Least Square Identification using Noise Reduction Disturbance Observer

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***Abstract*—in this article, the application of the identification algorithm of Recursive Least Squares with Forgetting Factor in conjunction with the Noise Reduction Disturbance Observer shows that the effects of noise, which affects input and output signals of the process, is reduced so that the identification process can be more effective and precise. In order to evaluate the effectiveness of this strategy the results of a case of study in which estimation of a first order process using the Noise Reduction Disturbance Observer is compare to an estimation without the Disturbance Observer**

Keywords—Identification, Least Square, Noise Reduction Disturbance Observer.

I. INTRODUCTION

It is well known that one of the most important setbacks in the identification of processes is the noise problem. In fact, due to existence of noise, which are n ormally assume random variables, led to the need to design strategies that minimize their effects resulting, in particular, in sch emes or al gorithms based on the least square minimization such as the Generalized Least Square or Recursive Least Square with Forgetting Factor (LSFF). However, these strategies assume that noise ϖ is "white", that is, u ncorrelated with $E\{\varpi\}=0$ and $E\{\varpi\varpi^T\}=\sigma^2 I$. Furthermore, as i t is show in [1], th e estimated parameters will have a d ispersion of at leas t σ . Therefore, the combined effect of noise in the measurements of the input and output signals of the process produces unacceptable dispersion or variations in th e estimated parameters. Moreover, in a r ealistic case noise is normally colored and correlated, which may induce also a polarization in the estimation. Under this condition, two strategies are usually applied: to assume a white noise filtered by a coloring filter resulting in a correlated white noise and the use of the Instrumental Variable Z, which is a two st age Least Square, [2].

Based on the setbacks induced by noise a n alternative to reduce its effects is to filter input/output signals before been used by any estimation technique. This should be without introducing additional modes or dy namics that can also alter the parameter estimation. In this context, the Noise Reduction Disturbance Observer (NR-DOB), [3-7], is an ex cellent alternative to filter the input/output signal of a process such that the effects of the noise are r educe. Although, NR-DOB was stablished for control purposes, its characteristics in estimating and reducing s ensor noise and input signal perturbations makes it a good alternative to signal filtering and hence to be applied in conjunction with any on line identification technique.

The paper is divided as follows: In Section 2, a brief description of the NR-DOB is present together with the well-known LSFF algorithm. In section 3, based on digital simulations, a case o f study, including NR-DOB design satisfying stability conditions shows the benefits of implementing NR-DOB in conjunction with LSFF. Finally, conclusions are in Section 4.

II. NR-DOB AND LSFF SUMMARY

The block diagram of Noise Reduction Disturbance Observer (NR-DOB) control system is depict in Fig. 1. Where the shadowed section represents the actual NR-DOB. $R(s)$, $\delta(s)$ and $\eta(s)$ are t he reference signal, input perturbation and sensor noise, respectively. $C(s)$, $G(s)$, $\tilde{G}(s)$, are the feedback controller, process, process model and, $F(s)$ a unity steady state gain low-pass filter, respectively. As it is common practice, at low frequency $|\eta(j\omega)|\approx 0$. Whereas at high frequency $|\eta(j\omega)|\neq 0$. This is a normal condition when $\eta(s)$ represents sensor noise. Also, it is assumed that $\delta(j\omega)\approx 0$ at high frequency $\omega>\omega_{B_F}$, where ω_{B_F} is filter $F(s)$ bandwidth.

