14}],[F_{21}],[F_{22}],[F_{31}]$. In these components is concentrated the main part of the energy from the three groups $[F_{1p}],[F_{2p}],[F_{3p}]$, for p=1,2,..,8. The concentration degree depends on the correlation in color- and time domains, existing between images in the original group. Higher concentration permits to reduce the number of the retained second principal components, and to get lower dimensionality of the new space got after the double transform of the original group of images. To achieve this, the number of images in the group (2^n) should be selected so that to ensure maximum mutual correlation in time- and color domains.

VIII. Evaluation of the Decorrelation Efficiency

The decorrelation efficiency for the group of 2^n color images could be evaluated as follows:

1) through analysis of the group of covariance matrices $[K_{L_p}]$ of the transformed color vectors $\vec{L}_{p,s}=[L_{1p,s},L_{2p,s},L_{3p,s}]^T$ got after the AC-PCA, for p=1,2,..,8. These matrices, defined in accordance with (2), are diagonal and as a result, full color decorrelation is got. This is confirmed by the results obtained from the modeling of the algorithm AC-PCA for color space transform used for various RGB images [16];

2) through analysis of the group of covariance matrices $[K^r_{L_p}]$ for r=1,2,3 of the 8-component vectors $\vec{L}^r_{p,s}$, got after HA-PCA for each group of 8 input images. Each matrix should also be diagonal, so that to guarantee the time decorrelation between all 8 eigen images in the group with sequential number r=1,2,3.

The decorrelation degree between the processed 2^n images (n=1,2,..) could be evaluated on the basis of their covariance matrix with elements $k_{i,j}$ and of size H×V. To calculate this matrix, S=H×V vectors of dimension 2^n are needed, each corresponding to one pixel of same spatial position in all images in the group. The criterion for the correlation evaluation is defined by the relation:

$$\mathrm{CovR} = \sum_{i=1}^{H} k^2_{i,i} \Big/ \sum_{i=1}^{H}\sum_{j=1}^{V} k^2_{i,j}(i \neq j) \qquad (13)$$

In case of full correlation is got CovR→∞, but to define the needed number of hierarchical levels n for HA-PCA is enough to satisfy the relation:

$$[\mathrm{CovR}(r=n)]^{-1} \leq \Delta, \qquad (14)$$

where Δ is a pre-defined threshold with a small value, defined experimentally. One example, which illustrates the decorrelation for a group of 8 computer tomography (CT) images got through execution of AC-PCA and HA-PCA, is shown on Fig. 7a,b. On the upper part of Fig. 7a is shown a single color CT image, and in the lower part - its first principal component, got after the AC-PCA execution. The first principal component is more than 100 times larger than next two components. The sequence from the upper part of Fig. 7b represents the first principal color components, got after the execution of AC-PCA on the group of 8 CT images. The sequence in the lower part of Fig. 7 b shows the second principal components of these CT images, got after the execution of HA-PCA.

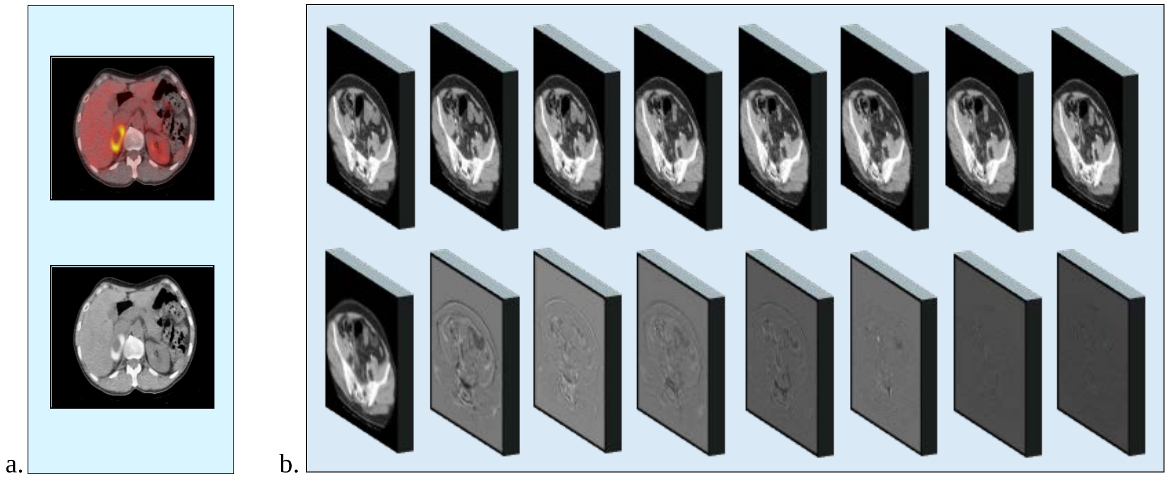

Fig. 7. Example for the double transform of 8 computer tomography images through AC-PCA and HA-PCA
a. Single color CT image (up) and its first principal component (below), got after AC-PCA;
b. The first color principal components for a group of CT images (up) and the result of their decorrelation through HA-PCA (below).

On Fig. 8 is shown the graphic power distribution of the second principal components from Fig. 7b got after the execution of HA-PCA and normalized towards the power value of the last component. This representation confirms the high power concentration in the first component, which corresponds to images shown on the lower part of Fig. 7b.

