A note on the Adomian decomposition method for generalized Burgers-Fisher equation

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***Abstract*—** The Adomian decomposition method (ADM) for Burgers-Fisher equation was introduced in. The generalized Burgers-Fisher equation is nonlinear partial differential equation which appears in fluid dynamics and other fields of applied physics. Burgers-Fisher equation describes different physical phenomena as convection effect, diffusion transport or interaction between the reaction mechanisms. The purpose of this paper is to improve numerical solution of the problems given in according the note given in and formulation given in. The calculated numerical results with just first few (two, three or four) terms in series decomposition show the improvement in efficiency and accuracy with presented algorithm. More terms used for numerical approximation of analytical solution result with significantly improved approximation. The proposed procedure and algorithm are expressed on wide number of examples. The comparison of numerical results according the given approach by using Adomian decomposition method with exact solution, with numerical results by decomposition method explained in and with numerical results obtained by exp-function method with heuristic computation presented in is provided..

***Keywords*—** Burgers-Fisher equation, nonlinear partial differential equation, Adomian decomposition method.

I. Introduction

We consider the nonlinear partial differential equation known as generalized Burgers-Fisher equation. This nonlinear partial differential equation appears in many fields of applied physics. Some of widely used applications are in fluid dynamics, turbulence and shock wave formation but also in financial mathematics. Burgers-Fisher equation is nonlinear partial differential equation that describes different physical phenomena as convection effect, diffusion transport or interaction between the reaction mechanisms**.** The equation has properties of convection from known Burgers differential equation and properties of diffusion transport from Fisher differential equation. This nonlinear partial differential equation was uncovered by J.M. Burgers and R.A. Fisher.

Large number of numerical procedure have been proposed to solve different nonlinear partial differential equations. The nonlinear partial differential equations are solved easily and efficient by using Adomian decomposition method (ADM). ADM is numerical method for solving nonlinear partial differential equations without transforming the equation and avoiding linearization and perturbation. The ADM for Burgers-Fisher was introduced in papers [1] and [2]. The generalized Burgers-Huxley equation was solved by using ADM in [1], but numerical results are improved in [3]. In this paper, the ADM approach shown in [3,4] was implemented for solving the generalized Burgers-Fisher equation. Wide number of numerical examples in this paper are taken from [1] and from [5] to show the improvement of numerical results calculated by decomposition method presented in [1] and exp-function method with heuristic computation presented in [5]. Numerical results show efficiency and accuracy of presented algorithm. By using more terms approximation of analytical solution could be significantly improved, what would be shown in examples. The convergence of sequence of numerical solutions calculating by ADM for Burgers-Fisher equation was demonstrated in [2].

II. Description of generalized Burgers-fisher equation

Consider the nonlinear partial differential equation known as the generalized Burgers-Fisher equation given in following form

$$u_t + \alpha u^{\delta} u_x - u_{xx} = \beta u\left(1 - u^{\delta}\right) \quad 0 \le x \le 1, t \ge 0\,, \tag{1}$$

with its initial condition

$$u(x,0) = u_0(x). \tag{2}$$

With simple supstitutions, $= \frac{\alpha\delta}{2(1+\delta)}$, $c = \frac{\alpha^2 + \beta(1+\delta)^2}{\alpha(1+\delta)}$, and given initial condition,

$$u_0(x) = \left[\frac{1}{2}\left(1 - \tanh(kx)\right)\right]^{1/\delta}. \tag{3}$$

follows the exact (analytical) solution of governing nonlinear partial differential equation (1),

$$u(x,t) = \left[\frac{1}{2}\left(1 - \tanh k(x - ct)\right)\right]^{1/\delta}. \tag{4}$$

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The governing nonlinear partial differential equation, (1), can be expressed in following different forms

$$u_t = u_{xx} - \alpha u^\delta u_x + \beta u\left(1-u^\delta\right)$$
$$= u_{xx} + \beta u - \beta u^{1+\delta} - \alpha\left(\frac{1}{1+\delta}u^{1+\delta}\right)_x . \quad (5)$$