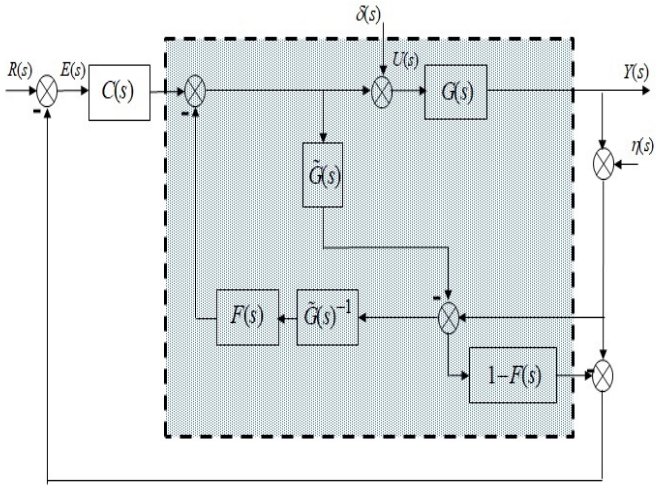

Fig. 1. NR-DOB control system

An important aspect of NR-DOB is the structure of filter $F(s)$, which is define by:

$$F(s)=\frac{1}{(\lambda s+1)^{n}} \tag{1}$$

That is, a s table unity state gain filter, where $n$is chosen such that $F(s)\tilde{G}(s)^{-1}$ is causal and $\lambda>0$. The time constant λ of $F(s)$ is tuned mainly to determine filtering conditions for sensor the noise, $\eta(s)$ and the exogenous perturbation $\delta(s)$. In addition, filter $F(s)$ satisfies the following conditions:

$$\begin{aligned} &|F(j\omega)|\approx 1,\ \ \omega\in\left[0,\omega_{B_F}\right] \\ &|F(j\omega)|\approx 0,\ \ \omega\in\left[\omega_{B_F}+\Delta_{\omega},\infty\right) \end{aligned} \tag{2}$$

The output of the closed-loop configuration of Fig. 1 results in:

$$Y(s)=T_R(s)R(s)+S_\delta(s)\delta(s)+S_\eta(s)\eta(s) \tag{3}$$

Where

$$T_R(s)=\frac{C(s)\,G(s)\,\tilde{G}(s)}{\left(1+\tilde{G}(s)C(s)\right)\left[\tilde{G}(s)+F(s)\left(G(s)-\tilde{G}(s)\right)\right]}\ ;$$

$$S_\delta(s)=\frac{G(s)\tilde{G}(s)\left(1-F(s)\right)}{\tilde{G}(s)+F(s)\left(G(s)-\tilde{G}(s)\right)} \quad \text{and} \tag{4}$$

$$S_\eta(s)=\frac{-G(s)F(s)}{\tilde{G}(s)+F(s)\left(G(s)-\tilde{G}(s)\right)}$$

Assuming denominator $\Lambda(s)=\left(1+\tilde{G}(s)C(s)\right)\left[\tilde{G}(s)+F(s)\left(G(s)-\tilde{G}(s)\right)\right]$ of equations (4) is Hurwitz and filter $F(s)$ comply with the characteristics defined by eq. (2) then, at low frequencies, $\omega\in\left[0,\omega_{B_F}\right]$, the NR-DOB control system defined by eqns. (4) satisfies:

$$|T_R(j\omega)|\approx\left|\frac{C(j\omega)\tilde{G}(j\omega)}{1+C(j\omega)\tilde{G}(j\omega)}\right|,\ |S_\delta(j\omega)|\approx 0 \ \text{ and} \\ |S_\eta(j\omega)|\approx -1 \tag{5}$$

Whereas, at high frequencies, i.e. $\omega\in\left[\omega_{B_F}+\Delta_\omega,\infty\right)$, filters magnitude reduces to $|F(j\omega)|\approx 0$; therefore, equation (4) is express as:

$$|T_R(j\omega)|\approx\left|\frac{C(j\omega)G(j\omega)}{1+C(j\omega)\tilde{G}(j\omega)}\right|,\ |S_\delta(j\omega)|\approx|G(j\omega)| \\ \text{and } |S_\eta(j\omega)|\approx 0 \tag{6}$$

