Facility location problem in extreme and uncertain environment. Part II: Model solution

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Abstract— In this work a new model of facility location-selection problem under uncertain and extreme environment is constructed. Uncertain factors which impact on the decision making process for the facility location planning are taken into consideration. Experts evaluate each humanitarian aid from distribution centers (HADC) against each of the uncertain factor. HADCs location problem is reduced to the bicriteria problem of partitioning the set of customers by the set of centers: (1) – Minimization of costs; (2) – Maximization of centers' selection ranking indexes (or Minimization of "not selecting" ranking indexes). Our approach for solving the constructed bicriteria partitioning problem consists of two phases: In the first phase, based on the coverings matrix, we generate a new matrix, columns of which allows us to find all possible partitionings of the HADCs with the service centers. Some constraints are also taken into consideration while generating the matrix. In the second phase, based on the matrix and using our exact algorithm we find the partitionings – allocations of the HADCs to the centers - which corresponds to the Pareto-optimal solutions. For illustration of the constructed model a numerical example is created.

Keywords— Facility location problem, multi-objective optimization problem, partitioning problem, Pareto-optimal solutions.

I. Introduction

In real-life situations the location problems are more complex, than their basic formulations consider. In the modern world, there are many different types of extreme situations that need to be taken into consideration for finding high reliable solutions. There are traffic jams, icy and snowy roads, various types of damages on roads, delays caused by strikes and demonstrations, etc. These factors can be divided into two main categories: 1. Factors that cause inaccuracies, imprecisions of time of movements on the roads between the demand points (e.g. overloaded traffic may significantly increase the time required to move from the HADC to the customer), 2. Factors that introduce uncertainty – question marks about feasibility of service delivery (e.g. if the road is expected to get closed due to weather conditions, or if there doesn't exist an accurate information about the state of the road and there is a possibility that the road is damaged as a result of a landslide or a terrorist attack or an explosion, etc.).

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We deal with the problems of facility location in extreme and uncertain environments. The models built for such problems can be used in extreme situations, for example for delivering humanitarian aid to the damaged region, as well as in daily business activities, as this model can consider and process more information and generate highly reliable solutions. In this regard, the model we have built is universal and is the generalization of classical models. However, for clarity and comprehension, we follow one line of examples below, specifically the problem of distributing humanitarian aid from distribution centers (HADCs) in a region, damaged as a result of earthquakes, floods, terrorist attacks or other factors. The problem solves the tasks of planning the recovery phase of a damaged region (some geographical area), which implies mobilization and deployment of emergency services (delivering first aid, supplying food and medicines and so forth) within the affected areas in order to avoid or reduce human and material damages. In such situations, the reaction time (the goods must be delivered to the demand points in minimum time, which is not always proportional to the distance between HADC and the demand point) and the reliability of the service plan is more important than minimizing different types of expenses, but the costs are also important dimension in order to effectively distribute required resources in the damaged region, so it's not possible to completely ignore this dimension.

As we discuss the tasks in the extreme and uncertain environment, we often deal with an incomplete information or / and with a lack of information. Therefore, to increase the accuracy of the model, objective data (such as the number of users, the volume of their demands, the capacity of the service centers, etc.) is enriched with subjective information that can be obtained from experts based on their knowledge and experience.

Timely servicing from emergency service centers to the affected geographical areas (demand points as customers, for example critical infrastructure objects) is a key task of the emergency management system. Scientific research in this area focuses on distribution networks decision-making problems, which are known as a Facility Location Problem (FLP) [2]. FLP's models have to support the generation of optimal locations of service centers in complex and uncertain situations. There are several publications about application of fuzzy methods in the FLP. However, all of them have a common approach. They represent parameters as fuzzy values (triangular fuzzy numbers [3] and others) and develop methods for facility location problems called in this case Fuzzy Facility Location Problem (FFLP) ([10,12,13] and others). Fuzzy TOPSIS approaches for facility location selection problem for different fuzzy environments are developed in [1,8].

In our model experts evaluate each HADC against each of the predefined factor. Examples of these factors can be: Accessibility by public and/or special transport; Connectivity with other types of transport (highways, railways, seaport, airport etc.); Security from accidents, theft and vandalism; Connectivity with the central locations; Impact on the environment; Availability of raw material and labor resources; Ability to conform to sustainable freight regulations imposed by emergency managers (e.g. restricted delivery hours, special delivery zones, etc.); Ability to increase size to accommodate growing demands; and more. Each of these factors may have its own weight. In addition, factors may not be independent. Two factors can have a higher or lower value (weight) together than the total weight of the same factors independently. In order to process these kinds of interactions and interdependences, it is important to use adequate measures. For these purposes, we have selected monotonous measures [5,9,11].

