Supervised Identification and Equalization of a Linear Systems Using Reproducing Kernel Hilbert Space

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***Abstract*—This work concerns the problem of the supervised identification of the parameters using a new mathematic tools based on a Positive-definite kernel on a Hilbert space using a Gaussian kernel. The input sequence is assumed to be independent and identically distributed (i.i.d), zero mean and must be non-Gaussian. The developed method is tested for different channel models. Simulation examples are provided to verify the performance of the developed method. The obtained results chowed the efficiency of the developped method.**

Indexing terms/Keywords: Wirless, networks, FIR channel, Reproducing Positive-definite kernel, Hilbert space, Gaussian kernel, code-division multiple access, MCCDMA, Equalization, identification, wirless communication, Spreading, Reproducing Kernel Hilbert Space.

I. INTRODUCTION

Wirless or cellular networks has been knowen a great evolution, which requires operators to improve their services offered. The vocals conversations were a major challenge for researchers, especially with the massive explosion of internet traffic in fixed networks, which imposes an effective improvement of techniques based on mobile network technology, several services has been introduced, such as 2G system including email, web access and online bank transaction services. Then the 3G architecture was held to solve the defects of the 2G, the first is based on Direct Sequence Code-Division Multiple Access (DSCDMA)[1]. However, the capabilities of 3G system remains insufficient due to the needs expressed in permanence. The 4th Generation that supports high-speed remains the solution adapted to the expectations of users. our work is situeted on the 4th Generation.

The identification of channe parameters has became a research topic mainly addressed by researchers in the signal processing field. In this work, we will apply a new method based on a positive-definite kernel on a hilbert space [2] [3][4] [5] using a gaussian kernel, in order to identify the channel parameters.

II. POSITIVE DEFINITE KERNELS

Kernel methods constitute an imoptant feild of machine learning for nonlinear function estimation [6]. Their popularity can be ascribed to their simplicity, flexibility, and good performance [**?**].

Kernel regression seeks an estimate of f_0 in an RKHS $\mathcal{H}$, which is the space of functions $f : \chi \longrightarrow \mathbb{R}$ defined as:

$$\mathcal{H} := \{f : f(x) = \sum_{i=0}^{m} \alpha_i k(x, x_i), \qquad \alpha \in \mathbb{R} \tag{1}$$

The kernel map $k : \chi\times\chi \to \mathbb{R}$ is any function defining a symmetric and positive semidefinite $N \times N$ matrix with entries $(K)_{x,x'} := k(x, x') \quad \forall x, x' \in \mathbb{R}$.

Symmetry and positivity are required. For any finite sequence $x_1, x_2, ..., x_n$ the matrix

K of elements $K(x_i, x_j)$ (matrix of Gram) is positive definite : $\forall \alpha \in \mathbb{R}^n$:$\alpha^T K \alpha \geq 0$.

In our work, we used the gaussien kernel $k(x, y) = exp[-\frac{\|x-y\|^2}{2\sigma^2}]$ in the estimation of the functions on the parameters, but there are many kind of kernelssuch as:

- Linear kernel
- Polynomial kernel
- Laplacian kernel .

A positive definite kernel K can build a space of functions, The function space with a scalar product is a Hilbert space H reproducing kernel called RKHS [7](Reproducing Kernel Hilbert Space).

The RKHS simply defined by its components (functions) and by a scalar product (between functions).

1) $\forall x \in \chi, k_x = k(x, .) \in \mathbb{H}_k$;
2) $\langle k_x, k_y \rangle = k(x, y)$;

III. Positif definite kernel and associated algorithms

We restrict ourselves to the case of a linear channel of communication and time invariant, and we supposed a finite impulse response. It can be described as a convolution filter $h(k)$ of the transmitted signal $x(k)$ [8][9].

We assume that we observe the output signal $y(k)$ as a discrete-time signal which is described by the following equation:

$$y(k) = h_m(i) * x(k) = \sum_{i=0}^{q} h(i)x(k-i) \quad (2)$$

and

$$s(k) = y(k) + n(k) \quad (3)$$

With:
-$x(n)$ is the input sequence
-h_m is the channel parameters
-$y(n)$ represents the system output in noiseless case and $s(n)$ is the observed system output corrupted by additive Gaussian noise $n(n)$.
-$n(k)$ is an additive white Gaussian noise.

The following conditions are assumed to be satisfied:
- The model order q is supposed to be known,
- The input sequence $x(n)$ is independent and identically distributed (i.i.d) zero mean, the variance is $\sigma_x^2 \equiv 1$, and non Gaussian,
- The system is causal and $hd(0) = 1$, - We assume taht the meseared noise $n(k)$ is zero mean, gaussian, i.i.d, with unknown variance σ_n^2.