In an operator form, equation (5) can be written as

$$Lu = u_{xx} + \beta u - \beta u^{1+\delta} - \alpha\left(\frac{1}{1+\delta}u^{1+\delta}\right)_x , \quad (6)$$

where the differential operator L is given as time derivative,

$$L = \frac{d}{dt} . \quad (7)$$

The inverse operator L^{-1} is an integral operator defined as $L^{-1}(\cdot) = \int_0^t(\cdot)dx$. Operating with inverse operator L^{-1} on (6) and using the initial condition (2) yields

$$\begin{aligned} u(x,t) &= u_0(x) \\ &\quad + L^{-1}\left(u_{xx} + \beta u - \beta u^{1+\delta} - \alpha\left(\frac{1}{1+\delta}u^{1+\delta}\right)_x\right) \\ &= u_0(x) \\ &\quad + L^{-1}\left(u_{xx} + \beta u - \beta u^{1+\delta} - \alpha\left(\frac{1}{1+\delta}u^{1+\delta}\right)_x\right). \end{aligned} \quad (8)$$

III. ADOMIAN DECOMPOSITION METHOD

The Adomian decomposition method defines the numerical solution u(x, t) as the decomposition series

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t), \quad (9)$$

where the components $u_n(x,t)$ will be determined reccurently. Finite part if infinite series is taken as numerical solution, numerical approximation of analytical solution,

$$u(x,t) = \sum_{n=0}^{N} u_n(x,t) \ , \quad (10)$$

Using the given initial condition (2), the reccurence relation for the components $u_k(x,t)$ is admitted by modified Adomian decomposition method in the form

$$u_0(x,t) = u_0(t) \ , \quad (11)$$

$$u_k(x,t) = L^{-1}(D(u_{k-1}(x))), \ k \geq 1 \ . \quad (12)$$

where $D(u(x))$ is governing nonlinear differential equation expressed in an operator form.
The nonlinear term in nonlinear partial differential equation can be expressed in operator form as $F(u(x))$ and further in the form of infinite series

$$F(u(x)) = \sum_{n=0}^{\infty} A_n \ , \quad (13)$$

where A_n are Adomian polynomials that can be constructed by algorithm derived in [4], as

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F\left(\sum_{k=0}^{\infty}\lambda^k u_k\right)\right]_{\lambda=0} . \quad (14)$$

The first few terms for Adomian polynomials, A_n, are expressed in generalized closed form as

$$A_0 = F(u_0) \ , \quad (15)$$

$$A_1 = u_1 F'(u_0) \ , \quad (16)$$

$$A_2 = u_2 F'(u_0) + \frac{1}{2!}u_1^2 F''(u_0) , \quad (17)$$

$$A_3 = u_3 F'(u_0) + u_1 u_2 F''(u_0) + \frac{1}{3!}u_1^3 F'''(u_0) \ ,$$

$$A_4 = u_4 F'(u_0) + \left(\frac{1}{2!}u_2^3 + u_1 u_3\right) F''(u_0)$$
$$+ \frac{1}{2!}u_1^2 u_2 F'''(u_0) + \frac{1}{4!}u_1^4 F^{(iv)}(u_0) \quad . \quad (18)$$

Other polynomials can be generated similar. First polynomial, A_0 , depends only on initial condition, u_0. Second polynomial, A_1, depends on u_0 and u_1. Generally, $n+1$-st polynomial depends on $u_0, u_1, \ldots, u_n$. Introduced Adomian polynomials show that sum of subscripts of the components of u of each term A_k is equal to k.
According the given algorithm, $A_0 = F(u_0)$ is identified and separated from the other terms. Remaining terms of $F(u)$can be expressed by using appropriate algebraic operation.