In the part I of this work (model construction) we constructed the bicriteria problem of partitioning for facility location problem:

Objective functions:

$$(1)\quad f_1 = \sum_{j=1}^{n} P_j r_j + \sum_{i=1}^{m}\sum_{j=1}^{n} \mu_{ij} x_{ij} \to min$$

$$(2)\quad f_2 = \mathop{\vee}_{j=1}^{n} \left(1-\delta_j\right) r_j \to min$$

Constraints:

- Considering the capacities of the HADCs: $\sum_{i=1}^{m} d_i x_{ij} \le C_j$, $j=\overline{1,n}$;
- Ensuring that single customer is fully satisfied from single HADC: $\sum_{j=1}^{n} x_{ij} = 1$, $i=\overline{1,m}$;

In this work (model solution) we solve the bicriteria problem of partitioning (1)-(2).

II. Solving the Bicriteria Partitioning Problem

Our approach for solving the constructed bicriteria partitioning problem consists of two phases: In the first phase, based on the coverings matrix A, we generate a new matrix A', columns of which allows us to find all possible partitionings of the customers with the centers. Some constraints are also taken into consideration while generating the matrix A'. In the second phase, based on the matrix A' and using exact algorithm constructed by authors of this work we find the partitionings – allocations of the demand points to the potential centers - which corresponds to the Pareto-optimal solutions [4]. Let's discuss each of the phases in details:

<u>*Phase I*</u>*:* Based on the coverings matrix A, we generate a new matrix $A' = a'_{il}$, $a'_{il} \in \{0;1\}$, $i=\overline{1,m}; l=\overline{1,s};$ columns of which allows us to find all partitionings of customers (demand points). This means that each and every customer must be covered by single center, so the matrix A must contain exactly one 1 in each row. Therefore, each customer is allocated to the single center despite the fact that the same customer could be covered by other centers in the τ time.

The principle of generating the columns of the matrix A' is as follows: based on the matrix A, we generate all possible columns, which can serve as a column for partitioning matrix. This means that any of 1s in the column of the matrix A can become 0, maybe several of 1s, but not all of them together (having this kind of columns in the matrix doesn't make any sense). In the case of the example which was presented above, the first column will generate following columns:

	1						
1	1	0	0	0	1	1	1
2	1	0	1	1	0	0	1
3	1	1	0	1	0	1	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0

As we can see in generated columns zeros are unchanged in the rows 4 - 8, which is logical since the center (HADC) corresponding to the first column can't cover these customers (in τ time, see part I).

While generating the columns of matrix A' we can additionally take into consideration the following factor: if the matrix A had single 1 in some row, it can't become 0. Therefore, in generated columns 1s must be unchanged in the corresponding row. To say the same in simpler words, if a specific customer is covered only by one HADC, this customer will be covered by this single HADC in every partition. In the example given above, we have single 1 in the first row (in the first column), thus the first column of the matrix A will generate not the columns which were listed above, but only following three columns (not including itself):

	1			
1	1	1	1	1
2	1	0	0	1
3	1	0	1	0
4	0	0	0	0
5	0	0	0	0
6	0	0	0	0
7	0	0	0	0
8	0	0	0	0

Finally, the matrix A gives us the following A' matrix:

	1				2				3	4				5			
1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	1
3	1	0	1	0	1	0	0	1	0	0	0	0	0	1	0	1	0
4	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0
5	0	0	0	0	1	0	1	0	0	1	0	0	1	0	0	0	0
6	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1

The columns of the matrix A' are grouped by the columns of the matrix A.

While generating the columns of the matrix A' we can also take into consideration C_j ($j = \overline{1,n}$) capacities of the HADCs. If for any specific column q, sum of the d_i demands of the customers corresponding to the 1s in this column, exceeds the C_j capacity of the HADC ($\sum_{i=1}^{m} a_{iq}' d_i > C_j$), which is the "parent" for the q-th column, we must remove (not include) this column from the matrix A', since this column can't exist in any solution.

Let s be the number of columns (after filtration) of the matrix A' and s_j be the number of columns in j-th column of the matrix A'. Obviously, $s = \sum_{j=1}^{n} s_j$.

<u>Phase II</u>: We translate the variables, constraints and objective functions of the initial problem to the language of the matrix A' and execute our exact algorithm of bicriteria partitioning. As a result, we find the partitionings – allocations of the customers to the HADCs – which will be the Pareto optimal solutions.

