# Improvement of a tool for the easy determination of control factor interaction in the Design of Experiments and the Taguchi Methods 

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#### Abstract

In recent years, the Design of Experiments (hereafter, DOE) have been widely used to decide optimum processing conditions. However, when large interactions between several control factors are present, since they behave as confounding variables, the estimation accuracy is significantly reduced and making the practical use of the DOE extremely difficult in some cases. As a common countermeasure, calculation accuracy is confirmed by comparing, through the final results, the best and worst results. This can be of great harm in terms of time and labor and, if the difference between the best and worst results is large, could result in the DOE estimations being ignored. Therefore, in previous studies, a usable tool for the easy determination of control factor interactions in the DOE was developed; here, said tool was able to determine control factor interactions in the DOE through several mathematical models. This research presented an improvement to the previous tool through an improved algorithm and more detailed mathematical models to evaluate complex control factor interactions. It was concluded that, (1) an improved tool for the determination of control factor interactions in the DOE and the Taguchi Methods was developed, (2) the tool was able to detect previously indistinguishable complex control factor interactions in the DOE or the Taguchi Methods, (3) a new algorithm was able to determine complex control factor interactions in models between control factors and functions.


Keywords-Experimental design, innovation, innovative tool, algorithm, optimum condition, control factor interactions

## I. Introduction

In recent years, the Design of Experiments (hereafter, DOE) and the Taguchi Methods have been widely used to decide optimum processing conditions [1], [2], [3]. Usually, the DOE procedure first defines the design parameters to be used. Second, the design parameters are used as control factors and all possible control factor combinations are compressed in an orthogonal table. Third, experimentation or CAE simulations are done according to the aforementioned orthogonal table. Finally, the obtained data is summarized and the optimum control factors are selected. Here, experiments or CAE

[^0]simulations are performed in the orthogonal table. However, when large interactions between control factors are present, since interactions behave as confounding variables, the estimation accuracy is significantly reduced and making the practical use of the DOE extremely difficult in some cases.

As a common countermeasure, the calculation accuracy is confirmed by comparing, through the final results, the best and worst results. This can be of great harm in terms of time and labor and, if the difference between the best and worst results is large, could result in the DOE estimations being ignored. In previous studies [4], a tool for the easy determination of control factor interactions in the DOE and the Taguchi Methods was developed and evaluated. Specifically, the Level 1 of each control factor was intentionally set to be zero (0) in the orthogonal table in order to obtain functions that are not influenced by control factor interactions. The comparison between the aforementioned functions and conventional DOE orthogonal table functions was then used as an assessment criterion, under several mathematical models, to prove the presence of control factor interaction as well as to determine interacting control factors. This research presented an improvement to the previous tool through an improved algorithm and more detailed mathematical models to evaluate complex control factor interactions. It was thought that this method was necessary in order to clarify the control factor interaction ambiguity present in the DOE and the Taguchi Methods [5], [6], [7], [8].

## II. Determination of Control Factor Interactions

## A. Previous Control Factor Interaction Detection Algorithm

In this section, the previous algorithm for the easy determination of control factor interaction in DOE is shown [4]. Here, the developed algorithm consisted in setting a level value of 0 (Zero) in each control factor in order to observe that the final function did not present any influence due to control factor

Table 1 Orthogonal table in experimental

| L4 | Control factors |  |  | Function <br>  |  | A | B | C | F |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $A_{1}$ | $B_{1}$ | $C_{1}$ | $F_{1}$ |  |  |  |  |  |
| No.2 | $A_{1}$ | $B_{2}$ | $C_{2}$ | $F_{2}$ |  |  |  |  |  |
| No.3 | $A_{2}$ | $B_{1}$ | $C_{2}$ | $F_{3}$ |  |  |  |  |  |
| No.4 | $A_{2}$ | $B_{2}$ | $C_{1}$ | $F_{4}$ |  |  |  |  |  |



Fig. 1 Effective figure of the control factor (control factor A)

Table 2 Trial data without control factor interaction

| Particularity of the trial set | Trial No. | Control factors |  |  | Function F |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | A | B | C |  |
| Trial data without interaction from control factor A | No.1' | 0 | $B_{1}$ | $C_{1}$ | $F_{\text {A01 }}$, |
|  | No.2' | 0 | $B_{2}$ | $C_{2}$ | $F_{\text {A02 }}{ }^{\prime}$ |
| Trial data without interaction from control factor B | No.3' | $A_{1}$ | 0 | $C_{1}$ | $F_{\text {B01 }}$, |
|  | No.4' | $A_{2}$ | 0 | $C_{2}$ | $F_{\text {B0 }}$ |
| Trial data without interaction from control factor C | No.5' | $A_{1}$ | $B_{1}$ | 0 | $F_{C 01}$, |
|  | No.6' | $A_{2}$ | $B_{2}$ | 0 | $F_{\text {C02 }}$ ' |

Table 3 Function $F_{A 0}$, $F_{B 0}$ ' and $F_{C O}$, without control factor interaction effects at level value $=0$

| Equation (4) | Control factors without interaction effects |
| :--- | :--- |
| $F_{A 0}{ }^{\prime}$ | $\mathrm{A}(\mathrm{A}$ and B or A and C) |
| $F_{B 0}{ }^{\prime}$ | $\mathrm{B}(\mathrm{A}$ and B or B and C) |
| $F_{C 0}{ }^{\prime}$ | $\mathrm{C}(\mathrm{A}$ and C or B and C) |

