INTERNATIONAL JOURNAL OF MATHEMATICS AND COMPUTERS IN SIMULATION
DOI: 10.46300/9102.2021.15.23 Volume 15, 2021

PID Controller design for multiple time delays system

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Received: August 5, 2019. Revised: August 22, 2021. Accepted: September 27, 2021. Published: November 27, 2021.

Abstract— This paper presents an approach of stabilization and control of time invariant linear system of an arbitrary order that include several time delays. In this work, the stability is ensured by PI, PD and PID controller. The method is analytical and needs the knowledge of transfer function parameters of the plant. It permits to find stability region by the determination of K_p, K_i and K_d gains.

***Keywords*— PID Controller, multiple delay systems, stability region**

I. Introduction

Time delay systems are often encountered in various engineering systems such as electrical and communication network, chemical process, turbojet engine, nuclear reactor, hydraulic system; it is frequently a source of instability, oscillation and poor performance in many dynamic systems. Furthermore, delay makes system analysis and control design much more complex [7], [17].

The PID controller is widely applied in control engineering applications for many industries. The choice of the PID controller parameters leads to obtain a closed loop stable system.

Many researches have been applied the PID controller to different classes of dynamical systems [1]. Among these the particular class of time delay system has been investigated by means of several methods [8], [14], [15], [16], of which the Nyquist criterion, a generalization of the Hermite-Biehler Theorem, and the root location method. The main objective to design the PID controller is to ensure closed loop stability. Indeed, by using the Hermit-Biehler theorem applicable to the quasi-polynomials [9], [10], [11], a characterization of all values of the PI/PID stabilization gains for stable first order delay system is addressed. However, these results are not applicable to the second order delay system. In [2], [3], the stabilizing problem of PI/PID controller for second order delay system is analysed and then used to obtain all PI and PID gains that stabilize an interval first and second order delay system [4], [5].

The design methods of PID controllers can be analytically determined in the case of knowledge of the transfer function parameters or numerically in the case of the knowledge of the delay system frequency response [6], [12], [13].

Since these methods have been developed mainly for the case of a single system delay, the contribution of our work concerns the stabilization of system with several time delays by using the PI, PD and PID controllers. The proposed approach is based on the extension of the analytical method developed in [6], [12], [13].

The considered feedback structure is depicted in Fig. 1 and the related transfer functions of the process $G(s)$ and the controller $C(s)$ are given by:

$$G(s)=\sum_{i=1}^{N} G_i(s)e^{-\tau_i s} \tag{1}$$

$$C(s)=K_p+\frac{K_i}{s}+K_d s \tag{2}$$

where N is the number of delays, τ_i is the time delay and G_i is a continuous linear system of any order, K_p, K_i and K_d are the PID parameters.

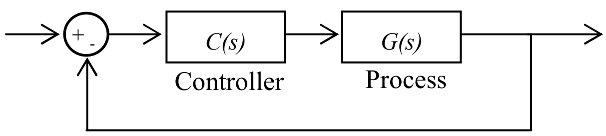

Figure 1: The closed-loop system with controller

The region of stability is found in the (K_p, K_i) plane for PI controller, (K_p, K_d) plane for PD controller and (K_p, K_i, K_d) space for PID controller. This approach has been applied to continuous time linear systems of any order, with multiple time delays.

The paper is organized as follows: In part II stabilization approach with PI controller is presented, similarly stabilization approach with PD controller is developed in section III. The case of PID controller is discussed in section IV. Finally, simulation results are given in section V.

II. Stabilization several time delays system using PI Controller

The considered plant is a continuous linear time-invariant system of any order that contains several time delays. It is described by its transfer function given by (1).

In this section, the stabilization of the plant is assured by the PI controller designed as follows:

$$C(s) = K_p + \frac{K_i}{s} = \frac{K_p s + K_i}{s} \tag{3}$$

The proposed method leads to an efficient calculation of the proportional and integral gains K_p and K_i achieving stability.

Let's note Δ(s) the closed loop characteristic polynomial of the process shown in Fig. 1.