From (5) and (6) equations, is clear that the effects of sensor noise $\eta(s)$, which as in dicated above is assume negligible at low frequency, can be effectively reduced at output $Y(s)$. Whereas, input perturbation $\delta(s)$ is also reduced. It must be noted that although $\delta(s)$ is not reduced at high frequency, as show n in (6), it was as sumed $\delta(j\omega)\approx 0$ when $\omega>\omega_{B_F}$. Hence, NR-DOB can reduce the effects of sensor noise and input perturbation at the input signal $\bar{U}(s)$ and output signal $\bar{Y}(s)$. In this context, LSFF will render better estimations $\hat{G}(z)$ of the discretization $G(z)$ of $G(s)$ using filtered signals $\bar{Y}(s)$ and $\bar{U}(s)$ rather than $Y(s)$ and $U(s)$, as describe in Figure (2).

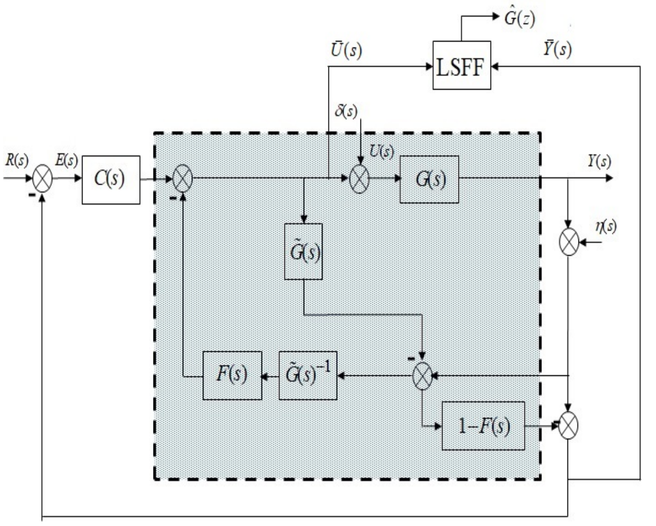

Fig. 2. NR-DOB control system with LSFF

Least square with forgetting factor (LSFF)

Let a discrete process or s ystem whose output $y(k)$ in instant k given by:

$$y(k) = \theta^T \phi(k-1) + \varpi(k) \tag{7}$$

Where, θ represents the parameters vector, $\phi(k-1)$ a vector of measurements of process input/output signals $u(k)$, $y(k)$ and, $\varpi(k)$ a perturbation.

The recursive estimation $\hat{\theta}(k)$ of θ that minimizes the quadratic error with forgetting factor criteria $J\{\hat{\theta}\}$, equation (8),

$$\begin{aligned} &J\left(\hat{\theta}(i)\right) = \sum_{i=1}^{k}\left(\prod_{j=i}^{k}\lambda(j)\right)\left(y(i)-\hat{\theta}(i)^T\phi(i-1)\right)^2; \\ &\lambda(k) = \lambda,\ \forall k \end{aligned} \tag{8}$$

results in:

$$\begin{aligned} &\hat{\theta}(k) = \hat{\theta}(k-1) + \lambda F(k+1)\phi(k-1)e(k) \\ &e(k) = y(k) - \hat{\theta}(k-1)^T\phi(k-1) \\ &\mathrm{F}(k+1) = \frac{1}{\lambda}\left(F(k) - \frac{F(k)\phi(k-1)\phi(k-1)^T F(k)}{1+\phi(k-1)^T F(k)\phi(k-1)}\right) \end{aligned} \tag{9}$$

Where $0 < \lambda \le 1$ is the forgetting factor and, $F(1) > 0$ a positive definite matrix.

The LSFF converges, that is $\hat{\theta}(k) \to \theta$ if the following conditions are satisfied, [8]

- The number of poles and zeros of $G(z)$ is known.
- $u(k)$ a persistent excitation signal.
- $u(k)$ and $y(k)$ bounded
- $\varpi(k)$ a uncorrelated white noise

III. Case of Study

Consider the following first order transfer function in the S domain.

$$\frac{Y(s)}{U(s)} = G(s) = \frac{36.9677}{(s+13.8629)} \tag{10}$$

From equations (4) - (6), the design of the NR-DOB control system for $G(s)$, which satisfies performance and stability specifications, assuming perfect match between process $G(s)$ and model process $\tilde{G}(s)$, provided $G(s)$ is stable and minimum phase, requires:

- Control $C(s)$ stabilizes process model $\tilde{G}(s)$ complying with performance specification.
- Filter $F(s)$ with sufficient roll-off to assure $F(s)G(s)^{-1}$ causal.