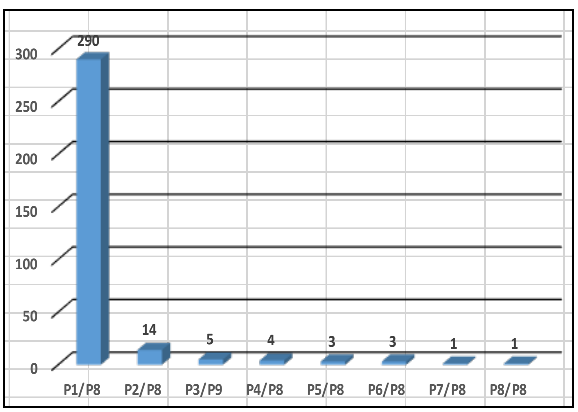

Fig. 8. Power distribution of the second principal components got after HA-PCA execution for the investigated group of CT images.

The results got from the HA-PCA modeling for the example above, and also for other investigated groups of correlated images of various kind, confirm its high decorrelation efficiency for the components in the time domain.

The analysis shows that the decorrelation of a group of images in the color- and time domains permits to achieve significant reduction of the existing information redundancy.

IX. Conclusions

In this work is offered new approach for two-dimensional decorrelation of groups of images through double PCA-based transform in color- and time domains. To achieve this, two algorithms are used: the Adaptive Color PCA for each color image in the group and the Hierarchical Adaptive PCA - for each group, got after the execution of the first transform (in the color space). The analysis of the efficiency of the new approach for processing of groups of correlated images shows its ability to achieve high decorrelation in two directions: in the color, and in the time domains (in theory, full decorrelation is got in both directions). This quality of the double transform opens wide abilities for its use in the efficient processing of correlated groups of images aimed at various applications, as:

- compression of correlated groups of images, based on the information redundancy reduction both in time- and color domains;
- reduction of color- and time feature space domains in information systems for processing, analysis, and recognition of objects in image sequences;
- protection of video information, based on watermarking of the low-informative second principal components, etc.

The presented approach for processing of correlated groups of images has lower computational complexity than the well-known iterative methods for calculation of PCA [16], and this opens new opportunities for large number of application areas.

- o the future development of the new approach is related to: development of algorithms for optimization of the number of images in the processed group in respect of their contents, respectively - their mutual correlation;
- o the choice of the retained principal components after both transforms;
- o the formats used for image data representation, which to ensure the needed accuracy and execution speed, etc.

References

[1] W. Pratt, *Digital Image Processing*, John Wiley & Sons, 2007.

[2] T. Reed, *Digital Image Sequence Processing, Compression, and Analysis*, CRC Press, 2004.

[3] I. Jolliffe, *Principal Component Analysis*, Second Edition, Springer, 2002.

[4] K. Rao, D. Kim, J. Hwang, *Fast Fourier Transforms: Algorithms and Applications*, Springer Science+Business Media, LLC 2010.

[5] H. Abdi, L. Williams, "Principal Component Analysis", *Wiley Interdisciplinary Reviews: Computational Statistics*, July/August 2010, Vol. 2, No. 4, 2010. pp. 433–459

[6] P. Fieguth, *Statistical Image Processing and Multidimensional Modeling*, Springer Science+Business Media, LLC 2011.

[7] J. Eichhorn, F. Sinz, M. Bethge, "Natural Image Coding in V1: How Much Use Is Orientation Selectivity? ", *PLoS Computational Biology*, Vol. 5, No. 4, April 2009, pp. 1-16.

[8] V. Esteban, S. Torres, "Adaptive Color Space Transform using Independent Component Analysis", in *Image Processing: Algorithms and Systems V*, J. Astola, K. Egiazarian, E. Dougherty (Eds.), Proc. of SPIE-IS&T Electronic Imaging, SPIE, Vol. 6497, 2007, pp. 64970P-1 - 64970P-12.

[9] R. Sadek, "SVD Based Image Processing Applications: State of The Art, Contributions and Research Challenges", *International Journal of Advanced Computer Science and Applications* (IJACSA), Vol. 3, No. 7, 2012, pp. 26-34.

[10] Y. Li, M. Wei, F. Zhang, J. Zhao, "Comparison of two SVD-based color image compression schemes", *PLoS ONE*, March 3, 2017, pp. 1-14.

[11] S. Sangwine, T. Ell and N. Bihan, "Hypercomplex Models and Processing of Vector Images", in *Multivariate image processing*, C. Collet, J. Chanussot, K. Chehdi (Eds.), John Wiley & Sons, Inc. 2009.

[12] I. Blanes, J. Sagristà, M. Marcellin, J. Rapesta, "Divide-and-conquer strategies for hyperspectral image processing", *IEEE Signal Processing Magazine*, Vol. 29, Issue 3, 2012, pp. 71-81.

[13] P. Hoyer and A. Hyvarinen, "Independent component analysis applied to feature extraction from color and stereo images", *Network: Computation in Neural Systems 11*, August 2000, pp. 191–210.

[14] R. Kountchev, R. Kountcheva, "Decorrelation of Multispectral Images, Based on Hierarchical Adaptive PCA", *Intern. Journal WSEAS Trans. on Signal Processing*, Vol. 9, No. 3, July 2013, pp. 120-137.

[15] R. Kountchev, P. Ivanov, "Decorrelation of Sequences of Medical CT Images based on the Hierarchical Adaptive KLT", Chapter in: *Advances in Intelligent Analysis of Medical Data and Decision Support Systems*, R. Kountchev and B. Iantovics (Eds.), Springer, 2013, pp. 41-55.

[16] P. Ivanov, R. Kountchev, "Hierarchical Principal Component Analysis based Transformation of Multispectral Images", *International Journal of Reasoning-Based Intelligent Systems (IJRIS)*, January 2013, pp. 1-15.