In frequency domain, the characteristic polynomial is defined by:

$$\Delta(j\omega) = 1 + C(j\omega)G(j\omega) = R_\Delta(\omega) + jI_\Delta(\omega) \tag{4}$$

the transfer function can be then written as:

$$\begin{aligned} G(j\omega) &= \sum_{i=1}^{N} R_i(\omega) + jI_i(\omega) \\ &= \sum_{i=1}^{N} R_i(\omega) + j\sum_{i=1}^{N} I_i(\omega) \end{aligned} \tag{5}$$

where R_Δ and I_Δ are the real and the imaginary parts of the characteristic polynomial, respectively. R_i and I_i are the real and the imaginary parts of the transfer function $G_i(j\omega)$.

The stability region is determined when $\Delta(j\omega)$ is equal to zero:

$$\Delta(j\omega) = 1 + (K_p - j\frac{K_i}{\omega})(\sum_{i=1}^{N} R_i(\omega) + j\sum_{i=1}^{N} I_i(\omega)) = R_\Delta(\omega) + jI_\Delta(\omega) = 0 \tag{6}$$

According to equations (4) and (5), the following results are obtained:

$$\begin{cases} R_\Delta(\omega) = 1 + K_p \sum_{i=1}^{N} R_i(\omega) + \frac{K_i}{\omega} \sum_{i=1}^{N} I_i(\omega) \\ I_\Delta(\omega) = K_p \sum_{i=1}^{N} I_i(\omega) - \frac{K_i}{\omega} \sum_{i=1}^{N} R_i(\omega) \end{cases} \tag{7}$$

Applying the real part and the imaginary part equal to zero leads to the following equations:

$$\begin{cases} K_p \sum_{i=1}^{N} R_i(\omega) + \frac{K_i}{\omega} \sum_{i=1}^{N} I_i(\omega) = -1 \\ K_p \sum_{i=1}^{N} I_i(\omega) - \frac{K_i}{\omega} \sum_{i=1}^{N} R_i(\omega) = 0 \end{cases} \tag{8}$$

Similar,

$$\begin{cases} \omega K_p \sum_{i=1}^{N} R_i(\omega) + K_i \sum_{i=1}^{N} I_i(\omega) = -\omega \\ \omega K_p \sum_{i=1}^{N} I_i(\omega) - K_i \sum_{i=1}^{N} R_i(\omega) = 0 \end{cases} \tag{9}$$

The K_p and K_i parameters are determined by solving the following system to ensure closed loop stability

$$\begin{bmatrix} \omega\sum_{i=1}^{N} R_i(\omega) & \sum_{i=1}^{N} I_i(\omega) \\ \omega\sum_{i=1}^{N} I_i(\omega) & -\sum_{i=1}^{N} R_i(\omega) \end{bmatrix} \begin{bmatrix} K_p \\ K_i \end{bmatrix} = \begin{bmatrix} -\omega \\ 0 \end{bmatrix} \tag{10}$$

The obtained expressions of K_p and K_i are:

$$\begin{cases} K_p(\omega) = -\frac{\sum_{i=1}^{N} R_i(\omega)}{|G(j\omega)|^2} \\ K_i(\omega) = -\omega \frac{\sum_{i=1}^{N} I_i(\omega)}{|G(j\omega)|^2} \end{cases} \tag{11}$$

where

$$|G(j\omega)|^2 = \left(\sum_{i=1}^{N} R_i(\omega)\right)^2 + \left(\sum_{i=1}^{N} I_i(\omega)\right)^2 \tag{12}$$

III. Stabilization several time delays system using PD Controller

In this section, the same plant (1) is stabilized with a PD controller as shown in Fig. 1.

The transfer function of this controller is given by:

$$C(s) = K_p + K_d s \tag{13}$$

To obtain stability region in terms of proportional and derivative gains K_p and K_d, the previous approach is applied in the case of several time delay system with PD controller

The closed loop characteristic polynomial $\Delta(s)$ is written as:

$$\Delta(j\omega) = 1 + (K_p + jK_d\omega)(\sum_{i=1}^{N} R_i(\omega) + j\sum_{i=1}^{N} I_i(\omega)) = R_\Delta(\omega) + jI_\Delta(\omega) \tag{14}$$

by setting $\Delta(j\omega)$ equal to zero:

$$\Delta(j\omega) = R_\Delta(\omega) + jI_\Delta(\omega) = 0 \tag{15}$$

where:

$$\begin{cases} R_\Delta(\omega) = 1 + K_p \sum_{i=1}^{N} R_i(\omega) - K_d \omega \sum_{i=1}^{N} I_i(\omega) \\ I_\Delta(\omega) = K_p \sum_{i=1}^{N} I_i(\omega) + K_d \omega \sum_{i=1}^{N} R_i(\omega) \end{cases} \tag{16}$$

the real part and the imaginary part equal to zero lead to the following equations:

$$\begin{cases} K_p \sum_{i=1}^{N} R_i(\omega) - K_d \omega \sum_{i=1}^{N} I_i(\omega) = -1 \\ K_p \sum_{i=1}^{N} I_i(\omega) + K_d \omega \sum_{i=1}^{N} R_i(\omega) = 0 \end{cases} \tag{17}$$