Notes:
$F_{A 0}$ 'and $F_{B 0}$ ' are not influenced in the presence of control factor interaction between control factors A and B.
$F_{B 0}$ 'and $F_{C 0}$ ' are not influenced in the presence of control factor interaction between control factors B and C.
$F_{A 0}$ 'and $F_{C 0}$, are not influenced in the presence of control factor interaction between control factors A and C.
interaction. For this, the algorithm used the properties present in the orthogonal array. For instance, in Table 1, control factors $A$, $B$ and $C$ were defined and two levels for each factor were defined as $A_{1}, A_{2}, B_{1}, B_{2}, C_{1}$ and $C_{2}$ respectively. The L4 orthogonal table shown uses the control factors and those levels for DOE. The function $F$ is the final result of the experiment or the CAE simulations in DOE and is included in the orthogonal table. The mathematical model in equation (1) defined function $F$ as,

$$
\begin{equation*}
F_{\mathrm{L}}=5 A_{\mathrm{S}}+2 B_{\mathrm{T}}+3 C_{\mathrm{M}}-0.1 A_{\mathrm{S}} B_{\mathrm{T}} \tag{1}
\end{equation*}
$$

In this case the equation is set so that $A_{\mathrm{S}}$ and $B_{\mathrm{T}}$ are the control factors with control factor interaction of a certain magnitude (0.1). The terms $S, T, M$ could take a value of 1 or 2 and the term $L$ could take values from 1 to 4 . The relationship between the
control factors and the final function was " $F_{\mathrm{L}}=5 A_{\mathrm{S}}+2 B_{\mathrm{T}}+$ $3 C_{\mathrm{M}}$ ". However, this relationship is affected by the control factor interaction, described as " $-0.1 A_{\mathrm{S}} B_{\mathrm{T}}$ " in equation (1). This equation is supposed to yield the most desirable conditions when the final function $F$ is obtained. Therefore the minus value " - " is used for the control factor interaction calculation. Final functions $F_{A 1}, F_{A 2}, F_{B 1}, F_{B 2}, F_{C 1}$ and $F_{C 2}$ for levels $A_{1}, A_{2}$, $B_{1}, B_{2}, C_{1}$ and $C_{2}$ was calculated by equations (2) due to the orthogonal table properties.


When each level value for the control factors $A, B$ and $C$ is set to be zero " 0 ", the final functions $F_{A 0}, F_{B 0}$ and $F_{C 0}$ can be approximated by equation (3) as depicted in Fig. 1.

$$
\left.\begin{array}{l}
F_{A 0}=F_{A 1}-\frac{F_{A 2}-F_{A 1}}{A_{2}-A_{1}} \quad A_{1}  \tag{3}\\
F_{B 0}=F_{B 1}-\frac{F_{B 2}-F_{B 1}}{B_{2}-B_{1}} B_{1} \\
F_{C 0}=F_{C 1}-\frac{F_{C 2}-F_{C 1}}{C_{2}-C_{1}} C_{1}
\end{array}\right\}
$$

Even though the number of each level was only two and poses a simple calculation, when the number becomes larger an array setup would be more convenient. From here, six trials, as shown in Table 2, to define control interaction cases were performed. On the other hand, except for the level value $=0$, the other levels kept the orthogonal property when looking for control factor interaction as shown in Table 2. Here, when the control factor level in $A, B$ and $C$ was set as zero " 0 ", the resultant final functions were denominated as $F_{A 0^{\prime}}, F_{B 0}$, and $F_{C 0}$, and were calculated by the equation (4) on the new trials.

$$
\left.\begin{array}{l}
F_{A 0}^{\prime}=\left(F_{A 01}{ }^{\prime}+F_{A 02}{ }^{\prime}\right) / 2  \tag{4}\\
F_{B 0}=\left(F_{B 01},+F_{B 02}\right) / 2 \\
F_{C 0}=\left(F_{C 01},+F_{C 02},\right) / 2
\end{array}\right\}
$$

In Table 3, it can be observed that, as zero values were set in Table 2, $F_{A 0}$ 'and $F_{B 0}$, have no control factor interaction with $A$

Table 4 Control factor interaction assessment criteria

| Conditions | Control factor interaction |
| :---: | :---: |
| $\begin{aligned} & F_{A 0}{ }^{\prime}=F_{A 0}, F_{B 0}{ }^{\prime}=F_{B 0} \text { and } \\ & F_{C 0},=F_{C 0} \end{aligned}$ | No interaction |
| $\begin{aligned} & F_{A 0},=F_{A 0}, F_{B 0}{ }^{\prime}=F_{B 0} \text { and } \\ & \boldsymbol{F}_{C 0} \neq \boldsymbol{F}_{C 0} \end{aligned}$ | Control factor interaction between control factors A and B. |
| $\begin{aligned} & \boldsymbol{F}_{A 0}, \neq \boldsymbol{F}_{A 0}, F_{B 0}{ }^{\prime}=F_{B 0} \\ & \text { and } F_{C 0}=F_{C 0} \end{aligned}$ | Control factor interaction between control factors B and C. |
| $\begin{aligned} & F_{A 0},=F_{A 0}, \boldsymbol{F}_{B 0}{ }^{\prime} \neq \boldsymbol{F}_{\mathrm{B} 0} \\ & \text { and } F_{C 0}{ }^{\prime}=F_{C 0} \end{aligned}$ | Control factor interaction between control factors A and C. |

and $B ; F_{B 0}$ 'and $F_{C 0}$ ' have no control factor interaction with $B$ and $C$; and $F_{A 0}$ 'and $F_{C 0}$ ' have no control factor interaction with control factors A and C.

