Parametric optimization of a second order time delay system with a PI-controller

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***Abstract*—**The paper addresses the problem of parametric optimization for the second order time delay system with a PI controller. The integral of squared error as a index of quality is considered. The value of the index is obtained using the quadratic Lyapunov functional. The quadratic Lyapunov functional for time delay system is determined by means of the Lyapunov matrix. In the paper the optimization results for varies values of delay and for varies damping factor are given.

***Index Terms*—**parametric optimization, time delay system, Lyapunov functional, Lyapunov matrix

I. Introduction

The Lyapunov quadratic functionals have many applications. The Lyapunov quadratic functionals are applied in investigation of time delay systems stability [1,7-10], in time delay systems robustness bounds calculation [13], in computation of the exponential estimates of the solutions of time delay systems [12]. The Lyapunov quadratic functionals are also used in the parametric optimization problem of time delay systems to calculation of the value of the quadratic performance index, see for instance [2-6]. The value of the index of integral of squared error is equal to the value of quadratic Lyapunov functional at the initial function of time delay system. The Lyapunov functional fulfills the following condition. Its time derivative computed along the solution of time delay system is equal to the negatively definite quadratic form of system momentary state. The Lyapunov quadratic functional can be determined by means of the Lyapunov matrix. The uniqueness condition of the Lyapunov matrix is given in [11]. In paper [2] is considered the parametric optimization problem for the first order system with a P-Controller. In papers [3-5] are considered parametric optimization problems for the first order system with delay and PD-Controller in [3], PI-Controller in [4] and PID-Controller in [5] respectively. In paper [6] is considered the second order time delay system with delay and with a P-Controller. The paper extends the earlier results to the case of second order system with time delay and PI-Controller. In the paper are also given results of optimization for two different values of damping factor and for varies values of delay.

II. Formulation of the Parametric Optimization Problem

Let us consider a second order time delay system with a PI-controller

$$\begin{cases} \frac{d^2x(t)}{dt^2}+2\zeta\omega_0\frac{dx(t)}{dt}+\omega_0^2x(t)=k_0u(t-h) \\ u(t)=-px(t)-\frac{1}{T_i}\int_0^t x(\xi)d\xi \\ x(\theta)=\varphi_1(\theta) \\ \frac{dx(t+\theta)}{dt}\mid_{t=0}=\varphi_2(\theta) \end{cases} \tag{1}$$

for $t\geq 0,\ \theta\in[-h,0]$.

Where $x(t)\in\mathbb{R}$ is a solution of initial value problem (1), $u(t)\in\mathbb{R}$ is the control, ζ is a damping factor, ω_0 is a natural frequency of oscillation, p is a gain of the PI-Controller and T_i is a time of isodrome of a PI controller, $\varphi_1,\varphi_2\in PC([-h,0],\mathbb{R})$ - the space of piece-wise continuous functions defined on the segment $[-h,0]$.

One introduces the state variables $x_1(t)$ and $x_2(t)$ as follows

$$\begin{cases} x_1(t)=x(t) \\ x_2(t)=\frac{dx_1(t)}{dt}=\frac{dx(t)}{dt} \end{cases} \tag{2}$$

The set of Eqs (1) takes a form

$$\begin{cases} \frac{dx_1(t)}{dt}=x_2(t) \\ \frac{dx_2(t)}{dt}=-\omega_0^2x_1(t)-2\zeta\omega_0x_2(t)+k_0u(t-h) \\ u(t)=-px_1(t)-\frac{1}{T_i}\int_0^t x_1(\xi)d\xi \\ x_1(\theta)=\varphi_1(\theta) \\ x_2(\theta)=\varphi_2(\theta) \end{cases} \tag{3}$$

for $t\geq 0,\ \theta\in[-h,0]$.