Equivalently the system can be written as:

$$\begin{bmatrix} \sum_{i=1}^{N} R_i(\omega) & -\omega \sum_{i=1}^{N} I_i(\omega) \\ \sum_{i=1}^{N} I_i(\omega) & \omega \sum_{i=1}^{N} R_i(\omega) \end{bmatrix} \begin{bmatrix} K_p \\ K_d \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \tag{18}$$

Solving (18), the results are as follows:

$$\begin{cases} K_p(\omega) = -\dfrac{\sum_{i=1}^{N} R_i(\omega)}{|G(j\omega)|^2} \\ K_d(\omega) = \dfrac{\sum_{i=1}^{N} I_i(\omega)}{\omega |G(j\omega)|^2} \end{cases} \tag{19}$$

where $|G(j\omega)|^2$ is given by (12).

IV. STABILIZATION SEVERAL TIME DELAYS SYSTEM USING PID CONTROLLER

Considering the same system (1) shown in Fig. 1, we attempt to achieve stabilization with PID controller presented by:

$$C(s) = K_p + \frac{K_i}{s} + K_d s = \frac{K_p s + K_i + K_d s^2}{s} \tag{20}$$

The same approach is applied in the case of several time delay system with PID controller.

The closed loop characteristic polynomial Δ(s) is written as:

$$\Delta(j\omega) = 1 - \left(\frac{-K_p\omega + j(K_i - K_d\omega^2)}{\omega}\right)\left(\sum_{i=1}^{N} R_i(\omega) + j\sum_{i=1}^{N} I_i(\omega)\right) = R_\Delta(\omega) + jI_\Delta(\omega) \tag{21}$$

The stabilization region is determined by setting $\Delta(j\omega)$ to zero:

$$\Delta(j\omega) = R_\Delta(\omega) + jI_\Delta(\omega) = 0 \tag{22}$$

where:

$$\begin{cases} R_\Delta(\omega) = \omega + K_p \omega \sum_{i=1}^{N} R_i(\omega) - K_d \omega^2 \sum_{i=1}^{N} I_i(\omega) + K_i \sum_{i=1}^{N} I_i(\omega) \\ I_\Delta(\omega) = K_p \omega \sum_{i=1}^{N} I_i(\omega) + K_d \omega^2 \sum_{i=1}^{N} R_i(\omega) - K_i \sum_{i=1}^{N} R_i(\omega) \end{cases} \tag{23}$$

Real part and the imaginary part are setting to zero to obtain equation system of three unknown variable

$$\begin{cases} K_p \omega \sum_{i=1}^{N} R_i(\omega) + K_i \sum_{i=1}^{N} I_i(\omega) = -\omega + K_d \omega^2 \sum_{i=1}^{N} I_i(\omega) \\ K_p \omega \sum_{i=1}^{N} I_i(\omega) - K_i \sum_{i=1}^{N} R_i(\omega) = -K_d \omega^2 \sum_{i=1}^{N} I_i(\omega) \end{cases} \tag{24}$$

In the first step, the K_d parameter is fixed. The (K_p, K_i) plane is then determined by solving the following system:

$$\begin{bmatrix} \omega \sum_{i=1}^{N} R_i(\omega) & \sum_{i=1}^{N} I_i(\omega) \\ \omega \sum_{i=1}^{N} I_i(\omega) & -\sum_{i=1}^{N} R_i(\omega) \end{bmatrix} \begin{bmatrix} K_p \\ K_i \end{bmatrix} = \begin{bmatrix} -\omega + K_d \omega^2 \sum_{i=1}^{N} I_i(\omega) \\ -K_d \omega^2 \sum_{i=1}^{N} R_i(\omega) \end{bmatrix} \tag{25}$$

leading to the K_p and K_i expressions:

$$\begin{cases} K_p(\omega) = -\dfrac{\sum_{i=1}^{N} R_i(\omega)}{|G(j\omega)|^2} \\ K_i(\omega) = \omega^2 K_d - \omega \dfrac{\sum_{i=1}^{N} I_i(\omega)}{|G(j\omega)|^2} \end{cases} \tag{26}$$

where $|G(j\omega)|^2$ is given by (12).