Finally, the calculated final functions $F_{A 0}, F_{B 0}$ and $F_{C 0}$ were compared with the trial final functions $F_{A 0^{\prime}}, F_{B 0^{\prime}}$ and $F_{C 0}$, respectively. Here, the control factor interaction and the control factors under reciprocal interaction can be determined under the criteria shown in Table 4.

## B. Final Stage Evaluation Through Defined Mathematical Models

In the previous algorithm for control factor interaction determination was evaluated by using several simple mathematical models such as equation (1). The tool was able to determine control factor interactions in DOE [4], and the defined assessment criteria allowed the relationship between the influence magnitude and the control factor interaction estimation to be handled to a certain degree [4].

## III. Evaluation Through More Detailed Models

## A. Evaluation Through A Cubic Equation Model

Here, a more complex model was used for evaluation of the algorithm. The mathematical model using equation (5) was then used.

$$
\begin{equation*}
F_{\mathrm{L}}=0.05 A_{\mathrm{S}}^{3}+0.02 B_{\mathrm{T}}^{3}+0.03 C_{\mathrm{M}}^{3}-0.1 A_{\mathrm{S}} B_{\mathrm{T}} \tag{5}
\end{equation*}
$$

(Where $\mathrm{L}=\mathrm{A}, \mathrm{B}$ or $\mathrm{C} . \mathrm{S}, \mathrm{T}$ and $\mathrm{M}=1,2,3$ or 4 .)
Table 5 Control factors

| Control factor | Level 1 | Level 2 | Level 3 | Level 4 |
| :---: | :---: | :---: | :---: | :---: |
| A | 1 | 5 | 9 | 13 |
| B | 20 | 30 | 40 | 50 |
| C | 3 | 9 | 15 | 21 |

Table 6 Orthogonal array and functions

| L16 | Control factor |  |  | Function |
| :---: | :---: | :---: | :---: | :---: |
|  | A | B | C | $F$ |
| No.1 | 1 | 20 | 3 | 158.9 |
| No.2 | 1 | 30 | 9 | 558.9 |
| No.3 | 1 | 40 | 15 | 1377.3 |
| No.4 | 1 | 50 | 21 | 2772.9 |
| No.5 | 5 | 20 | 9 | 178.1 |
| No.6 | 5 | 30 | 3 | 532.1 |
| No.7 | 5 | 40 | 21 | 1544.1 |
| No.8 | 5 | 50 | 15 | 2582.5 |
| No.9 | 9 | 20 | 15 | 279.7 |
| No.10 | 9 | 30 | 21 | 827.3 |
| No.11 | 9 | 40 | 3 | 1281.3 |
| No.12 | 9 | 50 | 9 | 2513.3 |
| No.13 | 13 | 20 | 21 | 521.7 |
| No.14 | 13 | 30 | 15 | 712.1 |
| No.15 | 13 | 40 | 9 | 1359.7 |
| No.16 | 13 | 50 | 3 | 2545.7 |

This cubic equation consists of the control factors $A, B$ and $C$. Here, the control factors $A$ and $B$ influence to the control factor interaction final function. The control factors, levels and each level value are shown in Table 5. As 4 sets of the combination between the control factors and the final function are required for solution of the cubic equation, the control factors used 4 levels. The final result functions $F$ were calculated using the equation (5) and (Orthogonal $\mathrm{L}_{16}$ ) Table 5 (See Table 6).

Then the calculated final functions $F_{A 0}, F_{B 0}$ and $F_{C 0}$ can be calculated by equations (2), (3) and Table 6, and the final functions $F_{A 0^{\prime}}, F_{B 0^{\prime}}$ and $F_{C 0}$, can be calculated by equation (4), Tables 2 and Table 3 with a new set of trials. Then, the calculated final functions $F_{A 0}, F_{B 0}$ and $F_{C 0}$ were compared with the final functions $F_{A 0}, F_{B 0}$, and $F_{C 0}$ as shown in Table 7. It was concluded from Table 7 that control factors $A$ and $B$ had a control factor interaction and that the algorithm can determine it, as well as the involved control factors ( $A$ and $B$ ).