The correlation and the spectrum of the output signal $y(k)$ are given respectively by:

$$r_y(k) = r_{h*x}(k) + r_x(k) = \sigma^2 \sum_{i=0}^{\infty} h(i)h(k+i) + \sigma^2\delta(k) \quad (4)$$

And

$$S_y(w) = \sigma_x^2 |H(w)|^2 + \sigma_n^2 \quad (5)$$

The problem addressed is to identify the channel parameters h_m using the positive definite kernels.

However, if we set:

$$F = h_m * x \quad (6)$$

The equation 2 becomes :

$$y(k) = F(k) + n(k) \quad (7)$$

According to the equation 7, we will focus our stady on the estimation parameters of the function F. If we apply the Laplace transforme on the equation 6, it becomes:

$$L(F) = L(h_m).L(x) \quad (8)$$

Historically, the approaches of kernels was mentioned since sixty years. Our idea is to use the learning examples $\langle (x_i, y_i) \rangle_{i=1,m}$ to make a parameter identification of the input space χ by weighting the outputs associated with inputs $(x_i)_{i=1,m}$.

The weight $K(x, x_i)$ which is associated with each output y_i depends on the relative position of x_i in the input space χ and the point x considered. The function $K(x, x')$ defining these weights is called, in these approaches [10], kernel function that is expressed by:

$$K(x, x') = g(\frac{d(x, x')}{\sigma}) \quad (9)$$

Where $d(x, x')$ is a scale factor defined on a set χ, and $g(.)$ is a decreasing function.

A frequent choice for this function is $g(z) = \exp^{-\frac{z^2}{2}}$. The use of this type of kernel function leads to estimate $\hat{y}$ as weighted average of $(y_i)_{i=1,m}$ with a stronger weight to examples where the distance $d(x, x_i)$ is small, the notion of being determined by the small value of σ.

Use the inductive principle of minimization of regularized empirical risk, taking the standard deviation as a function of loss in our problem of telecommunication, the estimated parameters are obtained by solving the functional minimization formulated as cited in ([6], [2])

$$R_F(F) = \frac{1}{m}\sum_{i=1}^{m}(y_i - F(x_i))^2 + \lambda \parallel F\|^2 \quad (10)$$

A. Theorem 1

(Representer theorem). The solution to the functional minimization in (10) can be expressed as

$$\begin{aligned} &F(x_i) = \sum_{i=0}^{m} \alpha_i k(x_i, x) \\ &for \quad same \quad \alpha_i \in \mathbb{R}, i = 1...m \end{aligned} \quad (11)$$

This result is important because it shows that the regularized problems of the form (10) naturally have solutions of the form (theorem 1):

$$F(x_i) = \sum_{i=0}^{m} \alpha_i k(x_i, x) \quad (12)$$

In our case, we try to solve this nonlinear regression problem in χ, for that, we need to define the fuction ϕ in a new hilbert space F, in order to make the problem a linear regression problem in the new space.

our approach is to estimate the parameters of the vector F, which amounts to estimate the parameters α.

The norm of the function $\phi = \sum_i \alpha_i k_{x_i}$ is given by the quadratic form defined by the similarity matrix K of size $n \times n$.

$$\|\phi\|^2_{H_k} = \alpha^T K \alpha \quad (13)$$

The optimization problem can be rewritten by replacing the search for F by the search for the coefficients α_i.

Lets pose Y the vector of components y_i , the probleme (10) becomes :

$$\begin{aligned} &R_\alpha(\alpha) = argmin_{\alpha\in\mathbb{R}^n}[\tfrac{1}{n}(K\alpha - Y)^T(K\alpha - Y) \\ &+\lambda\alpha^T\alpha] \end{aligned} \quad (14)$$

The function R is convex and differentiable on α. The coefficients α_i are obtained by solving the linear system :

$$(K + n\lambda I)\alpha = Y \quad (15)$$

Soon as $\lambda > 0$ the matrix $(K + n\lambda I)$ is invertible.

When searching for parameters α_i, several algorithms are possible, in our work we adopt the development of Cholisky.