We introduce a new state variable

$$x_3(t)=\frac{1}{T_i}\int_0^t x_1(\xi)d\xi \tag{4}$$

One can reshape equation (3) to a form

$$\begin{cases} \frac{dx_1(t)}{dt}=x_2(t) \\ \frac{dx_2(t)}{dt}=-\omega_0^2x_1(t)-2\zeta\omega_0x_2(t)-k_0px_1(t-h)-k_0x_3(t-h) \\ \frac{dx_3(t)}{dt}=\frac{1}{T_i}x_1(t) \\ x_1(\theta)=\varphi_1(\theta) \\ x_2(\theta)=\varphi_2(\theta) \\ x_3(\theta)=-\frac{1}{T_i}\int_\theta^0\varphi_1(\eta)d\eta=\varphi_3(\theta) \end{cases} \tag{5}$$

for $t\geq 0,\ \theta\in[-h,0]$.

System (5) can be written in an equivalent form

$$\begin{cases} \begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \\ \frac{dx_3(t)}{dt} \end{bmatrix} = A_0 \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + A_1 \begin{bmatrix} x_1(t-h) \\ x_2(t-h) \\ x_3(t-h) \end{bmatrix} \\ \begin{bmatrix} x_1(\theta) \\ x_2(\theta) \\ x_3(\theta) \end{bmatrix} = \begin{bmatrix} \varphi_1(\theta) \\ \varphi_2(\theta) \\ \varphi_3(\theta) \end{bmatrix} = \varphi(\theta) \end{cases} \tag{6}$$

for $t \geq 0,\ \theta \in [-h, 0]$, where

$$A_0 = \begin{bmatrix} 0 & 1 & 0 \\ -\omega_0^2 & -2\zeta\omega_0 & 0 \\ \frac{1}{T_i} & 0 & 0 \end{bmatrix} \tag{7}$$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 \\ -pk_0 & 0 & -k_0 \\ 0 & 0 & 0 \end{bmatrix} \tag{8}$$

In parametric optimization problem will be used the performance index of quality

$$J = \int_0^\infty \begin{bmatrix} x_1(t) & x_2(t) & x_3(t) \end{bmatrix} \begin{bmatrix} w_1 & 0 & 0 \\ 0 & w_2 & 0 \\ 0 & 0 & w_3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} dt \tag{9}$$

where w_1 , w_2 and w_3 are positive real numbers.

Problem 1. Determine the gain p_{opt} and $\frac{1}{T_{i\,opt}}$ which minimize the integral quadratic performance index (9).

The value of the performance index J is given by formula, see [2]

$$J = \varphi^T(0)U(0)\varphi(0) + 2\varphi^T(0)\int_{-h}^{0} U(-\theta-h)A_1\varphi(\theta)d\theta +$$

$$+\int_{-h}^{0}\int_{-h}^{0} \varphi^T(\theta)A_1^T U(\theta-\eta)A_1\varphi(\eta)d\eta d\theta \tag{10}$$

where U is a Lyapunov matrix.

III. The Lyapunov Matrix Determination

We need the Lyapunov matrix U to compute the value of the index of quality (9). To this end we solve the set of equations, see [2]

$$\frac{d}{d\xi}U(\xi) = U(\xi)A_0 + U(\xi-h)A_1 \tag{11}$$

$$U(-\xi) = U^T(\xi) \tag{12}$$

$$U(0)A_0 + U(-h)A_1 + A_0^T U(0) + A_1^T U(h) = -W \tag{13}$$

for $\xi \in [0, h]$.
Formula (12) implies

$$U(\xi-h) = U^T(h-\xi) = Z(\xi) \tag{14}$$

We calculate the derivative of $Z(\xi)$ with respect to ξ. Into this derivative and into Eq. (11) we substitute in place of matrices A_0 and $A_1$the terms (7) and (8). In this way we obtain the set of differential equations

$$\begin{bmatrix} \frac{dcolU(\xi)}{d\xi} \\ \frac{dcolZ(\xi)}{d\xi} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \begin{bmatrix} colU(\xi) \\ colZ(\xi) \end{bmatrix} \tag{15}$$

