Facility location problem in extreme and uncertain environment. Part I: Model construction

Gia Sirbiladze, Bidzina Matsaberidze, Bezhan Ghvaberidze, Bidzina Midodashvili

***Abstract*—**In this work a new model of facility location-selection problem under uncertain and extreme environment is constructed. Uncertain factors which impact on the decision making process for the facility location planning are taken into consideration. Experts evaluate each humanitarian aid from distribution centers (HADC) against each of the uncertain factors. HADCs location problem is reduced to the bicriteria problem of partitioning the set of customers by the set of centers: (1) – Minimization of transportation costs; (2) – Maximization of centers' selection ranking indexes (or Minimization of "not selecting" ranking indexes). Partitioning type constraints are also constructed.

***Keywords*—**Facility location problem, possibility measure, fuzzy numbers, multi-objective optimization problem, partitioning problem.

I. INTRODUCTION

IN real-life situations the location problems are more complex, than their basic formulations consider. In the modern world, there are many different types of extreme situations that need to be taken into consideration for finding high reliable solutions. There are traffic jams, icy and snowy roads, various types of damages on roads, delays caused by strikes and demonstrations, etc. These factors can be divided into two main categories: 1. Factors that cause inaccuracies, imprecisions of time of movements on the roads between the demand points (e.g. overloaded traffic may significantly increase the time required to move from the humanitarian aid from distribution centers HADC to the customer), 2. Factors that introduce uncertainty – question marks about feasibility of service delivery (e.g. if the road is expected to get closed due to weather conditions, or if there doesn't exist an accurate information about the state of the road and there is a possibility that the road is damaged as a result of a landslide or a terrorist attack or an explosion, etc.).

We deal with the problems of facility location in extreme and uncertain environments. The models built for such problems can be used in extreme situations, for example for delivering humanitarian aid to the damaged region, as well as in daily business activities, as this model can consider and process more information and generate highly reliable solutions. In this regard, the model we have built is universal and is a generalization of classical models. However, for clarity and comprehension, we follow one line of examples below, specifically the problem of distributing HADCs in a region, damaged as a result of earthquakes, floods, terrorist attacks or other factors. The problem solves the tasks of planning the recovery phase of a damaged region (some geographical area), which implies mobilization and deployment of emergency services (delivering first aid, supplying food and medicines and so forth) within the affected areas in order to avoid or reduce human and material damages. In such situations, the reaction time (the goods must be delivered to the demand points in minimum time, which is not always proportional to the distance between HADC and the demand point) and the reliability of the service plan is more important than minimizing different types of expenses, but the costs are also important dimension in order to effectively distribute required resources in the damaged region, so it's not possible to completely ignore this dimension.

As we discuss the tasks in the extreme and uncertain environment, we often deal with an incomplete information or / and with a lack of information. Therefore, to increase the accuracy of the model, objective data (such as the number of users, the volume of their demands, the capacity of the service centers, etc.) is enriched with subjective information that can be obtained from experts based on their knowledge and experience.

Timely servicing from emergency service centers to the affected geographical areas (demand points as customers, for example critical infrastructure objects) is a key task of the emergency management system. Scientific research in this area focuses on distribution networks decision-making problems, which are known as a Facility Location Problem (FLP) [2]. FLP's models have to support the generation of optimal locations of service centers in complex and uncertain situations. There are several publications about application of fuzzy methods in the FLP. However, all of them have a common approach. They represent parameters as fuzzy values

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(triangular fuzzy numbers [3] and others) and develop methods for facility location problems called in this case Fuzzy Facility Location Problem (FFLP) ([10,12,13] and others). Fuzzy TOPSIS approaches for facility location selection problem for different fuzzy environments are developed in [1,8] and others.

In our model experts evaluate each HADC against each of the uncertain factor. Examples of these factors can be: Accessibility by public and/or special transport; Connectivity with other types of transport (highways, railways, seaport, airport etc.); Security from accidents, theft and vandalism; Connectivity with the central locations; Impact on the environment; Availability of raw material and labor resources; Ability to conform to sustainable freight regulations imposed by emergency managers (e.g. restricted delivery hours, special delivery zones, etc.); Ability to increase size to accommodate growing demands; and more. Each of these factors may have its own weight. In addition, factors may not be independent. Two factors can have a higher or lower value (weight) together than the total weight of the same factors independently. In order to process these kinds of interactions and interdependences, it is important to use adequate measures. For these purposes, we have selected monotonous measures in multi-attribute/criteria decision making models [5,9,11].