Then two additional mathematical models were used for evaluation. In the two models, the control factors $A$ and $C$, or the control factors $B$ and $C$ influence to the control factor interaction final function, and the coefficients of the involved control factor interactions were changed. Those results were shown in Tables 8 and 9. It was concluded from those tables that

Table 7 Evaluation results using eq. (5) (Control factor interactionbetween the control factors A and B)

| The used model: <br> $F_{\mathrm{L}}=0.05 \mathrm{As}^{3}+0.02 B_{\mathrm{T}}{ }^{3}+0.03 C_{\mathrm{M}}{ }^{3}-0.1 A_{\mathrm{S}} B_{\mathrm{T}}$ <br> (Where $\mathrm{L}=\mathrm{A}, \mathrm{B}$ or $\mathrm{C} \mathrm{S}, \mathrm{T}$ and $\mathrm{M}=1,2,3$ or 4 ) |  |  |  |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & \hline F_{\mathrm{AO}}= \\ & 1220.4 \end{aligned}$ | $\begin{aligned} & \hline F_{\mathrm{AO}}{ }^{\prime}= \\ & 1220.4 \end{aligned}$ | Equal | Assessment: Control factors A and B have control factor interactions (see Table 4) |
| $\begin{aligned} & F_{\mathrm{B} 0}= \\ & 138.6 \end{aligned}$ | $\begin{aligned} & F_{\mathrm{B0}}{ }^{\prime}= \\ & 138.6 \end{aligned}$ | Equal |  |
| $\begin{aligned} & F_{\mathrm{CO}_{0}}= \\ & 1131.7 \end{aligned}$ | $\begin{aligned} & F_{\mathrm{CO}}{ }^{\prime}= \\ & 1128.7 \end{aligned}$ | Not Equal |  |

Table 8 Evaluation results using new model (Control factor interaction between the control factors A and C)

| The us $F_{\mathrm{L}}=0$. (Where | $\begin{aligned} & \text { model: } \\ & 5 \mathrm{As}^{3}+0.0 \\ & =\mathrm{A}, \mathrm{~B} \text { or } \end{aligned}$ |  | $\begin{aligned} & C_{\mathrm{M}}{ }^{3}-0.2 A_{\mathrm{S}} C_{\mathrm{M}} \\ & \mathrm{I}=1,2,3 \text { or } 4) \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & F_{\mathrm{A} 0}= \\ & 1220.4 \\ & \hline \end{aligned}$ | $\begin{aligned} & F_{\mathrm{AO}},= \\ & 1220.4 \end{aligned}$ | Equal | Assessment: <br> Control factors A and $C$ have control factor interactions (see Table 4) |
| $\begin{aligned} & F_{\mathrm{BO}}= \\ & 163.8 \end{aligned}$ | $\begin{aligned} & F_{\mathrm{B0}}{ }^{\prime}= \\ & 115.8 \\ & \hline \end{aligned}$ | Not Equal |  |
| $\begin{aligned} & \hline F_{\mathrm{C} 0}= \\ & 1158.2 \end{aligned}$ | $\begin{aligned} & \hline F_{\mathrm{Co}}{ }^{\prime}= \\ & 1158.2 \end{aligned}$ | Equal |  |

Table 9 Evaluation results using new model (Control factor interaction between the control factors B and C)

| The use $F_{\mathrm{L}}=0$. (Where | $\begin{aligned} & \text { model: } \\ & {A s^{3}+0 . C}^{=A, B \text { or }} \end{aligned}$ | $B_{\mathrm{T}}{ }^{3}+{ }_{\mathrm{ST}}$ | $\begin{aligned} & \mathrm{C}_{\mathrm{M}}{ }^{3}-0.3 B_{\mathrm{T}} C_{\mathrm{M}} \\ & =1,2,3 \text { or } 4) \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & \hline F_{\mathrm{AO}}= \\ & 1076.7 \end{aligned}$ | $\begin{aligned} & F_{\mathrm{AO}}{ }^{\prime}= \\ & 1071.9 \end{aligned}$ | Not Equal | Assessment: Control factors B and $C$ have control factor interactions (see Table 4) |
| $\begin{aligned} & F_{\mathrm{B0} 0}= \\ & 138.6 \end{aligned}$ | $\begin{aligned} & F_{\mathrm{BO}},= \\ & 138.6 \end{aligned}$ | Equal |  |
| $\begin{aligned} & F_{\mathrm{F}_{\mathrm{C}}}= \\ & 1158.2 \end{aligned}$ | $\begin{aligned} & F_{\mathrm{CO}}{ }^{\prime}= \\ & 1158.2 \end{aligned}$ | Equal |  |

the control factors $A$ and $C$, or the control factors $B$ and $C$ had control factor interactions.
In addition, the control factor interactions were confirmed in the several cases as follows; when the mathematical models were quadratic equations, and when the mathematical models were cubic equations with a quadratic and a linear terms, the coefficients of the control factor were changed and level numbers of the control factor were increased for evaluation.