The solution of the system (15) is given by the least squares method as the following equation:

$$\alpha = (K + n\lambda I)^{-1} \quad (16)$$

B. Reproducing Kernel Hilbert Space

Set $\mathbb{F}$ an Hilbert space [2] of real functions defined on an indexed set χ :

$$\begin{aligned} &F = \{\sum_{i=1}^{m} \alpha_i k(x_i, .) : m \in \mathbb{N}, x_i \in \chi, \\ &\alpha_i \in \mathbb{R}, i = 1...m\} \end{aligned} \quad (17)$$

$\mathbb{F}$ is called Hilbert space reproducing kernel [11] with a scalar product denoted $\langle ., .\rangle_F$ (with the norme $\|f\|_F = \sqrt{\langle f, f\rangle_F}$) if there exists a function $k : \chi \times \chi \to \mathbb{R}$ having the following properties:

1) for any element $x \in \chi$,k(x,.) belongs to F.
2) the function k is a reproducing kernel function, i.e, such that for any functions $f \in F$, we have :

$$\langle f, k(x, .)\rangle_{\mathbb{F}} = \sum^{i=1} m\alpha_i k(x_i, x) = f(x) \quad (18)$$

The fact that the kernel function is recurring means that, any function $f \ inF$ is equal to a scalar product, which is also a finite linear combination of basis functions.

The scalar product on F is defined as follows. Let have two functions $f, g \in F$ defined by :

$$\begin{aligned} f(x) &= \sum_{i=1}^{m} \alpha_i k(x_i, x) \\ g(x) &= \sum_{i=1}^{n} \beta_j k(x_j, x) \end{aligned} \quad (19)$$

Then :

$$\begin{aligned} &\langle f, g\rangle = \sum_{i=1}^{m}\sum_{i=1}^{n} \alpha_i\beta_j k(x_i, x_j) = \sum_{i=1}^{m} \alpha_i f(x_i) \\ &= \sum_{i=1}^{n} \beta_j g(x_j) \end{aligned} \quad (20)$$

It is interesting to note that while the basis of Mercer depends on the extent μ defined on χ, this is not the case of the reproducing base kernels which depends only on the function kernel.

Moreover, the Hilbert space of functions L_2 (with scalar product$\langle f, g\rangle_{L_2} = \int f(x)g(x)dx$) contains many irregular functions. In L_2, tha function of Dirac δ is the representation function, i.e. $f(x) = \int f(x')\delta(x-x')dx'$ The kernel functions play a role analogous to the function δ in a Reproducing Kernel Hilbert spaces , which are thus more regular, their degree of regularity depending on the regularity of the kernel function associated.

C. Mercer Theorem

If $k(.,.)$ is a symmetric and continuous function of an integral operator.

$$g(y) = Af(y) = \int_a^b k(x,y)f(y)dy + h(x) \tag{21}$$

Verifying :

$$\int_{\chi \times \chi} k(x,x')f(x)f(x')dxdx' \geq 0 \tag{22}$$

For any function $f \in L_2(\chi)$ (square integrable) (χ being a compact subspace $\mathbb{R}$), then the function $k(.,.)$ can be expanded into a series of uniformly convergent depending positive eigenvalues λ_i and eigenfunctions ψ_i :

$$k(x,x') = \sum_{j=1}^{N} \lambda_j \psi_j(x) \psi_j(x') \tag{23}$$

Where N is the number of positive eigenvalues (possibly infinite number).

IV. Philosophy of kernel methods

A. linear methods

This approach allows to use linear methods developed untel the sixty years to discover nonlinear relationships in the data.

The figure 1 summarizes the steps of the method generic. It is crucial to realize that, in this approach, the essential information about the data expressed in the kernel matrix K.

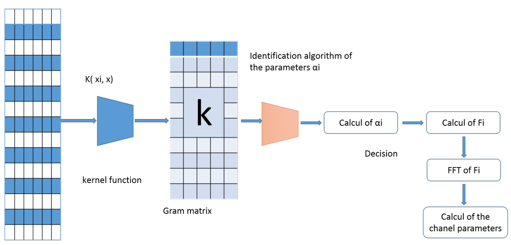

Figure 1: Chain generic treatment of kernel methods

B. Gram Matrix

The Gram matrix (or kernel matrix) contains all the information used by the kernel methods on data input. Elle est symétrique : $G_{ij} = G_{ji}$. Exclusive use of the information contained in this matrix about the training data, has the consequence that a part of the information on this data is lost. For exemple, This matrix is rotation invariant points in the input space χ.

C. Definition of a Gaussian kernel

1) Search of the parameters α: Lets have χ a non-empty set, and $\mathbb{H}$ a Hilbert space.

