# Fuzzy Information Measures in Decisiveness

Anjali Munde

***Abstract*—** Information Theory deals with the analysis of setbacks regarding some system. This comprises the dispensation of information, stowing information, repossession of information and decision making. Information Theory analyses all hypothetical problems associated with diffusion of information above transmission network. This comprises analysis of ambiguity quantities and numerous applied and efficient procedures of coding information for conduction. Information Theory has attained applications in numerous communal, substantial and biotic sciences. To quantify the extent of information numerically, *Shannon (1948)* applied the conception of entropy. In this paper, an innovative measure of fuzzy information has been projected and functions of fuzzy information measure in evaluation building have been conferred.

*Keywords*— Fuzzy information measures , Monotonicity, Decision making.

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## I. Introduction

The utmost outstanding facet of twentieth era has ensued to be the enlargement and utilization of novel transmission channels coexisting with the progression of tools of communicating and dealing information. In the course of last sixty years, particular innovative findings have been formulated in science and skill. Among them is the amount of entropy suggested by *Shannon (1948)*. The fundamental obstructions encountered by *Shannon (1948)* in the instigation of the subject, which was subsequently entitled as "Information Theory" were associated with engineering doubts allied to electrical and radio communications.

Information theory has found wide applications in statistics, information processing and computing instead of concerned with communication systems only.

If we consider entropy equivalent to uncertainty then an enormous deal of insight can be obtained. *Zadeh (1966)* introduced and enlightened about a generalized theory of vagueness or ambiguity. In order to observe about the external world, uncertainty plays a very important role. For understanding composite phenomena, any discipline that contributes in order to understand measure, regulate, maximize or minimize and control is considered as a significant input.

Uncertainty plays a significant role in our perceptions about the external world. Any discipline that can assist us in understanding it, measuring it, regulating it, maximizing or minimizing it and ultimately controlling it to the extent possible, should certainly be considered an important contribution to our scientific understanding of complex phenomena.

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Uncertainty is not a single monolithic concept. It can appear in several guises. It can arise in what we normally consider a probabilistic phenomenon. On the other hand, it can also appear in a deterministic phenomenon where we know that the outcome is not a chance event, but we are fuzzy about the possibility of the specific outcome. This type of uncertainty arising out of fuzziness is the subject of investigation of the relatively new discipline of fuzzy set theory.

Ambiguity has an important part in the contrary observations around peripheral world. The subject that aids in estimating, evaluating, maximizing or minimizing and eventually regulating it, should definitely be regarded as a critical input to technical consideration of the convoluted occurrences. The model of entropy was initiated to arrange numerical quantity of ambiguity.

*Shannon (1948)* originated a quantity

$$H(P) = -\sum_{i=1}^{n} p_i \log p_i \tag{1.1}$$

for the uncertainty of a probability distribution $(p_1,p_2,\ldots,p_n)$ and called it entropy.

*Chakrabarti (2005)*, *Aczel and Daroczy (1975)*, *Rathie and Taneja (1991)*, *Sharma and Mittal (1975)*, *Lavenda (2005)*, *Herremoes (2006)*, *Garbaczewski (2006)*, *Shigeru (2006)* and *Chen (2006)* conferred about categorization and simplification of various other measures of entropy. Some R-norm information measures are studied by *Singh and Kumar (2007)*.

*Zadeh (1966)* introduced fuzzy set theory which is associated with vagueness arising in human cognitive methods. The alteration for an element connecting membership and non-membership in the universe of classical sets is abrupt whereas in the universe of fuzzy sets the transition is gradual. Thus, the membership function describes the vagueness and ambiguity of an element and takes values in the interval [0, 1].