Expert can't always provide evaluations in the form of exact numbers. Often, it is convenient for them to provide evaluations in linguistic variables using natural language. They often use terms such as "very high", "high", "medium", etc. The model we have built can take similar evaluation as an input and translate them into fuzzy concepts (e.g. in triangular fuzzy numbers).

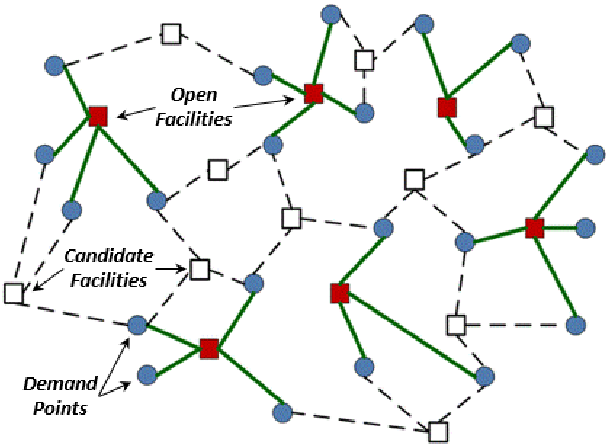

Fig.1 facility location network

As we have noted, imprecision and uncertainty emerge from the extreme situations while moving from centers to customers (see Fig. 1). Imprecision is mainly reflected in growth of inaccuracies of travel times – deviations from the average times required to deliver goods to customers in normal situations. For example, if in a normal situation it takes 20-minute to move between certain points, the expert can evaluate the movement time between the same points in the extreme situation as "about 25 minutes, ± 10 minutes", which can be written as triangular fuzzy number (15, 25, 35) [3]. As regards the uncertainty, in extreme and uncertain environment, the possibility of movement between specific points is questioned because of lack of complete information about road conditions or because of having information about some damages on the roads. We use a possibility measure (Pos) [3] to describe the feasibility of movement between points. However, due to the specifics of our problem, we consider not only the possibility of movement, but the possibility of movement in τ time. Therefore, for each candidate center cc_j (j -th HADC), we will define DP^j – a set of the customers, for which the goods can be supplied in τ time:

$$DP^j = \left\{ dp_i \in DP \mid Pos\left(\tilde{t}_{ij} \le \tau\right) \ge \lambda \right\}, 0 \le \lambda \le 1, j = \overline{1,n};$$

where λ is the minimal possibility level, which is defined by the emergency situation managers and by which the condition of on-time goods delivery ($\tilde{t}_{ij} \le \tau$) must be satisfied.

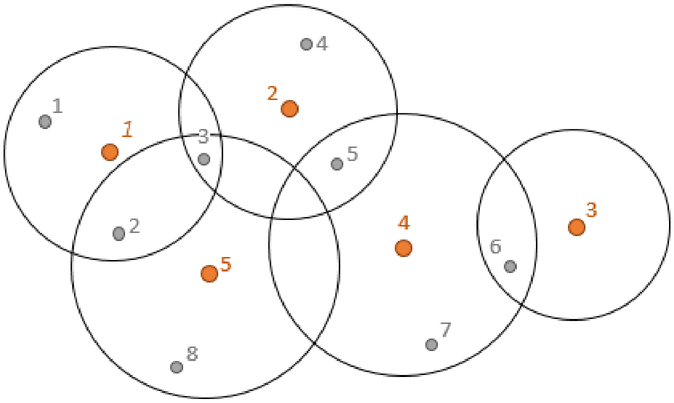

Fig. 2 Map of Coverings (considering τ time)

Finally, based on the subjective information received from the experts, we construct the map of coverings (for example see Fig. 2). Based on the results of our previous work [13] we calculate centers *selection ranking indexes* ($\delta = \left\{\delta_j\right\}, \delta_j \in [0,1]; j = \overline{1,n};$), which enable us to reflect in numbers the desire of choosing specific HADCs.

Briefly on the construction of *centers selection ranking indexes* [13]:

At first, we are focusing on a multi-attribute group decision making approach for *location planning for selection of service centers under uncertain and extreme environment*. a fuzzy multi-attribute decision making approach for the service center location selection problem for which a fuzzy probability aggregation operators' approach is used [13].