A simpler I controller $C(s)$ for model process $\tilde{G}(s)$ assuring adequate bandwidth and, gai n and phase margins results in:

$$C(s) = \frac{1.25}{s} \tag{12}$$

In Figure 3, B ode plots of $C(s)\tilde{G}(s)$ show a b andwidth $\omega_B = 3.49 rad/sec$ with appropriate gain and phase margins of $M_p = 75.9°$ and $M_g \to \infty dBs$, respectively. These conditions were set in ord er to obtain sufficient stability margins and a steady st ate response of approximately $t_{SS} = 4sec$. Therefore, the first requirement for performance and stability is complied.

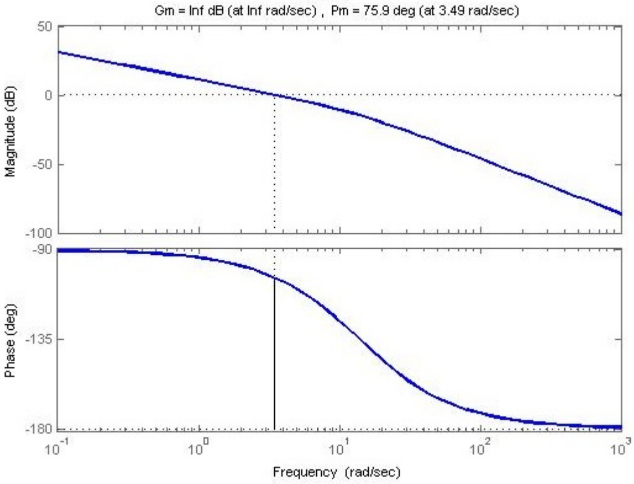

Fig. 3. Bode plots of $C(s)\tilde{G}(s)$

As model process $\tilde{G}(s)$ is first-degree filter $F(s)$ must have a degree equal or greater than two. In addition, in order to filter sensor noise at frequencies $\omega > 10 rad/sec$, $F(s)$ bandwidth is set $\omega_{B_F} = 10/rad/sec$ resulting in:

$$F(s) = \frac{1}{(0.1s+1)^2} \tag{13}$$

Time responses $\bar{y}(t)$ and $\bar{u}(t)$, Figure 4, resul ting from substituting $G(s)$, $\tilde{G}(s)$, $C(s)$ and $F(s)$ in the NR-DOB control system of Figure 2, with reference signal $R(s)$ a sequence of square pulses, input perturbation $\delta(t) = 0.5\cos(0.1t)$ and, sensor noise $\delta(t) = 0.5\cos(15t)$.