IV. ANALYSIS OF GENERALIZED BURGERS-FISHER EQUATION WITH ADOMIAN DECOMPOSITION METHOD

The nonlinear terms of governing nonlinear partial differential equation under integral operator, $u^{1+\delta}$ and $(u^{1+\delta})_z$, are expressed as an infinite series of polynomials

$$u^{1+\delta} = \sum_{n=0}^{\infty} A_n \ , \quad (19)$$

$$\left(u^{1+\delta}\right)_x = \sum_{n=0}^{\infty} A_{n,x} \ , \quad (20)$$

where A_n are Adomian polynomials that can be constructed by algorithm derived in [4], as

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F\left(\sum_{k=0}^{\infty}\lambda^k u_k\right)\right]_{\lambda=0} . \quad (21)$$

For the nonlinear term expressed in given nonlinear partial differential equation, $A(u) = u^{1+\delta}$, the first four terms for Adomian polynomials, A_n, are expressed in generalized closed form as

$$A_0 = u_0^{1+\delta} \ , \tag{22}$$

$$A_1 = (1+\delta)u_0^{\delta}u_1 \ , \tag{23}$$

$$A_2 = (1+\delta)u_0^{\delta}u_2 + \frac{1}{2}(1+\delta)\delta u_0^{\delta-1}u_1^2 \ , \tag{24}$$

$$A_3 = (1+\delta)u_0^{\delta}u_3 + (1+\delta)\delta u_0^{\delta-1}u_1u_2 + \frac{1}{6}(1+\delta)\delta(\delta-1)u_0^{\delta-2}u_1^3 \ . \tag{25}$$

$$A_4 = (1+\delta)u_0^{\delta}u_4 + (1+\delta)\delta u_0^{\delta-1}\left(\frac{1}{2}u_2^3 + u_1u_3\right) + \frac{1}{2}(1+\delta)\delta(\delta-1)u_0^{\delta-2}u_1^2u_2 + \frac{1}{24}(1+\delta)\delta(\delta-1)(\delta-2)u_0^{\delta-3}u_1^4 \ . \tag{26}$$

The needed further derivatives over x of Adomian polynomials, $A_{n,x}$, are then also expressed for the first four terms

$$A_{0,x} = (1+\delta)u_0^{\delta}u_{0,x} \ , \tag{27}$$

$$A_{1,x} = (1+\delta)\left[\delta u_0^{\delta-1}u_1u_{0,x} + u_0^{\delta}u_{1,x}\right] \ , \tag{28}$$

$$A_{2,x} = (1+\delta)\left[\delta u_0^{\delta-1}u_{0,x}u_2 + u_0^{\delta}u_{2,x}\right] + \frac{1}{2!}(1+\delta)\delta\left[(\delta-1)u_0^{\delta-2}u_{0,x}u_1^2 + 2u_0^{\delta-1}u_1u_{1,x}\right] \ , \tag{29}$$

$$A_{3,x} = (1+\delta)\left[\delta u_0^{\delta-2}u_{0,x}u_3 + u_0^{\delta}u_{3,x}\right] + (1+\delta)\delta\left[(\delta-1)u_0^{\delta-2}u_{0,x}u_1u_2 + u_0^{\delta-1}u_{1,x}u_2 + u_0^{\delta-1}u_1u_{2,x}\right] + \frac{1}{3!}(1+\delta)\delta(\delta-1)\left[(\delta-2)u_0^{\delta-3}u_{0,x}{u_1}^3 + 3u_0^{\delta-2}{u_1}^2{u_1}^2u_{1,x}\right] \ . \tag{30}$$

Using the given initial condition (2), the reccurence relation for the components $u_k(x,t)$ admitted by modified Adomian decomposition method in the form

$$u_0(x,t) = u_0(t) \ , \tag{31}$$

$$u_k(x,t) = L^{-1}\left(u_{k-1,xx} + \beta u_{k-1} - \beta A_{k-1} - \alpha\frac{1}{1+\delta}A_{k-1,x}\right), \quad k \geq 1 \ . \tag{32}$$

After determination of specific finite number of the components u_k, the approximation

$$\emptyset_n = \sum_{k=0}^{n} u_k \ ,$$

can be used to approximate solution of governing nonlinear partial differential equation.