The formation of expert's input data for construction of attributes is an important task of the centers' selection problem. To decide on the location of service centers, it is assumed that a set of candidate sites already exists. This set is denoted by $CC = \{cc_1, cc_2, \ldots, cc_n\}$, where we can locate service centers and $S = \{s_1, s_2, \ldots, s_k\}$ be the set of all uncertain factors (described above) which define CCs selection. For example: 1. $s_1 =$"post disaster access by public and special transport modes to the candidate site"; 2. $s_2 =$" post disaster security of the candidate site from accidents, theft and vandalism"; 3. $s_3 =$" post disaster connectivity of the location with other modes of transport (highways, railways, seaport, airport etc.)"; 4. $s_4 =$"costs in vehicle resources, required products and etc. for the location of CCs in candidate site"; 5. $s_5 =$ "impact of the candidate site on the environment, such as important objects of Critical Infrastructure and others"; 6. $s_6 =$"distances of the candidate site from the central locations"; 7. $s_7 =$"distances of the candidate site from demand points"; 8. $s_8 =$ "availability of raw material and labor resources in the candidate site"; 9. $s_9 =$"ability to conform to sustainable freight regulations imposed by emergency managers post disaster for e.g. restricted delivery hours, special delivery zones"; 10. $s_{10} =$"ability to increase size to accommodate growing demands post disaster" and others.

Let us assume that $DP = \{dp_1, dp_2, \ldots, dp_m\}$ is the set of all demand points (customers). Let $\tilde{W} = \{\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_k\}$ be the fuzzy weights of uncertain factors (attributes). For each expert e_k from invited group of experts (emergency service dispatchers and so on) $E = \{e_1, e_2, \ldots, e_t\}$, let $\tilde{a}_{ij}^{\prime l}$ be the fuzzy positive rating (presents in some fuzzy terms) of his/her evaluation for each candidate site cc_i, $(i = 1, \ldots, n)$, with respect to each attribute s_j, $(j = 1, \ldots, k)$. For the expert e_k we construct binary fuzzy relation $\tilde{A}_l' = \{\tilde{a}_{ij}^{\prime k},\ i = 1, \ldots, n;\ j = 1, \ldots, m\}$, elements of which are represented in fuzzy terms. We built an aggregation operators' approach, which for each candidate site cc_i, $(i = 1, \ldots, n)$ aggregates presented objective and subjective data into scalar values – *site's selection ranking index*. This aggregation formally can be represented as:

$$\delta_i \equiv \delta(cc_i) = Agreg(DP, CC, \tilde{g}, [\tilde{A}_l']_i,\ l = 1, \ldots, t;\ i = 1 \ldots, n).$$

where $\tilde{g}$ is - a fuzzy measure [5] which take into account fuzzy interaction indexes between attributes and fuzzy important values (weights) of attributes in its construction. *The bigger the centers selection ranking index δ_j is, it's more desirable to select the j-th center* (and open HADC there).

To summarize, the input data of our model can be divided into two groups: objective and subjective data. Objective data includes data such as the number of candidate centers, their opening expenses, their capacities, number of demand points, their demands, transportation costs, etc. and subjective data includes the possibility levels of customers covering (in τ time) and centers selection ranking indexes. Our model considers and synthesizes both types of data to increase the accuracy and reliability of solutions.

II. BICRITERIA MODEL FOR FLP IN EXTREME AND UNCERTAIN ENVIRONMENT

What is the solution of our model? Solution must allocate each customer to a single candidate center. It means that this candidate center will be opened, and the goods will be delivered from this center to the customer allocated to it. The allocation of customers to the centers should be done in a way that the costs are minimal and at the same time reliability of goods delivery is high.

Hereby we specify the assumptions made by our model: the model assumes distribution of uniform goods. For the example of delivering humanitarian aid, we can assume that this aid (e.g. food or various items) is packed in homogeneous boxes - in humanitarian packages and is delivered to customers in this form. For example, one customer may need 31 boxes, another 53. We also assume that all customers must be fully satisfied and at the same time each customer must be satisfied only from one center, because in emergency situations often there is no time for coordination between the centers to plan fulfilling customers' demands from several centers.

It is easy to see that finding the optimal solutions is not a trivial task, since it requires examining large number of combinations – different allocations of customers to centers. To analyze the number of such combinations, we can look at the map of coverings, which can easily be described by so called coverings matrix – rows of which correspond to the customers and columns of which correspond to candidate centers:

	1	2	3	4	5
1	1	0	0	0	0
2	1	0	0	0	1
3	1	1	0	0	1
4	0	1	0	0	0
5	0	1	0	1	0
6	0	0	1	1	0
7	0	0	0	1	0
8	0	0	0	0	1

Based on the map of coverings (and on the corresponding matrix), we can easily determine the number of partitionings (of customers by centers) – the number of different allocations. Our goal is to select the optimal one(s) out of these allocations. The number of the allocations is equal to the product of numbers of 1s in each row (not considering customers demands and candidate centers capacities). For our example, the number of allocations will be 1 x 2 x 3 x 1 x 2 x 2 x 1 x 1 = 24.