*Kapur (1997)* explained the concept of fuzzy entropy by considering the following vector, $\{\mu_A(x_1),\mu_A(x_2),\ldots,\mu_A(x_n)\}$. If $\mu_A(x_i) = 0$ then the i^th^ element does not belong to set A and if $\mu_A(x_i) = 1$, then the i^th^ element belongs to set A. If $\mu_A(x_i) = 0.5$, then highest ambiguity arises as to i^th^ element belongs to set A or not. Thus, $\{\mu_A(x_1),\mu_A(x_2),\ldots,\mu_A(x_n)\}$ is termed as fuzzy vector and the set A is identified as the fuzzy set. Thus crisp set are those sets in which each element is 0 or 1 and hence uncertainty does not arise in these sets whereas those sets in which elements are 0 or 1 and others lie among 0 and 1 are entitled as fuzzy sets. A fuzzy set A is represented as A = $\{x_i/\mu_A(x_i); i = 1,2,\ldots,n\}$ where $\mu_A(x_i)$ gives the degree of belongingness of the element $x_i$ to A.We explain the concept of membership function $\mu_A$: X$\rightarrow [0,1]$ as follows:

$$\mu_A(x_i) = \begin{cases} 0, if x \notin A \text{ and there is no ambiguity} \\ 1, if x \in A \text{ and there is no ambiguity} \\ 0.5, if \text{ there is maximum ambiguity} \\ \text{whether } x \in A \text{ or } x \notin A \end{cases}$$

De Luca and Termini [2] suggested that corresponding to Shannon's probabilistic entropy, the measure of fuzzy entropy should be:

$$H(A) = -\frac{1}{n\log 2} \cdot \sum_{i=1}^{n} [\mu_A(x_i) \log \mu_A(x_i) + (1 - \mu_A(x_i)) \log(1 - \mu_A(x_i))] \tag{1.2}$$

Further, *Gurdial et al.(2001)* introduced new parametric measures of information for fuzzy sets. *Kumar et al.(2011)* proposed some generalized measures of fuzzy entropy. *Bhandari and Pal (1993)* gave some new information measures for fuzzy sets.
*Kapur (2001)* proposed measures of information, generalized optimization principles and their applications. Also *Kapur and Sandip (2002)* introduced some new measures of M-Entropy.
*Deshmukh et al (2011)* introduced generalized measures of fuzzy entropy and their properties. *Prakash and Gandhi (2011)* gave new generalized measures of fuzzy entropy and their properties. *Prakash and Gandhi (2008) and (2010)* also proposed new measures of information based upon measures of central tendency and dispersion and applications of trigonometric measures of fuzzy entropy to geometry.
Further, *Prakash et al.(2011)* introduced information measures based on sampling distributions. *Parkash and Tuli (2006)* explained the concept of maximum fuzziness in different measures of entropy. Also, *Bajaj and Hooda (2010)* introduced some new generalized measures of fuzzy information. *Guo and Xin (2006)* gave some new generalized entropy formulas of fuzzy sets.

## II.NEW GENERALIZED MEASURE OF FUZZY INFORMATION

A new generalized fuzzy information measure of order α and type β has been suggested and their necessary and required properties are examined. Thereafter, its validity is also verified. Also, the monotonic behavior of fuzzy information measure of order α and type β has been conferred.

The generalized measure of fuzzy information of order $\alpha$ and type $\beta$ is given by,

$$H_\alpha^\beta(A) = \frac{1}{(1-\alpha)\beta} \sum_{i=1}^{n} \left( \{ \mu_A^{\alpha\mu_A(x_i)} + (1-\mu_A(x_i))^{\alpha(1-\mu_A(x_i))} \}^\beta - 2^\beta \right)$$

where $\alpha > 0, \alpha \nleq 1, \beta \neq 0.$ (2.1)

**PROPERTIES OF $H_\alpha^\beta(A)$**

We have supposed that, $0^{0.\alpha} = 1$ we study the following properties:

**Property1**: $H_\alpha^\beta(A) \geq 0$ i.e. $H_\alpha^\beta(A)$ is non negative.

**Property2**: $H_\alpha^\beta(A)$ is minimum iff A is a non fuzzy set.

For $\mu_A(x_i) = 0$ , it implies $H_\alpha^\beta(A) = 0$and $\mu_A(x_i) = 1$ we have $H_\alpha^\beta(A) = 0$.