where

$$Q_{11} =$$

$$\begin{bmatrix} 0 & 0 & 0 & -\omega_0^2 & 0 & 0 & \frac{1}{T_i} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\omega_0^2 & 0 & 0 & \frac{1}{T_i} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\omega_0^2 & 0 & 0 & \frac{1}{T_i} \\ 1 & 0 & 0 & -2\zeta\omega_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2\zeta\omega_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -2\zeta\omega_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{16}$$

$$Q_{12} = \begin{bmatrix} 0 & 0 & 0 & -pk_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -pk_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -pk_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -k_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -k_0 & 0 & 0 & 0 \end{bmatrix} \tag{17}$$

$$Q_{21} = \begin{bmatrix} 0 & pk_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & k_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & pk_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & k_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & pk_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_0 & 0 \end{bmatrix} \tag{18}$$

$$Q_{22} =$$

$$\begin{bmatrix} 0 & \omega_0^2 & -\frac{1}{T_i} & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2\zeta\omega_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_0^2 & -\frac{1}{T_i} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2\zeta\omega_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega_0^2 & -\frac{1}{T_i} \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2\zeta\omega_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{19}$$

for $\xi \in [0,h]$.

Solution of Eq. (15) is given by the term

$$\begin{bmatrix} colU(\xi) \\ colZ(\xi) \end{bmatrix} = \Phi(\xi) \begin{bmatrix} colU(0) \\ colZ(0) \end{bmatrix} \tag{20}$$

where the matrix

$$\Phi(\xi) = [\Phi_{ij}(\xi)] = \begin{bmatrix} M_{11}(\xi) & M_{12}(\xi) \\ M_{21}(\xi) & M_{22}(\xi) \end{bmatrix}$$

for $i,j = 1,...,18$ is a fundamental matrix of system (15)

We determine the initial conditions of Eq. (15) $colU(0)$, $colZ(0)$. Matrix $U(0)$ is a symmetric matrix, so

$$U(0) = \begin{bmatrix} U_{11}(0) & U_{12}(0) & U_{13}(0) \\ U_{12}(0) & U_{22}(0) & U_{23}(0) \\ U_{13}(0) & U_{23}(0) & U_{33}(0) \end{bmatrix}$$

Eq. (14) implies $Z(h) = U^T(0) = U(0)$.

From Eq. (20) we obtain

$$colZ(h) = colU(0) = M_{21}(h)colU(0) + M_{22}(h)colZ(0) \tag{21}$$

or in the equivalent form

$$[M_{21}(h) - I]colU(0) + M_{22}(h)colZ(0) = 0 \tag{22}$$

Formula (14) also implies

$$U(-h) = U^T(h) = Z(0) \tag{23}$$

Taking (23) into account Eq. (13) takes a form

$$U(0)A_0 + Z(0)A_1 + A_0^T U(0) + A_1^T Z^T(0) = -W \tag{24}$$

In Eq. (24) we substitute in place of matrices A_0 and $A_1$the terms (7) and (8).

Relation (24) and Eq.(22) form the set of algebraic equations

$$\begin{bmatrix} N_{11} & N_{12} \\ N_{21}(h) & M_{22}(h) \end{bmatrix} \begin{bmatrix} colU(0) \\ colZ(0) \end{bmatrix} = \begin{bmatrix} -w_1 \\ 0 \\ -w_2 \\ 0 \\ 0 \\ -w_3 \\ 0_{(9,1)} \end{bmatrix} \tag{25}$$

where

$$N_{11} = \begin{bmatrix} 0 & -2\omega_0^2 & \frac{2}{T_i} & 0 & 0 & 0 \\ 1 & -2\zeta\omega_0 & 0 & -\omega_0^2 & \frac{1}{T_i} & 0 \\ 0 & 2 & 0 & -4\zeta\omega_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\omega_0^2 & \frac{1}{T_i} \\ 0 & 0 & 1 & 0 & -2\zeta\omega_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{26}$$