We define a kernel k from : $\chi \times \chi \to \mathbb{R}$ by the following application :

$$k(x,y) = \exp(-\frac{\|x-y\|^2}{\sigma}) \tag{24}$$

par The vector x is the input signal,the function F follows the following form :

$$F(x) = \sum_{i=0}^{n} \alpha_i k(x_i, x) \tag{25}$$

From the equation 25, our goal amounts to finding the parameters α by minimizing the equation 10, we can represent the coefficients of the function F of a simplified manner:

$F(0) = \alpha_0 k(0,0) + \alpha_1 k(1,0) + ... + \alpha_i k(i,0) + ... + \alpha_n k(n,0)$

$F(1) = \alpha_0 k(0,1) + \alpha_1 k(1,1) + ... + \alpha_i k(i,1) + ... + \alpha_n k(n,1)$

$\vdots$

$F(i) = \alpha_0 k(0,i) + \alpha_1 k(1,i) + ... + \alpha_i k(i,i) + ... + \alpha_n k(n,i)$

$\vdots$

$F(n) = \alpha_0 k(0,n) + \alpha_1 k(1,n) + ... + \alpha_i k(i,n) + ... + \alpha_n k(n,n)$

This system can be represented in the following matrix form :

$$\begin{pmatrix} F(0) \\ \vdots \\ h(n) \end{pmatrix} = \begin{pmatrix} k(0,0) & k(1,0) & \cdots & k(i,0) & \cdots & k(n,0) \\ k(0,1) & k(1,1) & \cdots & k(i,1) & \cdots & k(n,1) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ k(0,i) & \cdots & \cdots & k(i,i) & \cdots & k(i,n) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ k(0,n) & k(1,n) & \cdots & k(i,n) & \cdots & k(n,n) \end{pmatrix} \times \begin{pmatrix} \alpha_0 \\ \vdots \\ \alpha_n \end{pmatrix} \tag{26}$$

D. Example of a channel of 2nd order

To validate this method of reproducing kernel Hilbert space, we will worked on the first time on a simple exemple, we will start with a chanel of the 2nd order, this last is defined by the folowing equations :

$$\left\{ \begin{array}{c} y(k) = x(k) - 0.85x(k-1) + 1.0x(k-2) \\ zros : 0.415 \pm j0.911, \\ s(k) = y(k) + w(k). \end{array} \right\} \tag{27}$$

The channel identification problem is to estimate hp based only on the received signal $r(k)$ and with knowledge of the energy of the transmitted data x(k). The output of the channel is characterized by its impulse response h(n), which we identify its parameters.

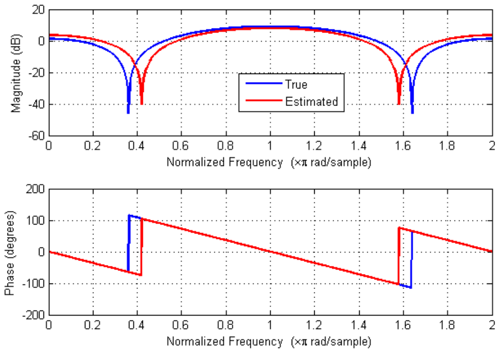

Figure 2: Estimation of the impulse responses in amplitude and phase with a Gaussian kernel

From the figure (2),we note that the estimated parameters follow the same shape as the true channel parameters.

V. MC-CDMA (BROADBAND) CHANNEL

There are many obstacles Between a mobile station (MS) and a base station, neighboring buildings causes a great loss of information during transmission between a fixed base and mobile station, and especially in vicinity of the MS. For this, the channel can be viewed as a time varying linear filter of impulse response $h(\tau)$ observed at time t, which can be expressed as Safi et al [8] [9] [12]

$$h(\tau) = \sum_{i=0}^{L-1} \psi_i \delta(\tau - \tau_i) \quad (28)$$

$\delta(n)$ is Dirac function, ψ_i is the magnitude of the target i, $L = 18$ the number of target and τ_i is the time delay (from the origin) of target i.

A. Mobile Channel Model (BRAN A)

For the development of the standard HiperLAN2, the ETSI BRAN Group worked out channel models in the 5 GHz band. There are many scenarios. In this paragraph we consider the ETSI BRAN A model representing the propagation in office.

Table I: Delays and magnitudes of the 18 trips BRAN A radio channel

Delays τ_i (ns)	magnitudes h_i (dB)	Delays τ_i (ns)	magnitudes h_i (dB)
0	0	90	-7.8
10	-0.9	110	-4.7
20	-1.7	140	-7.3
30	-2.6	170	-9.9
40	-3.5	200	-12.5
50	-4.3	240	-13.7
60	-5.2	290	-18
70	-6.1	340	-22.4
80	-6.9	390	-26.7

B. Mobile Channel Model (BRAN E)

As has treated the case of a channel in the interior of an office, it is imperative to handle the case in outdoor environment.