V. NUMERICAL EXAMPLES

Now, we shall show the efficiency and accuracy of presented numerical scheme and improved numerical solutions compared to the algorithm and results given in [1] and [5]. We consider the same parameter values for coefficients of the generalized Burger's-Fisher equation (1) with given initial condition (3) as considered specifically in [1] and [5].

Example 1
In the first presented example, we take coefficients of governing ewuation δ = 1, α = 0.001 and ß = 0.001. We consider nonlinear partial differential equation (1) with initial condition

$$u_0(x) = \left[\frac{1}{2}(1-\tanh(x/4000))\right] \ , \tag{34}$$

what leads to analytical solution

$$u(x,t) = \left[\frac{1}{2}(1-\tanh(x-2.0005t)/4000)\right] \ . \tag{36}$$

The numerical results are calculated and approximated with only first two series members, $\emptyset_1(x,t) = u_0(x,t) + u_1(x,t)$ and results are shown in Table I.

Table I
Numerical results with δ = 1, α = 0.001 and ß = 0.001, Ex. 1

x	t	exact	$\emptyset_1$	error	error in [1]	error in [5]
0.5	0.01	0.499940	0.499940	$2\cdot10^{-15}$	$2\cdot10^{-6}$	$4\cdot10^{-9}$
0.9	0.01	0.499888	0.499888	$3\cdot10^{-15}$	$2\cdot10^{-6}$	$2\cdot10^{-8}$
0.5	0.1	0.499963	0.499963	$1\cdot10^{-13}$		
0.5	1.0	0.500188	0.500188	$5\cdot10^{-12}$		

Numerical results calculated with only first two terms improved numerical solutions evaluated in [1] and [5].

Example 2
In the second example, we take values δ = 2, α = 1 and ß = 1. We consider nonlinear partial differential equation (1) with initial condition

$$u_0(x) = \left[\frac{1}{2}(1-\tanh(x/3))\right]^{1/2} \ , \tag{36}$$

what leads to analytical solution

$$u(x,t) = \left[\frac{1}{2}(1-\tanh(x-10t/3)/3)\right]^{1/3} . \ , \tag{37}$$

The numerical results in this example are calculated and approximated again with only first three series members,

$\emptyset_2(\mathrm{x,t}) = u_0(x,t) + u_1(x,t) + u_2(x,t)$ and results are shown in Table II

Table II
Numerical results with δ = 2, α = 1 and ß = 1, Ex. 2

x	t	exact	$\emptyset_2$	error	error in [1]	error in [5]
0.1	0.001	0.695625	0.695625	$1\cdot10^{-10}$	$3\cdot10^{-3}$	$1\cdot10^{-6}$
0.5	0.001	0.646506	0.646506	$1\cdot10^{-10}$	$3\cdot10^{-3}$	$1\cdot10^{-6}$
0.9	0.001	0.595695	0.595695	$1\cdot10^{-10}$	$3\cdot10^{-3}$	$4\cdot10^{-6}$
0.5	0.01	0.650264	0.650264	$1\cdot10^{-7}$		
0.5	0.1	0.687205	0.687323	$1\cdot10^{-4}$		

Numerical results calculated with only first three terms improved numerical solutions evaluated in [1] and [5]. We can also show that series with more terms gives better solution. Compared results for $\emptyset_2(\mathrm{x,t}) = u_0(x,t) + u_1(x,t) + u_2(x,t)$ and $\emptyset_3(\mathrm{x,t}) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t)$ are shown in Table III.

Table III
Numerical results with δ = 2, α = 1 and ß = 1, Ex. 2

x	t	$\emptyset_2$	error	$\emptyset_3$	error
0.5	0.01	0.650264	$1.2\cdot10^{-7}$	0.650264	$1.5\cdot10^{-10}$
0.5	0.1	0.687323	$1.2\cdot10^{-4}$	0.687205	$1.9\cdot10^{-6}$

Table III shows that by using more terms we get better approximation of analytical solution. With just one more term we get much better numerical solution.