## B. Evaluation Through Mathematical Models With One Control Factor Interaction Between Three Control Factors

As previously developed, a complex model of cubic nature was used for the evaluation of the algorithm. The mathematical model using equation (6) was then used.

$$
\begin{align*}
F_{\mathrm{L}}= & 0.395 A_{\mathrm{S}}^{3}+0.941 B_{\mathrm{T}}^{3}+0.013 C_{\mathrm{M}}^{3}+0.509 \mathrm{D}_{\mathrm{N}}^{3} \\
& +0.981 E_{\mathrm{O}}^{3}-B_{\mathrm{T}} C_{\mathrm{M}} \mathrm{D}_{\mathrm{N}} \tag{6}
\end{align*}
$$

(Where L=A, B, C, D or E. S, T, M, N and $\mathrm{O}=1,2,3$ or 4 .)
Table 10 Control factors

| Control <br> factors | Level <br> 1 | Level <br> 2 |  | Level <br> 3 | Level <br> 4 |
| :---: | :---: | :---: | :--- | :---: | :---: |
| A | 2.36 | 4.72 |  | 7.08 | 9.44 |
| B | 1.18 | 2.36 |  | 3.54 | 4.72 |
| C | 4.08 | 8.16 |  | 12.24 | 16.32 |
| D | 5.49 | 10.98 |  | 16.47 | 21.96 |
| E | 6.03 | 12.06 |  | 18.09 | 24.12 |

Table 11 Orthogonal array and functions

| L16 | Control factors |  |  |  |  | Functions |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | B | C | D | E | F |
| No.1 | 2.36 | 1.18 | 4.08 | 5.49 | 6.03 | 280.50 |
| No.2 | 2.36 | 2.36 | 8.16 | 10.98 | 12.06 | 2207.69 |
| No.3 | 2.36 | 3.54 | 12.24 | 16.47 | 18.09 | 7438.62 |
| No.4 | 2.36 | 4.72 | 16.32 | 21.96 | 24.12 | 17625.17 |
| No.5 | 4.72 | 1.18 | 8.16 | 16.47 | 24.12 | 15931.38 |
| No.6 | 4.72 | 2.36 | 4.08 | 21.96 | 18.09 | 11041.10 |
| No.7 | 4.72 | 3.54 | 16.32 | 5.49 | 12.06 | 1627.56 |
| No.8 | 4.72 | 4.72 | 12.24 | 10.98 | 6.03 | 418.86 |
| No.9 | 7.08 | 1.18 | 12.24 | 21.96 | 12.06 | 6959.44 |
| No.10 | 7.08 | 2.36 | 16.32 | 16.47 | 6.03 | 2063.85 |
| No.11 | 7.08 | 3.54 | 4.08 | 10.98 | 24.12 | 14463.80 |
| No.12 | 7.08 | 4.72 | 8.16 | 5.49 | 18.09 | 5926.41 |
| No.13 | 9.44 | 1.18 | 16.32 | 10.98 | 18.09 | 6660.12 |
| No.14 | 9.44 | 2.36 | 12.24 | 5.49 | 24.12 | 14059.91 |
| No.15 | 9.44 | 3.54 | 8.16 | 21.96 | 6.03 | 5352.16 |
| No.16 | 9.44 | 4.72 | 4.08 | 16.47 | 12.06 | 4109.71 |

Table 12 Evaluation results using eq.(6) (Control factor interaction between the control factors $\mathrm{B}, \mathrm{C}$ and D )

| The use $F_{\mathrm{L}}=0.3$ <br> (Where | $\begin{aligned} & \text { el : } \\ & +0.941 B \\ & E_{0}{ }^{3}-B \\ & \text { B, C, D o } \end{aligned}$ | $\begin{aligned} & .013 C_{\mathrm{N}} \\ & \mathrm{v} \\ & \text {, T, M, } \end{aligned}$ | $\begin{aligned} & .509 D_{\mathrm{N}}{ }^{3} \\ & \mathrm{dO}=1,2,3 \text { or } 4 .) \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & F_{\mathrm{A} 0}= \\ & 5891.6 \end{aligned}$ | $\begin{aligned} & F_{\mathrm{AO}}{ }^{\prime}= \\ & 6882.8 \end{aligned}$ | Not equal | Assessment: Control factors $B, C$ and $D$ have control factor interactions. |
| $\begin{aligned} & F_{\mathrm{BB}}= \\ & 7634.7 \end{aligned}$ | $\begin{aligned} & F_{\mathrm{Bo},}= \\ & 7634.7 \end{aligned}$ | Equal |  |
| $\begin{aligned} & \hline F_{\mathrm{CO}}= \\ & 7651.3 \end{aligned}$ | $\begin{aligned} & \hline F_{\mathrm{CO}}{ }^{\prime}= \\ & 7651.3 \end{aligned}$ | Equal |  |
| $\begin{aligned} & \hline F_{\mathrm{D} 0}= \\ & 5567.8 \end{aligned}$ | $\begin{aligned} & \hline F_{\mathrm{DD},}= \\ & 5567.8 \end{aligned}$ | Equal |  |
| $\begin{aligned} & \hline F_{\mathrm{EE} 0}= \\ & 1305.0 \end{aligned}$ | $\begin{aligned} & \hline F_{\mathrm{EO}}{ }^{\prime}= \\ & 1813.8 \end{aligned}$ | Not equal |  |

As shown, the mathematical model has one control factor interaction between three control factors. This cubic equation utilized control factors $A, B, C, D$ and $E$. Here, the control factors $B, C$ and $D$ influence to the control factor interaction final function. The control factors, levels and each level value are shown in Table 10. As 4 sets of the combination between the control factors and the final function are required for solution of the cubic equation, the control factors were used 4 levels. The final result functions $F$ were calculated using the equation (6) and (Orthogonal $\mathrm{L}_{16}$ ) Table 10 (See Table 11).