Table II: Delays and magnitudes of the 18 trips BRAN E radio channel

Delays τ_i (ns)	magnitudes h_i (dB)	Delays τ_i (ns)	magnitudes h_i (dB)
0	-4.9	320	0.0
10	-5.1	560	-2.8
20	-5.2	710	-5.4
40	-0.8	400	-11.7
70	-1.3	880	-7.3
100	-1.9	1070	-10.6
140	-0.3	1280	-13.4
190	-1.2	1510	-17.4
240	-2.1	1760	-20.9

C. MC-CDMA SYSTEM

In MC-CDMA systems, the codes are transmitted in the frequency domain on different subcarriers, toutfois, the subcarrier are sensitive to problems of synchronosations such as timing errors, the frequency offset of the carrier, also the emplitude and phase additive noise. In this section we describe an equalization based on a new concept using defined positive Kernal. It is one of the technologies considered candidates for wireless communication systems of the fourth generation [13].

The MC-CDMA signal is derived from the concatenation of operations spread spectrum by direct sequence modulation and multi-carrier [14]. The MC-CDMA modulator spreads the dataa_i of each user i in the frequency domain. The spreading is done by multiplying data a_i by each element of c_i code associated sprawl. The MC-CDMA modulator is in the case where the spreading code has a length Lc equal to the number of subcarriers N_p.

In this case, the MC-CDMA emetter system is given by :

$$x(t) = \frac{a_i}{\sqrt{N_p}} \sum_{q=0}^{N_u-1} \sum_{k=0}^{N_p-1} c_{i,k} \exp^{2jf_k t} \quad (29)$$

Avec $f_k = f_0 + \frac{1}{T_c}$ Where N_u the number of users and N_p the number of the subcarriers.

The impulse response h of the channel can be writen as 30. This expression takes fixed paths when the transmitter and receiver are moving.

$$h(\tau, t) = \sum_{p=0}^{P-1} \beta_p(t) \exp^{i(2\pi\nu_p t + \theta_p(t))} \delta(\tau - (\tau_p)) \quad (30)$$

The issued signal s and the receved one r are related by:

$$\begin{aligned} r(t) &= (h * s)(t) + n(t) \\ &= \int_{-\infty}^{+\infty} \sum_{p=0}^{P-1} \beta_p \exp^{i\theta_p} \delta(\tau - \tau_p) x(t-\tau) d\tau + n(t) \\ &= \sum_{p=0}^{P-1} \beta_p \exp^{i\theta_p} x(t-\tau) + n(t) \end{aligned} \quad (31)$$

In order to overcome disturbances introduced by the transmission channel, it is necessary to implement an equalization device. Two classes of detectors exist [14][15], single-user detectors and multi-user detectors. In the first case, only the sequence of the user assumed known, related interference to other users are then considered jammers. In the other, the sequences for all users are known, multiple access interference is then considered deterministic signals and not as random. So, where N_u users are active, from the relationship 30 of the impulse response h of the channel, the received signal may be expressed by :

$$\begin{aligned} r(t) = \frac{1}{\sqrt{N_p}} \sum_{p=0}^{P-1} \sum_{j=0}^{N_u-1} \sum_{k=0}^{N_p-1} \\ reel\{\beta_p \exp^{i\theta_p} a_j c_{k,j} \exp^{2i\pi(f_0 + \frac{k}{T_c})(t-\tau_p)}\} \end{aligned} \quad (32)$$

In order to facilitate the presentation of different detection techniques, we assume an adequate dimensioning of the system studied. So, the duration of the guard interval T_g is sufficient, ensuring no interference between symbols and subcarriers. Furthermore, the duration of the MC-CDMA symbol is sufficient to consider the channel as invariant over the duration of this symbol, well as the absence of frequency selectivity on each subcarrier. Then, the independence of random processes affecting each subcarrier will be guaranteed by the temporal and frequency interleaving. finally, and to facilitate the introduction of different detection techniques [16], we will take L_c = N_p. We can represent our matrix system:

$$r = HCa + n \quad (33)$$

Where r is a vector containing the values received on each subcarrier :

$$r = [r_0 ... r_{N_p-1}] \quad (34)$$

The matrix H is the matrix of size parameters of the channel $Np \times Np$.

The hypotheses previously assumed on the proper sizing of the system allow us to consider this matrix as diagonal matrix H:

$$H = \begin{pmatrix} h_0 & 0 & \cdots & 0 \\ 0 & h_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{N_p-1} \end{pmatrix} \quad (35)$$

The matrix C represente the spreading codes. The spreading operation can therefore be represented as the multiplication of the matrix C by the vector a, constituted of the data of each user. We can therefore write :

$$\begin{aligned} C &= [c_0, \cdots, c_{N_n-1}] \\ &= \begin{pmatrix} c_{0,0} & c_{0,1} & c_{0,2} & c_{0,N_n-1} \\ c_{1,0} & c_{1,1} & c_{1,2} & c_{1,N_n-1} \\ \vdots & \vdots & \ddots & \vdots \\ c_{N_p-1,0} & c_{N_p-1,1} & c_{N_p-1,2} & c_{N_p-1,N_n-1} \end{pmatrix} \end{aligned} \quad (36)$$

With : $c_i = [c_{0,i}, c_{1,i}, \cdots, c_{N_p-1,i}]^T$ and $a = [a_0, a_1, \cdots, a_{N_p-1}]^T$

The vector n presente the N_p components of the noise affecting each subcarrier, it is can be modeled as a Gaussian additive process: $n = [n_0, n_1, \cdots, n_{N_p-1}]^T$

In reception, the structure of the studied detectors based on the use of an equalization stage, followed despreading operations according to the user's sequence considered, see figure (3).