Let's denote the matrix of coverings by $A=a_{ij},\ a_{ij}\in\{0;1\};\ i=\overline{1,m};\ j=\overline{1,n};$ Then the number of partitionings can be calculated using the expression:

$$\prod_{i=1}^{m}\sum_{j=1}^{n}a_{ij}$$

In order to avoid generating all the partitionings, then calculating values of objective functions and selecting Pareto optimal solutions out of them, we propose an approach, which allows us to find Pareto optimal solutions without performing exhaustive search.

Remark: μ_{ij} can be defined (can be given) for any i and j ($i=\overline{1,m};\ j=\overline{1,n};$), but its value will be big for the customers, which are far from the HADCs. In this case all the elements of the covering matrix A would be equal to 1, but in real life problems this almost never happens because of the time limit τ .

Now we present the model in more formal way.

The objective data for the model is:

- m – Number of demand points (customers);
- $DP=\{dp_i\}$ – Set of demand points, $i=\overline{1,m};$
- d_i – Demand of i -th demand point, $i=\overline{1,m};$
- n – Number of potential centers (HADCs);
- $CC=\{cc_j\}$ – Set of potential (candidate) centers, $j=\overline{1,n};$
- C_j – Capacity of j -th center, $j=\overline{1,n};$
- P_j – Cost of opening j -th center;
- μ_{ij} – Cost of transporting (d_i goods) from j -th center to i -th demand point;
- τ – Maximum allowed time to deliver goods to demand points;

The subjective (expert) data for the model is:

- $DP^j=\{dp_i\in DP\mid Pos(\tilde{t}_{ij}\le\tau)\ge\lambda\}$ – Map of coverings, which is based on expert evaluations and satisfies the condition $\tilde{t}_{ij}\le\tau$, with λ minimal possibility level, $j=\overline{1,n};$
- $\delta=\{\delta_j\},\delta_j\in[0,1]$ – Centers' selection ranking indexes, $j=\overline{1,n};$

Variables:

- $r_j\in\{0;1\}$: 1 - if j -th HADC is opened, else 0;
- $x_{ij}\in\{0;1\}$: 1 - if i -th customer's d_i demand is fully satisfied by j -th HADC, else 0 (when i -th customer is not serviced from j -th HADC);

It's obvious, that $r_j=\overset{m}{\underset{i=1}{\vee}}x_{ij}\left(=\max_{i=\overline{1,m}}x_{ij}\right)$;

Constraints:

- Considering the capacities of the HADCs: $\sum_{i=1}^{m}d_ix_{ij}\le C_j,\ j=\overline{1,n};$
- Ensuring that single customer is fully satisfied from single HADC: $\sum_{j=1}^{n}x_{ij}=1,\ i=\overline{1,m};$

Objective functions:

$$(1)\quad f_1=\sum_{j=1}^{n}P_jr_j+\sum_{i=1}^{m}\sum_{j=1}^{n}\mu_{ij}x_{ij}\to min$$

$$(2^*)\quad f_2^*=\overset{n}{\underset{\substack{j=1\\ r_j\neq 0}}{\wedge}}\delta_jr_j\to max$$

$$(2)\quad f_2 = \overset{n}{\underset{j=1}{\vee}} \left(1-\delta_j\right) r_j \to min$$

where $\wedge$ and $\vee$ represent min and max operators respectively. (1) – Minimization of HAD's transportation costs; (2^*)/(2) – Maximization of HAD's selection indexes / Minimization of "not selecting" indexes.

Therefore, HADs' location problem is reduced to the bicriteria problem of partitioning ([4,6,7] and others) the set of demand points by the set of potential centers.

III. CONCLUSION

New fuzzy facility location-selection problem under uncertain and extreme environment is constructed. Bi-objective partitioning type optimization model is created. In this model experts evaluate each HADC against each of the uncertain factor. HADCs location problem is reduced to the bicriteria problem of partitioning the set of customers by the set of centers: (1) – Minimization of costs; (2) – Maximization of centers' selection ranking indexes (or Minimization of "not selecting" ranking indexes). More detailed and practical requirements will be considered in our future investigations.

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