Our algorithm of bicriteria partitioning is based on the extended algorithm of Dancing Links technique and DLX algorithm proposed by D. Knuth [6,7], also it is based on the ε-constraint method for multicriteria optimization problems [4]. The algorithm has high performance as it effectively uses RAM because of dynamic programming techniques, it can also run in parallel mode to handle the problems with big dimensions. The most important characteristic is that the algorithm finds exact Pareto front – all Pareto optimal solutions. The algorithm can solve in several seconds the problems having approximately 100 x 1000 dimensions (these dimensions can often be enough to solve the real-life problems but for bigger, very large dimensions, we switch to the approximate methods [2]).

Let's formulate the bicriteria partitioning problem in terms of the matrix A'. Let's introduce a Boolean variable $z_l \in \{0;1\}$: 1 – if the l-th column of the matrix A' is included in the solution, else 0, $l = \overline{1,s}$;

Objective functions will have the form:

$$(1')\quad f_1' = \sum_{l=1}^{s} \left(P_l' + \mu_l' \right) z_l \to min$$

$$(2')\quad f_2' = \mathop{\mathrm{W}}_{s}^{l=1} \left(1 - \delta_l' \right) z_l \to min$$

where:

$$P_l' = P_k,\ \mu_l' \sum_{i=1}^{m} \mu_{ik} a_{il}',\ \delta_l' = \delta_k;\ l = \overline{1 + \sum_{j=0}^{k-1} s_j, \sum_{j=0}^{k} s_j};$$

$$s_0 \equiv 0;\ k = \overline{1,n};$$

Indeed, l takes values from 1 to s ($s = \sum_{j=1}^{n} s_j$), therefore, l takes the values from the following intervals (corresponding to the column groups of the matrix A'):

$$[1, s_1],\ [1 + s_1, s_1 + s_2],\ [1 + s_1 + s_2, s_1 + s_2 + s_3],$$

$$\ldots\ ,\ \left[1 + \sum_{j=1}^{n-1} s_j, \sum_{j=1}^{n} s_j = s \right].$$

In the k-th group $P_l^{'} = P_k$ is fixed. Same is true for $\delta_l^{'}$ ($\delta_l^{'} = \delta_k$). As regards the second addend of the objective function (1) – transportation costs – it depends on the 1s, which will stay in the column of k-th group of the matrix A' from the k-th column of the initial matrix A.

As for the constraints, the HADCs' capacity constraints are already taken into consideration during the generating process of the matrix A' and solving the partitioning problem for the rows and columns of the matrix A' automatically means satisfying the second constraint – any customer must be fully satisfied from the single HADC.

If we execute our exact algorithm on the matrix A' to solve the problem $(1')$-$(2')$, we find the partitionings, which correspond to the Pareto optimal solutions. Each of them will have the form of a matrix, rows of which corresponds to the customers and columns represent the subset of the columns of the matrix A'.

If we analyze the matrix A', we can notice, that the following proposition is true (without proof):

Proposition 1: *Each solution (Pareto optimal partitioning) can contain no more than one column from each column group of the matrix* A'.

Indeed, in each column group contains all possible coverings of customers by specific HADC (we mean the customers, which can be covered by the HADC in τ time). While searching the minimal partitioning, if we find out that the specific HADC covers for example p number of customers, from the column group corresponding to this HADC, the algorithm will always choose the column which contains p number of 1s and will not choose the separate columns (from the same column group), union of which is identical of the above mentioned column (containing p number of 1s). *Note:* the columns with intersecting 1s can't be included in the partitioning solution.

As a summary, we can note that our approach for solving (1)-(2) bicriteria problem is better than exhaustive search approach, because based on the matrix A' it generates only the admissible (satisfying the constraints) and interesting (not worse that already identified in previous steps of the algorithm) partitionings.

III. Model illustration: Numerical Example

To illustrate the model, we have described, and the approach to its solution, let us consider a small dimensional example. Suppose we have 8 demand points and 5 potential locations where humanitarian aid distribution centers can be opened (Fig. 1)

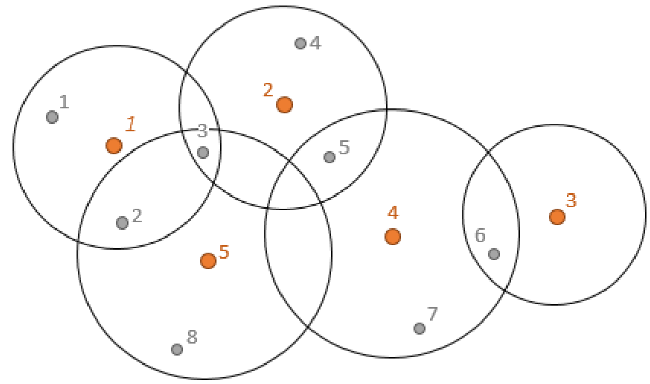

Fig. 1 Map of Coverings (considering τ time)

Below is given the specific numerical data relevant to the problem.