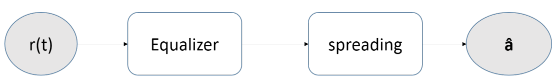

Figure 3: Principle of the single user detection

Single-user detectors consider only the active user signal, other users are treated as jammers. Single-user detectors typically encountered using a linear equalization structure [17], [18], consisting of an equalizer into an outlet. Using the above matrix notation, it is possible to express G,the diagonal matrix composed of coefficients equalization G_K:

$$G = \begin{pmatrix} g_0 & 0 & \cdots & 0 \\ 0 & g_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & g_{N_p-1} \end{pmatrix} \tag{37}$$

After equalization and despreading in the sequence of the user C_J considered, the estimated transmitted symbol $\hat{a_i}$ can be expressed as :

$$\begin{aligned} \hat{a_i} &= {c_i}^{-T} Gr \\ &= {c_j}^{-T} GHCa + {c_j}^{-T} n \\ &= \underbrace{\sum_{k=0}^{N_p-1} {c_{i,k}}^2 g_k h_k a_i} \\ &+ \underbrace{\sum_{q=0}^{N_u-1} \sum_{k=0}^{N_p-1} c_{i,k} c_{q,k} g_k a_q} \\ &+ \underbrace{\sum_{k=0}^{N_p-1} {c_{i,k}}^2 g_k n_k} \end{aligned} \tag{38}$$

VI. The optimum detector in single-user context

A. The detector combination and orthogonality restoration COR(Zéro Forcing(ZF))

This technique COR, for Orthogonality Restoring Combining, cancels completely the dispersion introduced by the channel. The use factor applied to each subcarrier is given by:

$$g_k = \frac{1}{h_k} avec h_k \neq 0. \tag{39}$$

In this case, the expression of the estimate $\hat{a_i}$ becomes :

$$\hat{a_i} = \underbrace{\sum_{k=0}^{N_p-1} {c_{i,k}}^2 a_i} + \underbrace{\sum_{q=0}^{N_u-1} \sum_{k=0}^{N_p-1} c_{i,k} c_{q,k} a_q} + \underbrace{\sum_{k=0}^{N_p-1} c_{i,k} \frac{1}{h_k} n_k} \tag{40}$$

The use of orthogonal spreading codes to the levels of the transmitter guarantees :

$$\sum_{k=0}^{N_p-1} c_{i,k} c_{q,k} = 0 \forall i \neq q \tag{41}$$

B. The combination detector minimum mean square error (MMSE)

This MMSE technical [19][8], for Minimum Mean Square Error, offers a compromise between minimizing the term multipath interference and maximizing signal to noise ratio. It comes from the application of the Wiener filter [16][20]. The calculation of the equalization coefficients has to minimize the mean square error for each subcarrier between the transmitted signal and the equalized signal. This resolution leads to the expression of the coefficients g_k :

$$g_k = \frac{h_k^*}{|h_k|^2 + \frac{1}{\gamma_k}} \tag{42}$$

With the channel normalization hypothesis in power :$E[|h_k|^2] = 1$. The coefficient γ_k is calculated from the estimated signal to noise ratio per subcarrier, inducing additional complexity.

In this case, the expression of the estimate $\hat{a_i}$ becomes :

$$\begin{aligned} \hat{a_i} = & \underbrace{\sum_{k=0}^{N_p-1} {c_{i,k}}^2 \frac{|h_k|^2}{|h_k|^2 + \frac{1}{\gamma_k}} a_i} \\ & + \underbrace{\sum_{q=0}^{N_u-1} \sum_{k=0}^{N_p-1} c_{i,k} c_{q,k} \frac{|h_k|^2}{|h_k|^2 + \frac{1}{\gamma_k}} a_q} \\ & + \underbrace{\sum_{k=0}^{N_p-1} {c_{i,k}}^2 \frac{|h_k|^*}{|h_k|^2 + \frac{1}{\gamma_k}} n_k} \end{aligned} \tag{43}$$