$$N_{12} = \begin{bmatrix} 0 & 0 & 0 & -2pk_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -pk_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k_0 & 0 & -pk_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -k_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2k_0 & 0 & 0 & 0 \end{bmatrix} \tag{27}$$

$$N_{21}(h) = \begin{bmatrix} N_{211}(h) & N_{212}(h) & N_{213}(h) \end{bmatrix} \tag{28}$$

$$N_{211}(h) = \begin{bmatrix} \Phi_{10,1}(h) - 1 & \Phi_{10,2}(h) + \Phi_{10,4}(h) \\ \Phi_{11,1}(h) & \Phi_{11,2}(h) + \Phi_{11,4}(h) - 1 \\ \Phi_{12,1}(h) & \Phi_{12,2}(h) + \Phi_{12,4}(h) \\ \Phi_{13,1}(h) & \Phi_{13,2}(h) + \Phi_{13,4}(h) - 1 \\ \Phi_{14,1}(h) & \Phi_{14,2}(h) + \Phi_{14,4}(h) \\ \Phi_{15,1}(h) & \Phi_{15,2}(h) + \Phi_{15,4}(h) \\ \Phi_{16,1}(h) & \Phi_{16,2}(h) + \Phi_{16,4}(h) \\ \Phi_{17,1}(h) & \Phi_{17,2}(h) + \Phi_{17,4}(h) \\ \Phi_{18,1}(h) & \Phi_{18,2}(h) + \Phi_{18,4}(h) \end{bmatrix} \tag{29}$$

$$N_{212}(h) = \begin{bmatrix} \Phi_{10,3}(h) + \Phi_{10,7}(h) & \Phi_{10,5}(h) \\ \Phi_{11,3}(h) + \Phi_{11,7}(h) & \Phi_{11,5}(h) \\ \Phi_{12,3}(h) + \Phi_{12,7}(h) - 1 & \Phi_{12,5}(h) \\ \Phi_{13,3}(h) + \Phi_{13,7}(h) & \Phi_{13,5}(h) \\ \Phi_{14,3}(h) + \Phi_{14,7}(h) & \Phi_{14,5}(h) - 1 \\ \Phi_{15,3}(h) + \Phi_{15,7}(h) & \Phi_{15,5}(h) \\ \Phi_{16,3}(h) + \Phi_{16,7}(h) - 1 & \Phi_{16,5}(h) \\ \Phi_{17,3}(h) + \Phi_{17,7}(h) & \Phi_{17,5}(h) \\ \Phi_{18,3}(h) + \Phi_{18,7}(h) & \Phi_{18,5}(h) \end{bmatrix} \tag{30}$$

$$N_{213}(h) = \begin{bmatrix} \Phi_{10,6}(h) + \Phi_{10,8}(h) & \Phi_{10,9}(h) \\ \Phi_{11,6}(h) + \Phi_{11,8}(h) & \Phi_{11,9}(h) \\ \Phi_{12,6}(h) + \Phi_{12,8}(h) & \Phi_{12,9}(h) \\ \Phi_{13,6}(h) + \Phi_{13,8}(h) & \Phi_{13,9}(h) \\ \Phi_{14,6}(h) + \Phi_{14,8}(h) & \Phi_{14,9}(h) \\ \Phi_{15,6}(h) + \Phi_{15,8}(h) - 1 & \Phi_{15,9}(h) \\ \Phi_{16,6}(h) + \Phi_{16,8}(h) & \Phi_{16,9}(h) \\ \Phi_{17,6}(h) + \Phi_{17,8}(h) - 1 & \Phi_{17,9}(h) \\ \Phi_{18,6}(h) + \Phi_{18,8}(h) & \Phi_{18,9}(h) - 1 \end{bmatrix} \tag{31}$$

$$M_{22}(h) = \begin{bmatrix} \Phi_{10,10}(h) & \cdots & \Phi_{10,18}(h) \\ \vdots & \ddots & \vdots \\ \Phi_{18,10}(h) & \cdots & \Phi_{18,18}(h) \end{bmatrix} \tag{32}$$

The Lyapunov matrix $U(\xi)$ we obtain from Eq. (20). The initial conditions $colU(0)$ and $colZ(0)$ are attained by solving the set of algebraic equations (25).