**Property3**:$H_\alpha^\beta(A)$ is maximum iff A is most fuzzy set.

We have,

$$\frac{\partial H_\alpha^\beta(A)}{\partial \mu_A(x_i)} = \frac{\alpha}{1-\alpha}[\{\mu_A(x_i)^{\alpha\mu_A(x_i)} + (1-\mu_A(x_i))^{\alpha(1-\mu_A(x_i))}\}]^{\beta-1} \cdot [\mu_A(x_i)^{\alpha\mu_A(x_i)}(1 + \log\mu_A(x_i)) - (1-\mu_A(x_i))^{\alpha(1-\mu_A(x_i))}(1 + \log(1-\mu_A(x_i)))]$$

Taking, $\frac{\partial H_\alpha^\beta(A)}{\partial \mu_A(x_i)} = 0$ which is possible iff $\mu_A(x_i) = 1 - \mu_A(x_i)$ that is iff $\mu_A(x_i) = \frac{1}{2}$.

Consider,

$$\frac{\partial^2 H_\alpha^\beta(A)}{\partial^2 \mu_A(x_i)} = \frac{\alpha^2(\beta-1)}{1-\alpha}[\mu_A(x_i)^{\alpha\mu_A(x_i)} + (1-\mu_A(x_i))^{\alpha(1-\mu_A(x_i))}]^{\beta-2}[\mu_A(x_i)^{\alpha\mu_A(x_i)}(1 + \log\mu_A(x_i)) - (1-\mu_A(x_i))^{\alpha(1-\mu_A(x_i))}(1 + \log(1-\mu_A(x_i)))]^2 + \frac{\alpha}{1-\alpha}[\mu_A(x_i)^{\alpha\mu_A(x_i)} + (1-\mu_A(x_i))^{\alpha(1-\mu_A(x_i))}]^{\beta-1}[\mu_A(x_i)^{\alpha\mu_A(x_i)-1} + \alpha\mu_A(x_i)^{\alpha\mu_A(x_i)}\{1 + \log\mu_A(x_i)\}^2 + (1-\mu_A(x_i))^{\alpha(1-\mu_A(x_i))-1} + \alpha(1-\mu_A(x_i))^{\alpha(1-\mu_A(x_i))}\{1 + \log(1-\mu_A(x_i))\}^2]$$

Thus, at $\mu_A(x_i) = \frac{1}{2}$.

$$\frac{\partial^2 H_\alpha^\beta(A)}{\partial^2 \mu_A(x_i)} = \frac{-1}{1-\frac{1}{\alpha}}(2^{1-\frac{\alpha}{2}})^{\beta-1}[2^{2-\frac{\alpha}{2}} + \alpha.2^{1-\frac{\alpha}{2}}(1 - \log 2)^2]< 0$$

Hence, the maximum value exists at $\mu_A(x_i) = \frac{1}{2}$.

**Property4**: $H_\alpha^\beta(A^*) \leq H_\alpha^\beta(A)$, where $A^*$ be sharpened version of A.

When $\mu_A(x_i) = \frac{1}{2}$,

$$H_\alpha^\beta(A) = \frac{n.2^\beta}{(1-\alpha).\beta}\left(\frac{1-2^{\frac{\alpha\beta}{2}}}{2^{\frac{\alpha\beta}{2}}}\right)$$

When $\mu_A(x_i)$lies between 0 and $\frac{1}{2}$ then $H_\alpha^\beta(A)$is an increasing function whereas when $\mu_A(x_i)$lies between $\frac{1}{2}$ and 1 then $H_\alpha^\beta(A)$is a decreasing function of $\mu_A(x_i)$

Let $A^*$ be sharpened version of A which means that

If $\mu_A(x_i) < 0.5$ then $\mu_A*(x_i) \leq \mu_A(x_i)$ for all i = 1,2,…,n

If $\mu_A(x_i) > 0.5$ then $\mu_A*(x_i) \geq \mu_A(x_i)$ for all i = 1,2,…,n

Since $H_\alpha^\beta(A)$is an increasing function of $\mu_A(x_i)$ for $0 \leq \mu_A(x_i) \leq \frac{1}{2}$ and decreasing function of $\mu_A(x_i)$ for $\frac{1}{2} \leq \mu_A(x_i) \leq 1$, therefore

$\mu_A*(x_i) \leq \mu_A(x_i)$this implies $H_\alpha^\beta(A^*) \leq H_\alpha^\beta(A)$ in [0,0.5]

$\mu_A*(x_i) \leq \mu_A(x_i)$this implies $H_\alpha^\beta(A^*) \leq H_\alpha^\beta(A)$ in [0.5,1]

Hence, $H_\alpha^\beta(A^*) \leq H_\alpha^\beta(A)$

**Property5**: $H_\alpha^\beta(A) = H_\alpha^\beta(\bar{A})$, where $(\bar{A})$ is the compliment of A i.e. $\mu_A(x_i) = 1 - \mu_A(x_i)$

Thus when $\mu_A(x_i)$ is varied to (1 - $\mu_A(x_i)$ ) then $H_\alpha^\beta(A)$ does not change.