Example 3
In the third example, we take values δ = 3, α = 1 and ß = 0. We consider nonlinear partial differential equation (1) with initial condition

$$u_0(x) = \left[\frac{1}{2}(1-\tanh(3x/8))\right]^3 \quad , \tag{38}$$

what leads to analytical solution

$$u(x,t) = \left[\frac{1}{2}(1-\tanh 3(x-t/4)/8)\right]^{1/3}. \tag{39}$$

The numerical results in this example are calculated and approximated again with first three series members, $\emptyset_2(\mathrm{x,t}) = u_0(x,t) + u_1(x,t) + u_2(x,t)$ and results are shown in Table IV.

Table IV
Numerical results with δ = 3, α = 1 and ß = 0, Ex. 3

x	t	exact	$\emptyset_2$	error	error in [1]	error in [5]
0.1	0.001	0.783683	0.783683	$3.5\cdot10^{-13}$	$4\cdot10^{-4}$	$5\cdot10^{-7}$
0.5	0.001	0.741309	0.741309	$4.5\cdot10^{-14}$	$2\cdot10^{-3}$	$6\cdot10^{-7}$
0.9	0.001	0.696183	0.696183	$4.6\cdot10^{-14}$	$9\cdot10^{-4}$	$7\cdot10^{-7}$
0.5	0.01	0.741566	0.741566	$4.5\cdot10^{-11}$		
0.5	0.1	0.741022	0.741022	$4.5\cdot10^{-8}$		

Numerical results calculated with only first three terms improved numerical solutions evaluated in [1] and [5]. We can also in this example show that series with more terms gives better solution. Compared results for $\emptyset_3(\mathrm{x,t}) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t)$ and $\emptyset_4(\mathrm{x,t}) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + u_4(x,t)$ are shown in Table V.

Table V
Numerical results with δ = 3, α = 1 and ß = 0, Ex. 3

x	t	$\emptyset_3$ error	$\emptyset_3$error	$\emptyset_4$ error
0.5	0.1	$4.5\cdot10^{-8}$	$1.0\cdot10^{-10}$	$1.2\cdot10^{-12}$
0.5	0.5	$5.5\cdot10^{-6}$	$6.7\cdot10^{-8}$	$3.7\cdot10^{-9}$
0.5	1.0	$4.7\cdot10^{-5}$	$1.7\cdot10^{-8}$	$1.2\cdot10^{-7}$

Table V shows that by using more terms we get better approximation of analytical solution. In this example we take even five terms to show quality of approximation for larger value of time variable.

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Example 4
In the fourth example, we take values as in example presented in [5] δ = 1, α = 0.5 and ß = 0.5. We consider nonlinear partial differential equation (1) with initial condition

$$u_0(x) = \left[\frac{1}{2}(1-\tanh(x/4))\right] \quad , \tag{40}$$

what leads to analytical solution

$$u(x,t) = \left[\frac{1}{2}(1-\tanh(x-2.25t)/8)\right] \quad . \tag{41}$$

The numerical results in this example are calculated and approximated again with only first three series members, $\emptyset_2(\mathrm{x,t}) = u_0(x,t) + u_1(x,t) + u_2(x,t)$ and results are shown in Table VI.

Table VI
Numerical results with δ = 1, α = 0.5 and ß = 0.5, Ex. 4

x	t	exact	$\emptyset_2$	error	error in [5]
0.4	0.1	0489064	0489064	$3.7\cdot10^{-6}$	$1.6\cdot10^{-5}$
0.8	0.1	0.464124	0.464128	$3.6\cdot10^{-6}$	$3.5\cdot10^{-5}$
0.4	0.8	0586618	0588501	$1.9\cdot10^{-3}$	$6.2\cdot10^{-4}$
0.8	0.8	0.562177	0.564046	$1.9\cdot10^{-3}$	$6.3\cdot10^{-4}$

We can also show that series with more series members gives better solution. Compared numerical results calculated with three $\emptyset_2(\mathrm{x,t}) = u_0(x,t) + u_1(x,t) + u_2(x,t)$ and $\emptyset_3(\mathrm{x,t}) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t)$ with four terms are shown in Table VII.