Then the calculated final functions $F_{A 0}, F_{B 0}, F_{C 0}, F_{D 0}$ and $F_{E 0}$ were calculated by equations (2), (3) and Table 11, and the final functions $F_{A 0}$, $F_{B 0}$ ', $F_{C 0}{ }^{\prime}, F_{D 0}$ ' and $F_{E 0}$ ' were calculated by equation (4), Tables 2 and Table 3 with a new set of trials. Then, the calculated final functions $F_{A 0}, F_{B 0}, F_{C 0}, F_{D 0}$ and $F_{E 0}$ were compared with the final functions $F_{A 0}, F_{B 0}, F_{C 0}, F_{D 0}$ ' and $F_{E 0}$, as shown in Table 12. It was concluded from Table 12 that the control factors $B, C$ and $D$ had a control factor interaction and that the algorithm was able to determine the interaction and the involved control factors ( $B, C$ and $D$ ).
However, it was observed that when the control factor interaction $B_{T} C_{\mathrm{M}} \mathrm{D}_{\mathrm{N}}$ in the equation (6) was changed to $A_{\mathrm{S}} B_{\mathrm{T}} \mathrm{C}_{\mathrm{M}}$ or $A_{\mathrm{S}} B_{\mathrm{T}} \mathrm{E}_{\mathrm{O}}$, in equations (7) or (8), the final results became indistinguishable as shown in Tables 13 and 14.

$$
\begin{align*}
F_{\mathrm{L}}= & 0.395 A_{\mathrm{S}}{ }^{3}+0.941 B_{\mathrm{T}}{ }^{3}+0.013 C_{\mathrm{M}}{ }^{3}+0.509 \mathrm{D}_{\mathrm{N}}{ }^{3}  \tag{7}\\
& +0.981 \mathrm{E}_{\mathrm{O}}{ }^{3}-A_{\mathrm{S}} B_{\mathrm{T}} \mathrm{C}_{\mathrm{M}} \\
F_{\mathrm{L}}= & 0.395 A_{\mathrm{s}}{ }^{3}+0.941 B_{\mathrm{T}}{ }^{3}+0.013 C_{\mathrm{M}}{ }^{3}+0.509 \mathrm{D}_{\mathrm{N}}{ }^{3}  \tag{8}\\
& +0.981 \mathrm{E}_{\mathrm{O}}{ }^{3}-A_{\mathrm{S}} B_{\mathrm{T}} \mathrm{E}_{\mathrm{O}}
\end{align*}
$$

(Where L= A, B, C, D or E. S, T, M, N and O = 1, 2, 3 or 4.)
Table 13 Evaluation results using other model (Control factor interaction between the control factors $\mathrm{A}, \mathrm{B}$ and C )

| The used model :$F_{\mathrm{L}}=0.395 A_{\mathrm{s}}{ }^{3}+0.941 B_{\mathrm{T}}{ }^{3}+0.013 C_{\mathrm{M}}{ }^{3}+0.509 D_{\mathrm{N}}{ }^{3}$$+0.981 E_{\mathrm{O}}{ }^{3}-A_{\mathrm{S}} B_{\mathrm{T}} C_{\mathrm{M}}$,(Where $L=\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ or $\mathrm{E} . S, T, M, N$ and $O=1,2,3$ or 4 .) |  |  |  |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & F_{\mathrm{AD}}= \\ & 7634.5 \end{aligned}$ | $\begin{aligned} & F_{\mathrm{AO},}= \\ & 7543.6 \end{aligned}$ | Not equal | Assessment: Indistinguishable |
| $\begin{aligned} & \hline F_{\text {B0 }}= \\ & 7725.6 \end{aligned}$ | $\begin{aligned} & \hline F_{\text {B0 }}= \\ & 7634.7 \end{aligned}$ | Not equal |  |
| $\begin{aligned} & \hline F_{\mathrm{F} 0}= \\ & 7742.2 \end{aligned}$ | $\begin{aligned} & F_{\mathrm{CO}},= \\ & 7651.3 \end{aligned}$ | Not equal |  |
| $\begin{aligned} & \hline F_{\mathrm{DD}}= \\ & 5190.0 \end{aligned}$ | $\begin{aligned} & \hline F_{\mathrm{D} 0}= \\ & 5360.4 \end{aligned}$ | Not equal |  |
| $\begin{aligned} & \hline F_{\mathrm{E0} 0}= \\ & 1918.3 \end{aligned}$ | $\begin{aligned} & F_{\mathrm{EO}}{ }^{\prime}= \\ & 2088.8 \end{aligned}$ | Not equal |  |