In the second step, the K_i parameter is now fixed. The (K_p, K_d) plane is then determined by solving the following system:

$$\begin{bmatrix} -\omega \sum_{i=1}^{N} R_i(\omega) & \omega^2 \sum_{i=1}^{N} I_i(\omega) \\ \omega \sum_{i=1}^{N} I_i(\omega) & \omega^2 \sum_{i=1}^{N} R_i(\omega) \end{bmatrix} \begin{bmatrix} K_p \\ K_d \end{bmatrix} = \begin{bmatrix} \omega + K_i \sum_{i=1}^{N} I_i(\omega) \\ K_i \sum_{i=1}^{N} R_i(\omega) \end{bmatrix} \tag{27}$$

K_p is given by the same equations (26) and:

$$K_d(\omega) = \frac{K_i}{\omega^2} + \frac{\sum_{i=1}^{N} I_i(\omega)}{\omega |G(j\omega)|^2} \tag{28}$$

V. SIMULATION RESULTS

The proposed approach is illustrated on a linear system defined by two parallel subsystems, having two different time delays. The first subsystem is of order one and the second is of order three given by:

$$G(s) = G_1(s)e^{-\tau_1 s} + G_2(s)e^{-\tau_2 s} \tag{29}$$

where

$$G_1(s) = \frac{K_1}{1 + a_1 s}, \quad G_2(s) = \frac{K_2 + K_3 s}{1 + b_1 s + b_2 s^2 + b_3 s^3} \tag{30}$$

We suppose that: $\begin{cases} K_1 = 0.5 \\ a_1 = 2 \\ \tau_1 = 1.5 \end{cases}$ and $\begin{cases} K_2 = 1 \\ K_3 = -0.5 \\ b_1 = 1 \\ b_2 = 3 \\ b_3 = 2 \\ \tau_2 = 0.6 \end{cases}$

noting:

$$G(j\omega) = G_1(j\omega)\left[\cos(\tau_1\omega) - j\sin(\tau_1\omega)\right] + G_2(j\omega)\left[\cos(\tau_2\omega) - j\sin(\tau_2\omega)\right] \quad (31)$$

Developing (31), the results are as follows:

$$\begin{cases} R_1(\omega) = \dfrac{1}{1+A_1(\omega)^2}[K_1\cos(\tau_1\omega) - K_1A_1(\omega)\sin(\tau_1\omega)] \\ I_1(\omega) = -\dfrac{1}{1+A_1(\omega)^2}[K_1\sin(\tau_1\omega) + K_1A_1(\omega)\cos(\tau_1\omega)] \end{cases} \quad (32)$$

where

$$A_1(\omega) = a_1\omega$$

and

$$\begin{cases} R_2(\omega) = \dfrac{1}{B_2(\omega)^2 + B_1(\omega)^2}[X_1(\omega)\cos(\tau_2 w) + X_2(\omega)\sin(\tau_2 w)] \\ I_2(\omega) = \dfrac{1}{B_2(\omega)^2 + B_1(\omega)^2}[X_2(\omega)\cos(\tau_2 w) - X_2(\omega)\sin(\tau_2 w)] \end{cases} \quad (33)$$

where

$$\begin{cases} B_1(\omega) = b_1\omega - b_3\omega^3 \\ B_2(\omega) = 1 - b_2\omega^2 \\ X_1(\omega) = K_2B_2(\omega) + \omega K_3B_1(\omega) \\ X_2(\omega) = \omega K_3B_2(\omega) - K_2B_1(\omega) \end{cases}$$

R_1, I_1 are the real and imaginary parts of G_1, respectively

R_2, I_2 are the real and imaginary parts of G_2, respectively

A. Stabilization with PI controller

K_p and K_i parameters are given by:

$$\begin{cases} K_p(\omega) = -\dfrac{R_1(\omega) + R_2(\omega)}{(R_1(\omega) + R_2(\omega))^2 + (I_1(\omega) + I_2(\omega))^2} \\ K_i(\omega) = -\omega\dfrac{I_1(\omega) + I_2(\omega)}{(R_1(\omega) + R_2(\omega))^2 + (I_1(\omega) + I_2(\omega))^2} \end{cases} \quad (34)$$