As well it is assumed that the spreading codes are orthogonal, so we can deduce that :

$$\sum_{k=0}^{N_p-1} c_{i,k} c_{q,k} = 0 \forall i \neq q \tag{44}$$

So, the equation 43 becomes:

$$\begin{aligned} \hat{a_i} = & \underbrace{\sum_{k=0}^{N_p-1} {c_{i,k}}^2 \frac{|h_k|^2}{|h_k|^2 + \frac{1}{\gamma_k}} a_i} \\ & + \underbrace{\sum_{k=0}^{N_p-1} {c_{i,k}}^2 \frac{|h_k|^*}{|h_k|^2 + \frac{1}{\gamma_k}} n_k} \end{aligned} \tag{45}$$

VII. Simulation results

In this section we show the performance results obtained by computer simulation for different SNR and assuming that the input channel is driven by non Gaussian signal $x(n)$. The output channel $y(n)$ is corrupted by a gaussian noise $N(n)$. The equalization performance of

MC-CDMA systems are evaluated using algorithm previously studied.This assessment is made by calculating the Binary Error Rate (BER), for both ZF and MMSE equalizers, using the measured and estimated parameters of both channels A and BRAN BRAN E.

A. BRAN A Identification using the Reproducing Kernal Hilbert Space algorithm (RKHS)

The figures (Fig. 4 and 5) shows that the estimated parameters of the channel impulse response approach to real model parameters. These results are performed for SNR = 32 dB and a number of samples $N = 4096$.

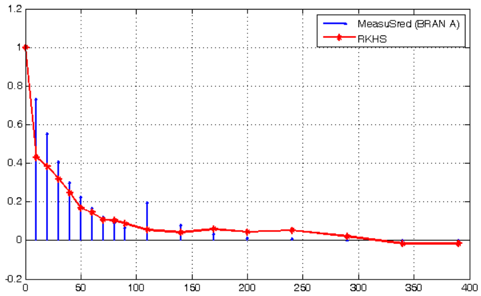

Figure 4: Parameter estimation of the impulse response of BRAN A channel using the RKHS algorithm for SNR=16 db

We represente the estimation of the BRAN A parmaeters usign the RKHS algorithme, for an $SNR = 16db$ the data length is 2048 and for 100 iterations, we observe a very lwo variation due to the noise on the parameters estimation.

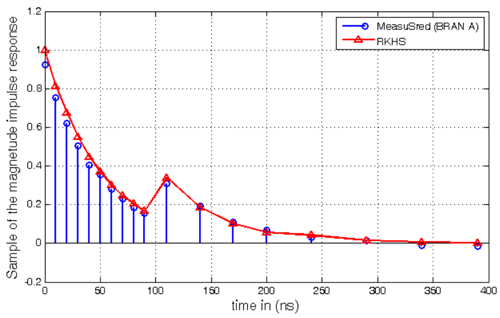

Figure 5: Parameter estimation of the impulse response of BRAN A channel using the RKHS algorithm for SNR=32 db

In this section we consider the BRAN E channel model. The fig 6 represente the impulse response estimation for BRAN E channel model usign RKHS algorithme for $SNR = 16db$. These last shows that there is a slight influence of noise in impulse response parameters estimation, whereas for a $SNR = 32db$, the effect of noise decreases.

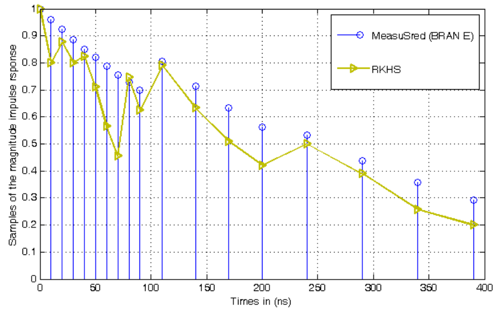

Figure 6: Parameter estimation of the impulse response of BRAN E channel using the RKHS algorithm for SNR=16 db

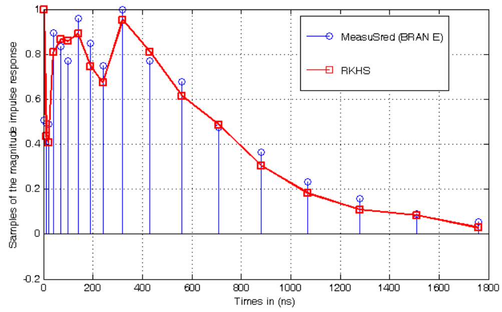

Figure 7: Parameter estimation of the impulse response of BRAN E channel using the RKHS algorithm for SNR=32 db

VIII. MC-CDMA SYSTEM PERFORMANCE

To evaluate the performance of the MC-CDMA system, using the Reproducing Kerne Hilbert Space methods. These performances are evaluated by calculation of the Bit Error Rate (BER), for ZF and MMSE equalizers, using the measured and estimated BRAN A and BRAN E channel impulse response. These stadie was evaluated for different SNR values.