Under the above conditions, the generalized measure proposed in (2.1) is a valid measure of fuzzy information measure.

***MONOTONIC BEHAVIOR OF FUZZY INFORMATION MEASURE***

In this section we study the monotonic behavior of the fuzzy information measure. For this, diverse values of $H_\alpha^\beta(A)$ by assigning various values to $\alpha$and $\beta$ has been calculated and further the generalized measure has been presented graphically.

Case I: For $\alpha > 1, \beta = 1$, we have compiled the values of $H_\alpha^\beta(A)$ in Table(2.1) and presented the fuzzy entropy in Fig (2.1) which clearly shows that the fuzzy information measure is a concave function.

For $\alpha = 6, \beta = 1$

| $\mu_A(x_i)$ | $H_\alpha^\beta(A)$ |
|---|---|
| 0.0 | 0.0 |
| 0.1 | 0.2365 |
| 0.2 | 0.3024 |
| 0.3 | 0.3323 |
| 0.4 | 0.3460 |
| 0.5 | 0.35 |
| 0.6 | 0.3460 |
| 0.7 | 0.3323 |
| 0.8 | 0.3024 |
| 0.9 | 0.2365 |
| 1.0 | 0.0 |

Table 2.1

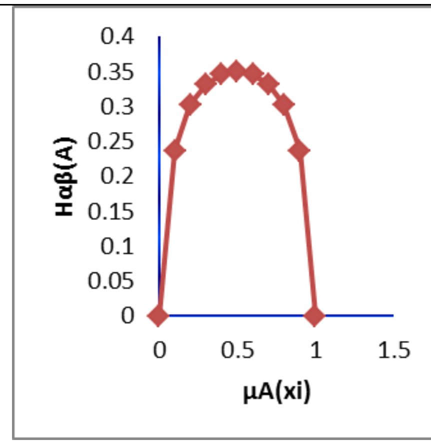


Figure 2.1

Case II: For $\alpha > 1$ and $0 < \beta < 1$.we have compiled the values of $H_\alpha^\beta(A)$ in Table (2.2) and presented the fuzzy entropy in Fig (2.2) which clearly shows that the fuzzy entropy is a concave function.

Let $\alpha$ = 2.0 and $\beta$ = 0.2

| $\mu_A(x_i)$ | $H_\alpha^\beta(A)$ |
|---|---|
| 0.0 | 0.0 |
| 0.1 | 0.3519 |
| 0.2 | 0.5358 |
| 0.3 | 0.6538 |
| 0.4 | 0.7210 |
| 0.5 | 0.7430 |
| 0.6 | 0.7210 |
| 0.7 | 0.6538 |
| 0.8 | 0.5358 |
| 0.9 | 0.3519 |
| 1.0 | 0.0 |

Table 2.2

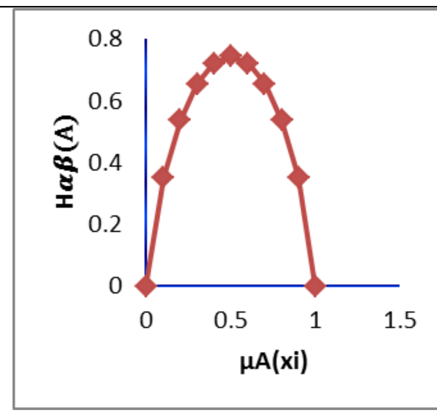


Figure 2.2

Case III: For $\alpha > 1$ and $\beta > 1$ we have compiled the values of $H_\alpha^\beta(A)$ in Table (2.3) and presented the fuzzy entropy in Fig (2.3) which clearly shows that the fuzzy entropy is a concave function.