Table VII
Numerical results with δ = 1, α = 0.5 and ß = 0.5, Ex. 4

x	t	$\emptyset_2$	error	$\emptyset_3$	error
0.4	0.1	0489064	$3.7\cdot10^{-6}$	0489064	$9.2\cdot10^{-9}$
0.8	0.1	0.464128	$3.6\cdot10^{-6}$	0.464124	$6.3\cdot10^{-9}$
0.4	0.8	0588501	$1.9\cdot10^{-3}$	0586618	$4.3\cdot10^{-6}$
0.8	0.8	0.564046	$1.9\cdot10^{-3}$	0.52223	$4.6\cdot10^{-5}$

Table VII shows that by using more terms we get better approximation of analytical solution. By using four terms we get significantly better approximation of analytical solution.

Example 5
In the fifth example, we take values as in example presented in [5], δ = 1, α = 0.1 and ß = 0.1. We consider nonlinear partial differential equation (1) with initial condition

$$u_0(x) = \left[\frac{1}{2}(1 - \tanh(x/40))\right] \quad , \tag{42}$$

what leads to analytical solution

$$u(x,t) = \left[\frac{1}{2}(1 - \tanh(x - 2.05t)/40)\right] \quad . \tag{43}$$

The numerical results in this example are calculated and approximated again with only first three series members, $\emptyset_2(\mathrm{x,t}) = u_0(x,t) + u_1(x,t) + u_2(x,t)$ and results are shown in Table VIII.

Table VIII
Numerical results with δ = 1, α = 0.1 and ß = 0.1, Ex. 5

x	t	exact	$\emptyset_2$	error	error in [5]
0.4	0.1	0.497563	0.497563	$2.2\cdot10^{-8}$	$4.1\cdot10^{-7}$
0.8	0.1	0.492563	0.492563	$2.2\cdot10^{-8}$	$2.4\cdot10^{-7}$
0.4	0.8	0.515495	0.515507	$1.1\cdot10^{-5}$	$3.1\cdot10^{-6}$
0.8	0.8	0.510498	0.510510	$1.1\cdot10^{-5}$	$5.3\cdot10^{-6}$

We can also show that series with more terms gives in this example significantly improved numerical solution. Compared results for $\emptyset_2(\mathrm{x,t}) = u_0(x,t) + u_1(x,t) + u_2(x,t)$ and $\emptyset_3(\mathrm{x,t}) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t)$ are shown in Table IX.

Table IX
Numerical results with δ = 1, α = 0.1 and ß = 0.1, Ex. 5

x	t	$\emptyset_2$	error	$\emptyset_3$	error
0.4	0.1	0.497563	$2.2\cdot10^{-8}$	0-497563	$2.1\cdot10^{-12}$
0.8	0.1	0.492563	$2.2\cdot10^{-8}$	0.492563	$4.4\cdot10^{-12}$
0.4	0.8	0.515507	$1.1\cdot10^{-5}$	0.515495	$1.7\cdot10^{-9}$
0.8	0.8	0.510510	$1.1\cdot10^{-5}$	0.510498	$1.7\cdot10^{-8}$

Table IX shows that by using more terms we get better approximation of analytical solution. By using four terms we get significantly better approximation of analytical solution.

VI. CONCLUSION

In this paper, we have improved numerical solutions of a generalized Burgers-Fisher equation given in [1] through an accurate, efficient and more convenient form of the Adomian recursive scheme according the note given in [3] and recursive scheme given in [4]. The proposed improved decomposition algorithm resulted with reliable and efficient computational method for governing nonlinear partial differential equation known as Burgers-Fisher equation. We have also presented that more series members used in decomposition leads to improved numerical approximation of analytical solution. The numerical solutions are very close to the analytical with only first few (two, three or four) terms in the series decomposition. Wide number of numerical examples are presented and used to describe proposed procedure and algorithm. Numerical examples show improvement of numerical solution calculated with proposed numerical procedure and algorithm in comparison with numerical solution presented in papers [1] and]5]

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