IV. Optimization results

We compute the value of the performance index (10) for initial function φ given by terms

$$\varphi_1(\theta) = \begin{cases} x_{01} & for\ \theta = 0 \\ 0 & for\ \theta \in [-h, 0) \end{cases} \tag{33}$$

$$\varphi_2(\theta) = \begin{cases} x_{02} & for\ \theta = 0 \\ 0 & for\ \theta \in [-h, 0) \end{cases} \tag{34}$$

$$\varphi_3(\theta) = \begin{cases} x_{03} & for\ \theta = 0 \\ 0 & for\ \theta \in [-h, 0) \end{cases} \tag{35}$$

After calculations one obtains

$$J = \begin{bmatrix} x_{01} & x_{02} & x_{03} \end{bmatrix} \begin{bmatrix} U_{11}(0) & U_{12}(0) & U_{13}(0) \\ U_{12}(0) & U_{22}(0) & U_{23}(0) \\ U_{13}(0) & U_{23}(0) & U_{33}(0) \end{bmatrix} \begin{bmatrix} x_{01} \\ x_{02} \\ x_{03} \end{bmatrix} \tag{36}$$

We search for an optimal gain p_{opt} and optimal $1/T_{i\,opt}$ which minimize the index (36). Optimization results, obtained by means of Matlab function *fminsearch*, are given in Table I and Table II. These results are obtained for $x_{01} = 1$, $x_{02} = 1$, $x_{03} = 1$ $w_1 = 1$, $w_2 = 1$, $w_3 = 1$, $k_0 = 1$, $\omega_0 = 0,1$ and for varies values of time delay h and damping factor ζ.

TABLE I. OPTIMIZATION RESULTS FOR ζ=0.9

Delay h	Optimal p	Optimal 1/Ti	Index value
0.5	0.2186	0.0062	943.6
1.0	0.1144	0.0033	2905,.1
1.5	0.0778	0.0023	5758.7
2.0	0.0592	0.0018	9360.4
2.5	0.0479	0.0015	13590

TABLE II. OPTIMIZATION RESULTS FOR ζ=1.5

Delay h	Optimal p	Optimal 1/Ti	Index value
0.5	0.3498	0.0153	289.7
1.0	0.1912	0.0080	831,8
1.5	0.1331	0.0054	1643.4
2.0	0.1029	0.0040	2712.3
2.5	0.0844	0.0032	4026.7

Fig. 1 shows the value of $x_1(t) = x(t)$ for $h = 1.5$, $\zeta = 1.5$ and optimal values of a PI-Controller parameters $p_{opt} = 0.1331$ and $1/T_{i\,opt} = 0.0054$.

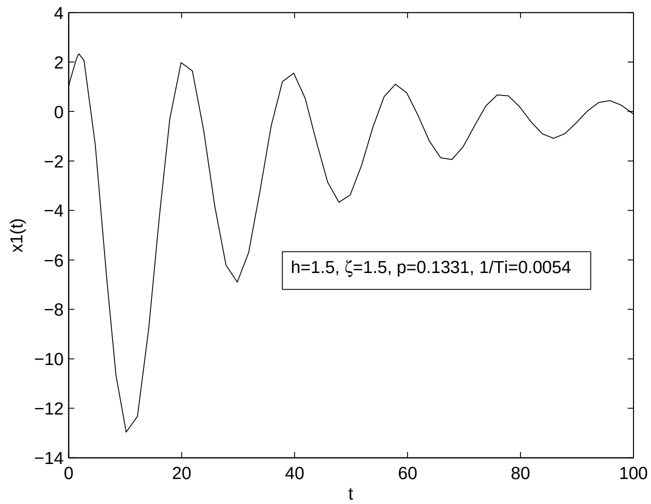

Fig.1 The value of x_1(t)

Fig. 2 shows the value of $x_2(t) = \frac{dx(t)}{dt}$ for $h = 1.5$, $\zeta = 1.5$ and optimal values of a PI-Controller parameters $p_{opt} = 0.1331$ and $1/T_{i\,opt} = 0.0054$.