Let $\alpha = 2.5$, $\beta = 2$

| $\mu_A(x_i)$ | $H_\alpha^\beta(A)$ |
|---|---|
| 0.0 | 0.0 |
| 0.1 | 0.7247 |
| 0.2 | 0.9393 |
| 0.3 | 1.0381 |
| 0.4 | 1.0840 |
| 0.5 | 1.0976 |
| 0.6 | 1.0840 |
| 0.7 | 1.0381 |
| 0.8 | 0.9393 |
| 0.9 | 0.7247 |
| 1.0 | 0.0 |

Table 2.3

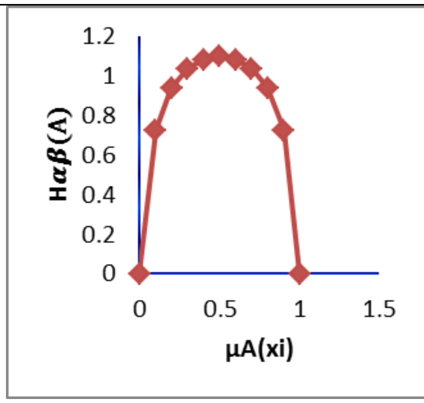


Figure 2.3

## III. APPLICATION TO DECISION PROBLEM

Decision making plays an important role in business, finance, management, economics, social and political science, engineering and computer science, biology and medicine etc. Multiple Criteria Decision Making is the examination of processes and systems that are related to numerous divergence conditions and are properly integrated into organization development procedures. In the early 1970, Multiple Criteria Decision Making has been established as an undertaking and vital field of examination. Thus, the amount of involvement to hypothesis and patterns which can be applied to methodical and coherent decision making with multi criterion has sustained to develop at a constant condition.

In order to make decision in the existence of multiple generally inconsistent condition Multiple Criteria Decision Making was established. A Multiple Criteria Decision Making problem obtains the preeminent solution from all the viable options measured on Multiple Criterion. The two categories of Multiple Criteria Decision Making methods are compensatory and non-compensatory. In compensatory method, there is condition of transaction among measure, i.e. minor decay of one measure is adequate if it is remunerated with certain improvement in one or more criterion. It is not necessary that Multiple Criteria Decision Making problem constantly have categorical or exclusive solution. Thus brittle data is deficient in reproducing many existent life decision problems. Certainly, human decisions are unclear in quality and it is unfeasible to denote them by precise mathematical values. So to represent human decision, an additional accurate method could use linguistic variables.

Example: Suppose the department of mathematics in a university wants to appoint outstanding mathematics teachers. The appointment is done by a committee of decision makers, Chairperson $A_1$, Secretary $A_2$, Director $A_3$ and Principal $A_4$. Panel of decision makers made strict evaluation for the teachers according to the following five attributes, the past experience $P_1$, the research capability $P_2$, subject knowledge $P_3$, teaching skill $P_4$ and the industry exposure $P_5$. For evaluating four alternatives, the decision markers formed four fuzzy sets as:

| | ($P_1$) | ($P_2$) | ($P_3$) | ($P_4$) | ($P_5$) |
|---|---|---|---|---|---|
| ($A_1$) | 0.4 | 0.7 | 0.6 | 0.2 | 0.2 |
| ($A_2$) | 0.3 | 0.2 | 0.4 | 0.6 | 0.1 |
| ($A_3$) | 0.1 | 0.1 | 0.6 | 0.7 | 0.5 |
| ($A_4$) | 0.4 | 0.7 | 0.5 | 0.3 | 0.3 |

Fuzzy information measure for each given option ($A_1$) , ($A_2$) , ($A_3$) , ($A_4$) is given as
For $\alpha = 2$, $\beta = 0.6$ we have,

$H(A_1) = 3.7306$
$H(A_2) = 3.5231$
$H(A_3) = 3.7306$
$H(A_4) = 4.0028$

Optimal solution with maximum entropy is $A_4$ with preference order given as $A_4$,$A_1$,$A_3$ and $A_2$.

## IV.CONCLUSION

The recent degrees anticipated have augmented the analysis in an extensive method and specified means and procedures for Multi Criteria Decision Making problems that are desired significantly in current society corresponding numerical and in addition qualitative and technical character. The suggested quantities have numerous well designed properties that enrich the employability of these quantities. Further, certain recent outcome formulation procedures have been established for explaining Multi Criteria Decision making problems with fuzzy information and supported with exemplified instances

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