Table 14 Evaluation results using other model (Control factor interaction between the control factors $\mathrm{A}, \mathrm{B}$ and E )

| The used $F_{\mathrm{L}}=0$ <br> (Where |  | $\begin{aligned} & { }^{3}{ }^{3}+0 \\ & -A_{s} B \\ & S, T, 1 \end{aligned}$ | $\mathrm{n}^{3}+0.509{D_{\mathrm{N}}}^{3}$ <br> nd $O=1,2,3$ or 4 .) |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & \hline F_{\mathrm{AA}}= \\ & 7006.2 \end{aligned}$ | $\begin{aligned} & F_{\mathrm{A} 0}= \\ & 7543.6 \end{aligned}$ | Not equal | Assessment: Indistinguishable |
| $\begin{aligned} & \hline F_{\mathrm{BB}}= \\ & 7769.1 \end{aligned}$ | $\begin{aligned} & \hline F_{\mathrm{BO}}{ }^{\prime}= \\ & 7634.7 \end{aligned}$ | Not equal |  |
| $\begin{aligned} & \hline F_{\mathrm{CO}}= \\ & 7072.0 \end{aligned}$ | $\begin{aligned} & \hline F_{\mathrm{co},}= \\ & 7311.3 \end{aligned}$ | Not equal |  |
| $\begin{aligned} & \hline F_{\mathrm{D}}= \\ & 4988.4 \end{aligned}$ | $\begin{aligned} & \hline F_{\mathrm{D},}= \\ & 5227.7 \end{aligned}$ | Not equal |  |
| $\begin{aligned} & F_{\mathrm{ED} 0}= \\ & 2430.5 \end{aligned}$ | $\begin{aligned} & F_{\text {EO }}= \\ & 2296.1 \end{aligned}$ | Not equal |  |



Fig. 2 Difference between the function $\mathrm{F}_{0}$ at level value $=0$ using the approximate equation and the function $\mathrm{F}_{0}$ ' by new trial using the value of level $1=0$.

As a result, it was decided to use the mathematical models shown in equations (6), (7) and (8) for the evaluation of the previous algorithm. The results of said evaluation are shown in Fig. 2. It was concluded that the interaction with control factors $B, C$ and $D$ in equation (6) was determined by the previous algorithm, but the interaction with of control factors $A, B$ and $C$ in equation (7) and the interaction of control factors $A, B$ and $E$ in equation (8) were not identifiable. This, since in equations (7) and (8), set at level value $=$ zero " 0 " , the final functions $F_{A 0}$, $F_{B 0}$ and $F_{C 0}$ were calculated by equation (3) a comparison with final functions $F_{A 0}$, $F_{B 0}$, and $F_{C 0}$, calculated by equation (4) was not possible.

## IV. Improvement of The Algorithm and Its Evaluatio

As the presented limitation of the algorithm needs to be addressed the following must be observed. The control factor A in the presented orthogonal array (Table 11) could not calculate with precision the final function at the level value $=0$, and the difference between $F_{0}$ and $F_{0}{ }^{\prime}$ on the control factors $\mathrm{A}, \mathrm{B}$ and C in Fig. 2 (b) and on the control factors A, B and E in Fig. 2 (c) were not 0 . This can be asserted because, in this case, the angle of the approximation curve was not 0 . Therefore, it was concluded that both the control factor interaction with the control factors $A, B$ and $C$ in equation (7) and the control factor interaction with the control factors $A, B$ and $E$ in equation (8) were not possible to detect with the previous algorithm. Here, in order to corroborate the previous results two orthogonal arrays were devised as shown in Table 16. The first orthogonal array, located in the left, was set to have control factor level values that were selected in a random fashion and zero " 0 " values were located in a way similar as in the previous algorithm. The second orthogonal array, located in the right, was systematically set to have control factor level values in ascending order (1 to 4). Then, the mathematical models using equations (6), (7) and (8) were used for the evaluation of the improved algorithm. Here, the values and functions $F$ shown in Table 11 were used. Subsequently, the final functions $F_{0}\left(=F_{A 0}, F_{B 0}, F_{C 0}, F_{D 0}\right.$ and $F_{E 0}$ ) can be calculated by equations (2) and (3).

In order to compare the new orthogonal arrays as the difference between $F_{0}$ and $F_{0}{ }^{\prime}, F_{0}$ was described by Table 15 ,
$F_{0-1}{ }^{\prime}\left(=F_{A 0-1}{ }^{\prime}, F_{B 0-1}, F_{C 0-1}{ }^{\prime}, F_{D 0-1}{ }^{\prime}\right.$ and $\left.F_{E 0-1}{ }^{\prime}\right)$ described the left orthogonal array in Table 16 and $F_{0-2}{ }^{\prime}\left(=F_{A 0-2}, F_{B 0-2^{\prime}}, F_{C 0-2}\right.$, $F_{D 0-2}$ ' and $F_{E 0-2}$ ') described the right orthogonal array in Table 16. The final functions $F_{0-1}$ ' and $F_{0-2}$ ' were can be calculated by equation (4) with a new set of trials. Upon comparison it was observed that independently, the differences between $F_{0}$ and $F_{0-1}$, the differences between $F_{0}$ and $F_{0-2}$, shown in Fig. 3, still offered an undistinguishable control factor interaction assessment. However, it was also observed that the results of said differences were the same. Here, it was considered that a subtraction between both orthogonal array differences could lead to a final assessment of control factor interaction in complex control factor arrangements as represented by the