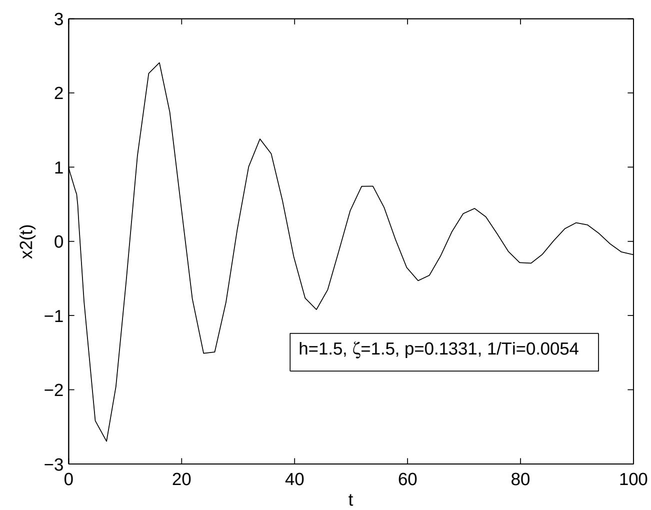

Fig. 2 The value of x_2(t)

Fig. 3 shows the value of $x_3(t)$ for $h = 1.5$, $\zeta = 1.5$ and optimal values of a PI-Controller parameters $p_{opt} = 0.1331$ and $1/T_{i\,opt} = 0.0054$.

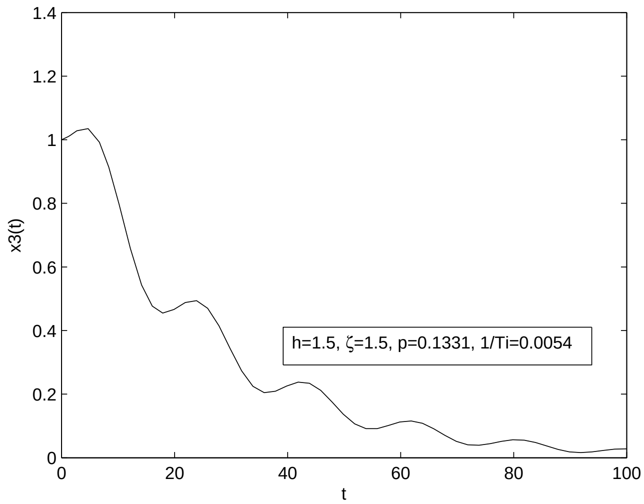

Fig. 3 The value of x_3(t)

Fig. 4 shows the value of $x_1(t) = x(t)$ for $h = 1.5$, $\zeta = 0.9$ and optimal values of a PI-Controller parameters $p_{opt} = 0.0778$ and $1/T_{i\,opt} = 0.0023$.

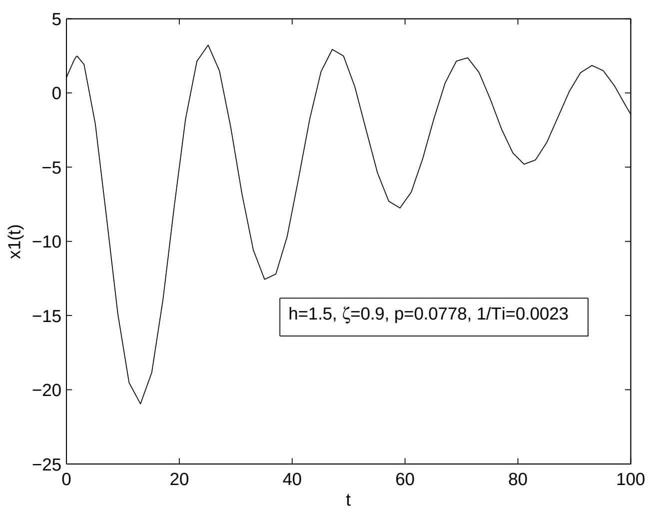

Fig. 4 The value of x_1(t)