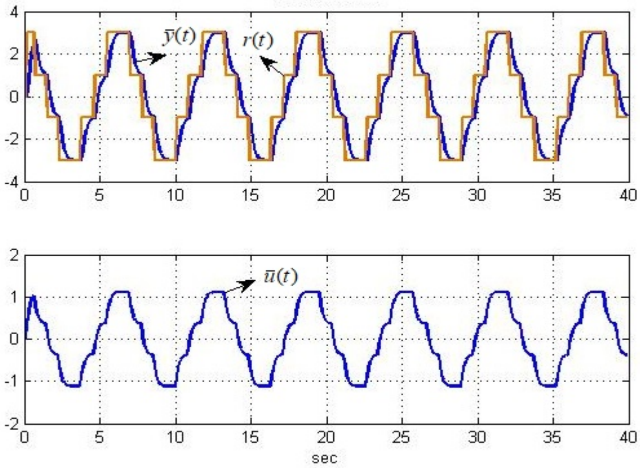

Fig. 4. Time responses of $\bar{y}(t)$ and $\bar{u}(t)$ with NR-DOB

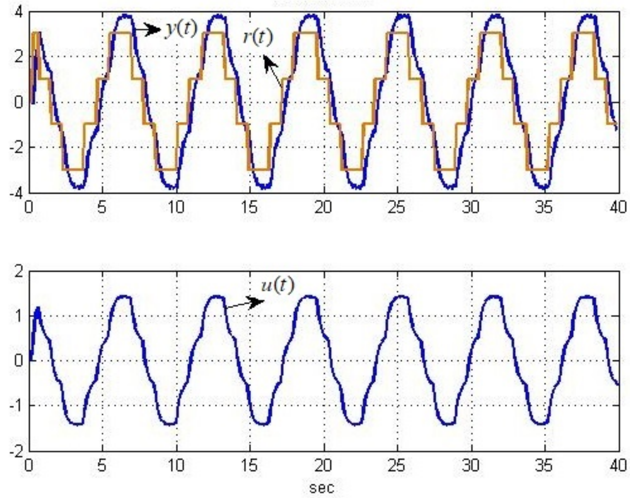

Fig. 5. Time responses of $y(t)$ and $u(t)$ without NR-DOB

On the other hand, Figure 5 shows time responses $u(t)$ and $y(t)$ of control system without NR-DOB; that is, a control system based only on controller $C(s)$ and process $G(s)$ of equations (12) and (13).

Figure 4 show s how the NR-DOB highly reduces t he perturbations in the input/output signals, compared with the responses without NR-DOB of Figure 5. Therefore, it is projected that using the filtered signals $\bar{y}(t)$ and $\bar{u}(t)$, rather than $u(t)$ and $y(t)$, will render a better estimation of process.

From equation (9), is clear that LSFF identifies the parameters of discrete models based on difference equations. In this context, discretization $G(z)$ of $G(s)$ including zero order hold with a sampling period $T = 0.1sec$ results in:

$$\frac{Y(z)}{U(z)} = G(z) = \frac{2}{(z-0.25)} \tag{11}$$

The difference equations of $G(z)$ result in:

$$y(k) = 0.25y(k-1) + 2u(k-1)$$
$$y(k) = \begin{bmatrix} 0.25, & 2 \end{bmatrix} \begin{bmatrix} y(k-1) \\ u(k-1) \end{bmatrix} \tag{12}$$

That is, parameter vector θ and regression vector $\phi(k-1)$ are given by:

$$\theta^T = [\theta_1, \quad \theta_2] = [0.25, \quad 2]$$
$$\phi(k-1) = [y(k-1), \quad u(k-1)]^T \tag{13}$$

On line samples of $\bar{y}(t)$ and $\bar{u}(t)$, Figure 2, with sampling period $T = 0.1sec$, were used in the LSFF assuming the following initial conditions:

$$F_1 = \begin{bmatrix} 40 & 0 \\ 0 & 50 \end{bmatrix}; \qquad \theta(0) = [0.1 \quad 0.1]^T \text{ and}$$

λ=0.95

Figure 6, shows the evolution of the estimated parameter vector $\hat{\theta}(k) = [\hat{\theta}_1(k) \quad \hat{\theta}_2(k)]^T$; meanwhile, Figure 7 shows the estimation of $\hat{\theta}(k)$ without NR-DOB under the same conditions; that is, with the same initial conditions, forgetting factor sampling period and, using samples of $u(t)$ and $y(t)$