Customer requirements vector:

$$D = \{d_i\} = \{120, 215, 145, 110, 181, 210, 172, 168\};$$

Centers capacity vector:

$$C = \{C_j\} = \{460, 450, 400, 470, 500\};$$

Centers opening costs vector:

$$P = \{P_j\} = \{1100, 1000, 800, 1200, 900\};$$

Transportation costs matrix $M = \|\mu_{ij}\|$:

	1	2	3	4	5
1	503	0	0	0	0
2	451	0	0	0	681
3	782	387	0	0	612
4	0	445	0	0	0
5	0	524	0	603	0
6	0	0	319	1426	0
7	0	0	0	357	0
8	0	0	0	0	589

Suppose that centers selection indices are:

$$\delta = \{\delta_j\} = \{0.72, 0.44, 0.36, 0.70, 0.67\};$$

To solve the problem, in accordance with Phase I of the above considered methodology, let us generate the matrix A' based on the matrix A of coverings and remove from it those columns that violate the capacity constraints of the centers (Fig. 2, columns to be removed are marked with a different color).

C_j		460				450				400	470				500			
d_i		1				2				3	4				5			
120	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
215	2	1	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	1
145	3	1	0	1	0	1	0	0	1	0	0	0	0	0	1	0	1	0
110	4	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0
181	5	0	0	0	0	1	0	1	0	0	1	0	0	1	0	0	0	0
210	6	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0
172	7	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0
168	8	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
The sum		480	120	265	335	436	110	291	255	210	563	172	382	353	528	168	313	383

Fig.2

Finally, we obtain $A' = \| a'_{il} \|,\ a'_{il} \in \{0;1\},\ i = \overline{1,8};\ l = \overline{1,14};$ i.e. 8x14 dimensional matrix (the values of the objective functions are written in the last two lines, Fig.3):

	CC_1			CC_2				CC_3	CC_4			CC_5		
	CC_{11}	CC_{12}	CC_{13}	CC_{21}	CC_{22}	CC_{23}	CC_{24}	CC_{31}	CC_{41}	CC_{42}	CC_{43}	CC_{51}	CC_{52}	CC_{53}
1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0	0	0	0	0	1
3	0	1	0	1	0	0	1	0	0	0	0	0	1	0
4	0	0	0	1	1	1	1	0	0	0	0	0	0	0
5	0	0	0	1	0	1	0	0	0	0	1	0	0	0
6	0	0	0	0	0	0	0	1	0	1	0	0	0	0
7	0	0	0	0	0	0	0	0	1	1	1	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	1	1	1
f_1'	1603	2385	2054	2356	1445	1969	1832	1119	1557	2983	2160	1489	2101	2170
f_2'	0.28	0.28	0.28	0.56	0.56	0.56	0.56	0.64	0.30	0.30	0.30	0.33	0.33	0.33

Fig.3.

According to the Phase II of our approach, let us apply on matrix A' the exact bicriteria partitioning algorithm [6, 7] which is worked out by the authors of this paper. As a result, we obtain two partitionings which correspond to the optimal solutions:

Solution 1: CC_{13}, CC_{21}, CC_{31}, CC_{41}, CC_{51}

Total cost: 8575;

Total selection ranking index: 0.36 (= 1 - 0.64);

Solution 2: CC_{13}, CC_{21}, CC_{42}, CC_{51}

Total cost: 8882;

Total selection ranking index: 0.56 (= 1 - 0.44);

Given the essence of Pareto-optimality, as we can see, the first solution has the better value of Total cost than the second. However, the second solution has a higher reliability because the Total selection ranking index of the centers is better.

As for interpreting solutions based on matrix A' and translating into the initial problem terms, it is a simple task, since the columns in the solution give information about which centers should be opened as well as which customer should belong to which center. For example, if we consider the second solution, it turns out that the third potential center should not open and the assignments should be as follows (the centers that should open are darkened).

I. Conclusion

Fuzzy facility location selection problem under uncertain and extreme environment is constructed. Bi-objective partitioning type optimization model is created. In this model experts evaluate each HADC against each of the uncertain factor. Our approach for solving the constructed bicriteria partitioning problem consists of two phases: In the first phase, based on the coverings matrix, we generate a new matrix, columns of which allows us to find all possible partitionings of the HADCs with the service centers. Some constraints are also taken into consideration while generating the matrix. In the second phase, based on the matrix and using our exact algorithm we find the partitionings – allocations of the HADCs to the centers - which corresponds to the Pareto-optimal solutions. For illustration of constructed model, a numerical example is considered.

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