Fig. 5 shows the value of $x_2(t) = \frac{dx(t)}{dt}$ for $h = 1.5$, $\zeta = 0.9$ and optimal values of a PI-Controller parameters $p_{opt} = 0.0778$ and $1/T_{i\,opt} = 0.0023$.

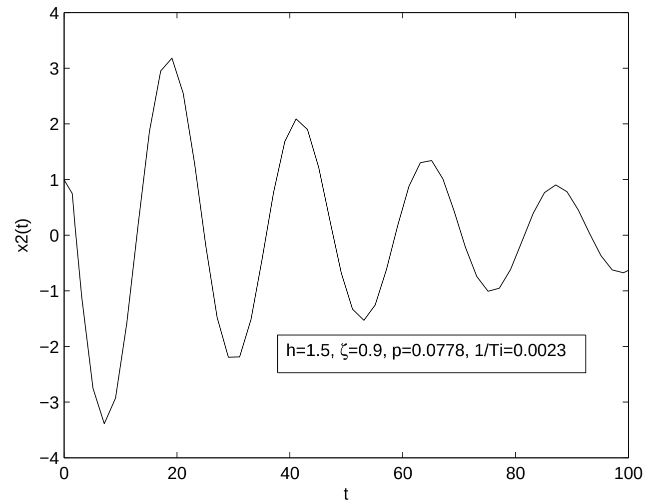

Fig. 5 The value of x_2(t)

Fig. 6 shows the value of $x_3(t)$ for $h = 1.5$, $\zeta = 0.9$ and optimal values of a PI-Controller parameters $p_{opt} = 0.0778$ and $1/T_{i\,opt} = 0.0023$.

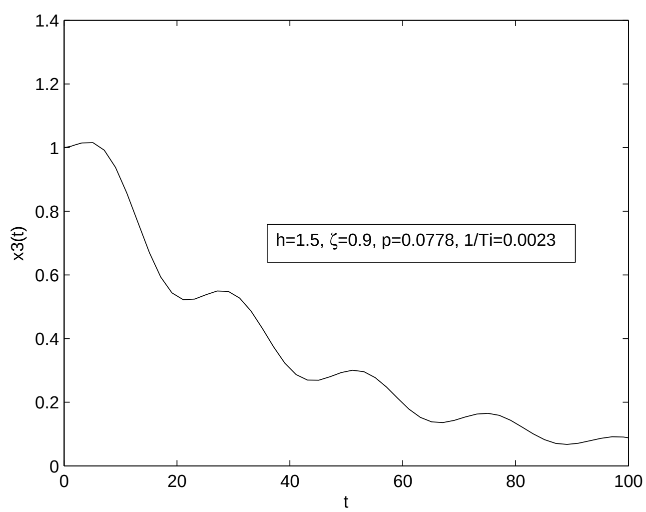

Fig. 6 The value of x_3(t)

Analyzing graphs on figures we can see oscillations with big overshoot and long setting time. The overshoot is less for greater values of damping factor. Oscillations are typical for performance index of type integral of squared error. When we compare results of that paper to results published in [6] we can see that using PI-Controller causes oscillations with greater overshoot and longer setting time than using P-Controller. The values of the index of quality in case of PI-Controller are also greater than in case of P-Controller.

V. CONCLUSIONS

In the paper the parametric optimization problem of the second order time delay system with PI-Controller was presented. The quadratic performance index was considered. The value of the index was determined by means of the Lyapunov matrix. The results of optimization were given.

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