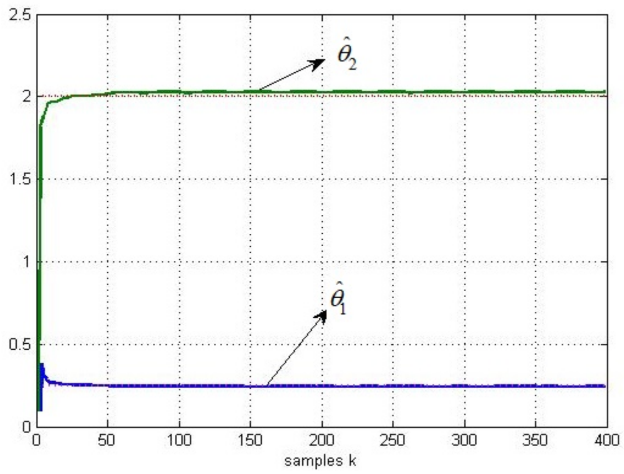

Fig. 6. Estimation of $\hat{\theta}_1$ and $\hat{\theta}_2$ with NR-DOB

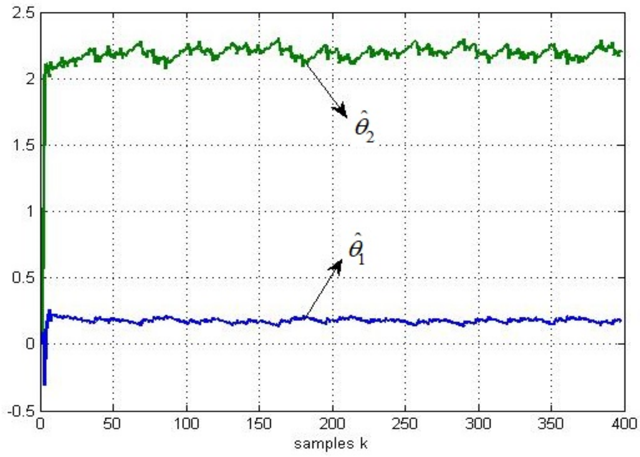

Fig. 7. Estimations of $\hat{\theta}_1$ and $\hat{\theta}_2$ without NR-DOB

Comparison between Figures 6 and 7 shows that thanks to the reduction of noise and perturbations by NR-DOB th e estimation of a process via LSFF results more effective and precise.

However, input/output signals $\bar{y}(t)$ and $\bar{u}(t)$ were obtained based on the unrealistic case of perfect match between process $G(s)$ and model process $\hat{G}(s)$ in the design of the NR-DOB control system.

In order t o assess the effects of mismatch between $G(s)$ and model process $\hat{G}(s)$, process model is modified as:

$$\tilde{G}(s) = \frac{40}{(s+10)} \tag{14}$$

That is, with an in crement in gai n and t he pole at l ower frequency rendering a reduction in the phase margin.

The resulting Bode plots of modified process model $\tilde{G}(s)$ and controller $C(s)$ of equation (12) depicted in Figure 8 shows that stability is preserved. Therefore, and assuming filter $F(s)$ as described in equation (13), the NR-DOB control system is stable.

Figure 9 s hows NR-DOB control system responses $\bar{y}(t)$, $\bar{u}(t)$ and $r(t)$, using the modified model process of equation (14). These responses show a small performance degradation due to the reduction of phase margin $M_p = 67.1°$. These, responses are due also because NR-DOB is in fact an Internal Model Control (IMC) control system were output $\bar{y}(t)$ is driven, at frequencies $\omega \in [0, \omega_{B_F}]$, by model process $\tilde{G}(s)$ as indicated by transfer function $T_R(s)$ in equation (5).