Table 15 Basic orthogonal arrays with L16

| No. | Factor |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | B | C | D | E |
| No.1 | 1 | 1 | 1 | 1 | 1 |
| No.2 | 1 | 2 | 2 | 2 | 2 |
| No.3 | 1 | 3 | 3 | 3 | 3 |
| No.4 | 1 | 4 | 4 | 4 | 4 |
| No.5 | 2 | 1 | 2 | 3 | 4 |
| No.6 | 2 | 2 | 1 | 4 | 3 |
| No.7 | 2 | 3 | 4 | 1 | 2 |
| No.8 | 2 | 4 | 3 | 2 | 1 |
| No.9 | 3 | 1 | 3 | 4 | 2 |
| No.10 | 3 | 2 | 4 | 3 | 1 |
| No.11 | 3 | 3 | 1 | 2 | 4 |
| No.12 | 3 | 4 | 2 | 1 | 3 |
| No.13 | 4 | 1 | 4 | 2 | 3 |
| No.14 | 4 | 2 | 3 | 1 | 4 |
| No.15 | 4 | 3 | 2 | 4 | 1 |
| No.16 | 4 | 4 | 1 | 3 | 2 |

Table 16 Confirmation run orthogonal arrays

| Random level value OA ( $F_{0-1}$ ') |  |  |  |  |  | Ascending level value OA$\left(F_{0-2}{ }^{\prime}\right)$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. | Factor |  |  |  |  | No. | Factor |  |  |  |  |
|  | A | B | C | D | E |  | A | B | C | D | E |
| No. 1 | 0 | 1 | 1 | 1 | 1 | No. 1 | 0 | 1 | 1 | 1 | 1 |
| No. 2 | 0 | 2 | 2 | 2 | 2 | No. 2 | 0 | 2 | 2 | 2 | 2 |
| No. 3 | 0 | 3 | 3 | 3 | 3 | No. 3 | 0 | 3 | 3 | 3 | 3 |
| No. 4 | 0 | 4 | 4 | 4 | 4 | No. 4 | 0 | 4 | 4 | 4 | 4 |
| No. 5 | 1 | 0 | 1 | 1 | 1 | No. 5 | 1 | 0 | 1 | 1 | 1 |
| No. 6 | 2 | 0 | 2 | 3 | 4 | No. 6 | 2 | 0 | 2 | 2 | 2 |
| No. 7 | 3 | 0 | 3 | 4 | 2 | No. 7 | 3 | 0 | 3 | 3 | 3 |
| No. 8 | 4 | 0 | 4 | 2 | 3 | No. 8 | 4 | 0 | 4 | 4 | 4 |
| No. 9 | 1 | 1 | 0 | 1 | 1 | No. 9 | 1 | 1 | 0 | 1 | 1 |
| No. 10 | 2 | 2 | 0 | 4 | 3 | No. 10 | 2 | 2 | 0 | 2 | 2 |
| No. 11 | 3 | 3 | 0 | 2 | 4 | No. 11 | 3 | 3 | 0 | 3 | 3 |
| No. 12 | 4 | 4 | 0 | 3 | 2 | No. 12 | 4 | 4 | 0 | 4 | 4 |
| No. 13 | 1 | 1 | 1 | 0 | 1 | No. 13 | 1 | 1 | 1 | 0 | 1 |
| No. 14 | 2 | 3 | 4 | 0 | 2 | No. 14 | 2 | 2 | 2 | 0 | 2 |
| No. 15 | 3 | 4 | 2 | 0 | 3 | No. 15 | 3 | 3 | 3 | 0 | 3 |
| No. 16 | 4 | 2 | 3 | 0 | 4 | No. 16 | 4 | 4 | 4 | 0 | 4 |
| No. 17 | 1 | 1 | 1 | 1 | 0 | No. 17 | 1 | 1 | 1 | 1 | 0 |
| No. 18 | 2 | 4 | 3 | 2 | 0 | No. 18 | 2 | 2 | 2 | 2 | 0 |
| No. 19 | 3 | 2 | 4 | 3 | 0 | No. 19 | 3 | 3 | 3 | 3 | 0 |
| No. 20 | 4 | 3 | 2 | 4 | 0 | No. 20 | 4 | 4 | 4 | 4 | 0 |



Fig. 3 Results for the assessment of the control factor interactions using an improved algorithm for equations (6), (7) and (8)
difference between $F_{0-1}$ ' and $F_{0-2}$ ' in Fig. 3. Consequently, it was concluded that control factors $\mathrm{B}, \mathrm{C}$ and D had an interaction in equation (6), control factors $A, B$ and $C$ had an interaction in equation (7), control factors $A, B$ and $E$ had an interaction in equation (8) and that the improved algorithm can determine the interactions and the involved factors $B_{T} C_{M} D_{N}$ in equation (6), $A_{\mathrm{S}} B_{\mathrm{T}} \mathrm{C}_{\mathrm{M}}$ in equation (7) or $A_{\mathrm{S}} B_{\mathrm{T}} \mathrm{E}_{\mathrm{O}}$ in equation (8).

## V. Conclusion

It was concluded that,
(1) An improved tool for the determination of control factor interactions in the DOE and the Taguchi Method was developed.
(2) The tool was able to detect previously indistinguishable complex control factor interactions present in the DOE or the Taguchi Methods.
(3) A new algorithm was able to determine complex control factor interactions in models between control factors and
functions.

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