A. ZF and MMSE equalizers : case of BRAN A channel

We represent in the figure Fig. 8 the simulation results using the measured parameters and the parameters estimated by the reproducing kernel hilbert space method appleid to BRANA channel. The equalization presente a acceptable results using ZF equalizer.

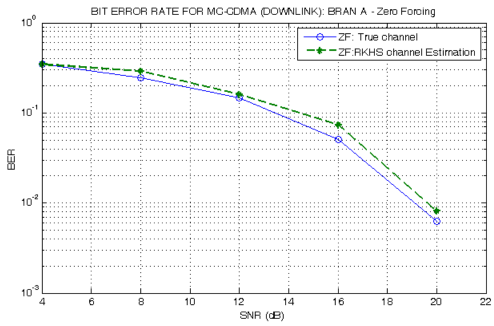

Figure 8: BER of the estimated and measured BRAN A channel, for different SNR, using the ZF equalizer

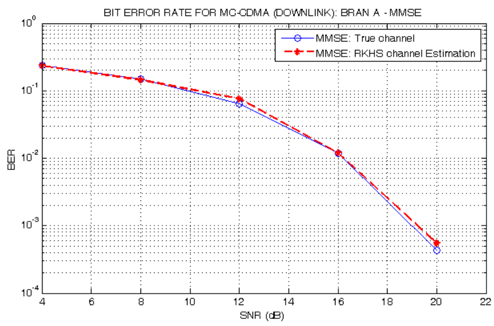

Figure 9: BER of the estimated and measured BRAN A channel, for different SNR, using the MMSE equalizer

the figures (8 and 9) shows that for defferent SNR the results obtained by RKHS folow the same form comparing with those obtained using mesured data. From thes same figures we conclude that: the equalization results using MMSE equalizer is better than those obtained by ZF equalizer, especially where the $SNR > 20db$, in these case we have only a BER of 10^{-4}.

B. ZE and MMSE equalizer : case of BRAN E channel

We represent in the Fig. (10 and 11), the simulation results of BER estimation using the measured and estimated of the BRAN E channel impulse response. The equalization is performed using ZF and MMSE equalizers.

From the results obtained by the equalizers ZF and MMSE (Fig. 10 and 11), it is clear that the BER obtained, based on the estimated

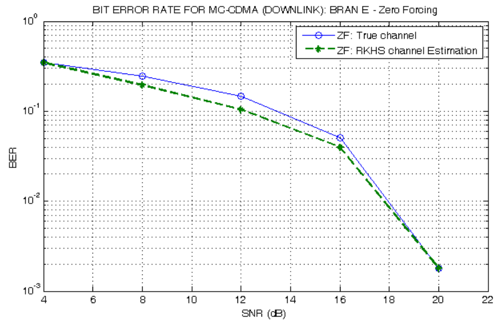

Figure 10: BER of the estimated and measured BRAN E channel, for different SNR, using the ZF equalizer

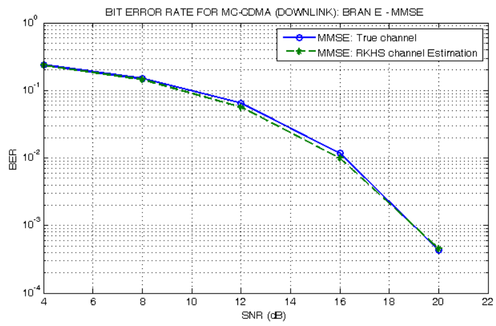

Figure 11: BER of the estimated and measured BRAN E channel, for different SNR, using the MMSE equalizer

parameters, and year using the RKHS method gives good results per compared to those obtained in using the measured values of BRAN E channal. So, if the SNR values are superior to $20db$, we observe that 1 bit error occurred when we receive 103 bit, but if the $SNR > 20db$ we obtain only one bit error for 104 bit received.

IX. Conclusion and Perspectives

The development of kernel methods, especially for the identification and equalization of the channel parameters, marks the point of convergence of several essential concepts: passing the nonlinear, thanks to the astuteness kernels, a large family of linear algorithms relying only on scalar products in the space of inputs χ.

In this paper, it has been applied the method of defined positive kernels for the parameter identification of a wirless transmission channel, for a single user of a finite impulse response signal, and for a stationary system and time invariant. We applied this approach to a different channels, and it was observed from the results of the simulation parameters in phase and amplitude that follows the shape of the measured response. The RKHS algorithm show their efficiency in the impulse response channel (BRAN (A and E) normalized for the MC-CDMA system) identification with high precision, MMSE has demonstrated its effectiveness compared to ZF, knowing that both of them give very satisfactory results.

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