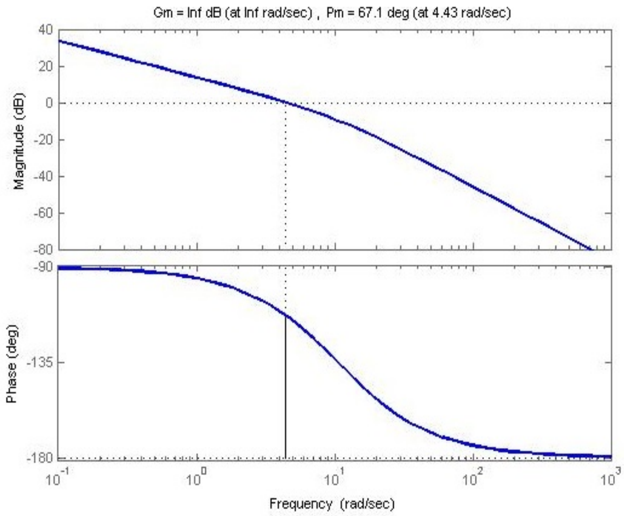

Fig. 8. Bode plots of $C(s)\tilde{G}(s)$ with modified $\tilde{G}(s)$

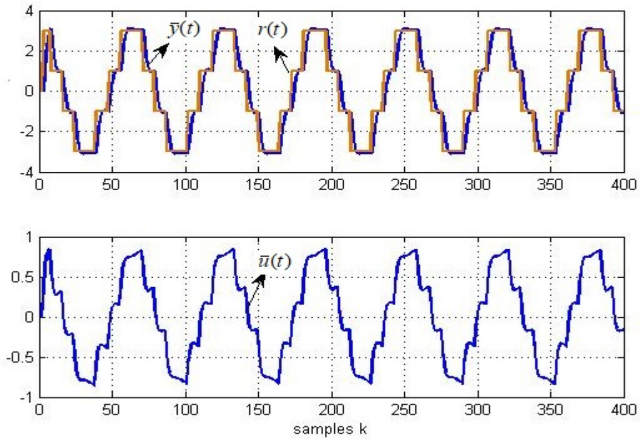

Fig. 9. NR-DOB $\bar{y}(t)$ and $\bar{u}(t)$ time responses with modified $\tilde{G}(s)$

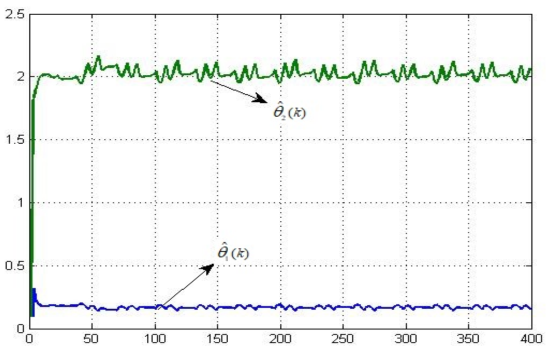

Fig. 10. Estimations of $\hat{\theta}_1$ and $\hat{\theta}_2$ with modified $\tilde{G}(s)$

Setting conditions for the LSFF identical to p revious experiments, estimated parameters $\hat{\theta}(k) = \left[\hat{\theta}_1(k) \quad \hat{\theta}_2(k)\right]^T$, sing the modified model process $\tilde{G}(s)$, are show in Figure 10.

As expected, Figure 10 shows a det erioration in the estimation of parameter vector $\hat{\theta}(k)$ due to th e reduction of NR-DOB filtering capabilities by mismatch between process and model process. Nevertheless, $\hat{\theta}(k)$ estimation is still more accurate than in the case without NR-DOB.

IV. Conclusions

Noise or perturbations at p rocesses input/output signals introduce problems in the identification of processes, especially when the perturbations do n ot comply with been "withe noise" and if its variance is high. Although several approaches can be used to reduce these problems (Maximum Likelihood or Instrumental Variable) implementing a filtering scheme that reduces noise or disturbances without introducing additional modes could improve parameter estimation, regardless of the identification strategy used. In this articl e, the proposed filtering strategy is the Noise Reduc tion Disturbance Observer (NR-DOB) which has proven to be an excellent control strategy capable of reduc ing the effects of noise and dist urbances. This characteristic is use to f ilter process input/output signals so th at the i dentification is less affected. Through a case study, based on the identification of a first order sys tem, the u sed of the NR-DOB and the Least Square with Forgetting Factor identification algorithm, it is shown that the use of NR-DOB helps to gen erate a more accurate and effective process identification.

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