Substituting (32) and (33) into (34) leads to determine stability region in (K_p, K_i) plane shown in the following figure:

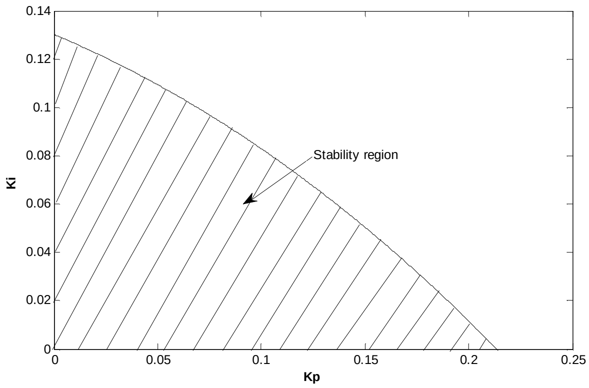

Figure 2: Stability region with PI Controller

In (K_p, K_i) plane, the curve described by values of K_p and K_i given by equations (34) bounds a zone that represents stability region, which is displayed as shaded in Fig. 2.

Closed loop responses of this system are represented by the following figures:

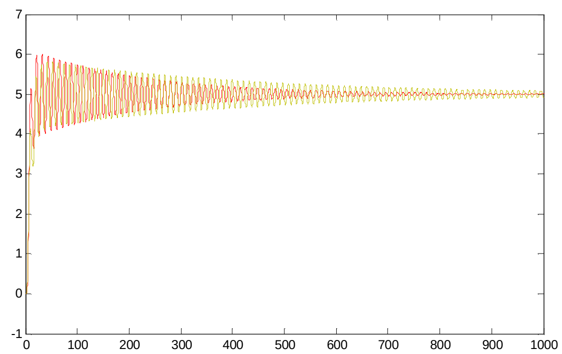

Figure 3: Closed loop step responses with PI Controller (Kp= 0.05, Ki=0.1: red response; Kp= 0.1, Ki=0.077: green response): stable case

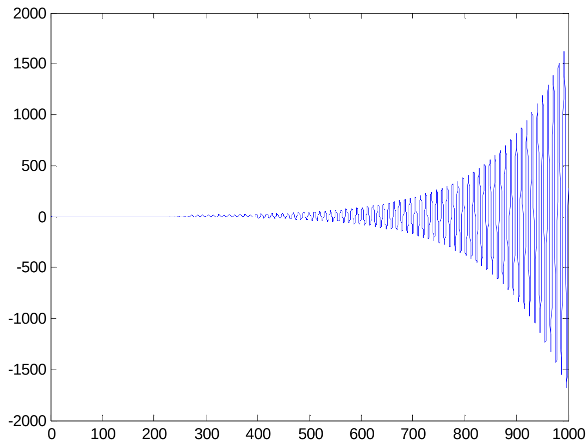

Figure 4: Closed loop step response with PI Controller (Kp= 0.2, Ki=0.04): unstable case

The parameters of the PI controller are altered according to their belonging to stability region.

Starting with K_p = 0.05, K_i = 0.1, the closed loop response system is described by the red response shown in Fig. 3.

Corresponding response for these gains displays that the system is stable in the shaded zone.

For K_p = 0.1, K_i = 0.077, these proportional and integral gains define a point belonging to the border of the shaded zone. Corresponding closed loop behavior system is shown by the green response in Fig. 3. In the transient regime, the system presents oscillations and it is stabilized in the steady state which represents the limit of the stability.

Finally, K_p and K_i parameters are chosen so that they define a point out of the region of stability. For K_p = 0.2, K_i = 0.04, the closed loop response system is presented in Fig. 4. The result shows that in the not shaded region, the system becomes unstable.

B. *Stabilization with PD controller*

The same approach is applied in the case of stabilization with PD controller.

The K_p and K_d expressions are given by:

$$\begin{cases} K_p(\omega) = -\dfrac{R_1(\omega)+R_2(\omega)}{(R_1(\omega)+R_2(\omega))^2+(I_1(\omega)+I_2(\omega))^2} \\ K_d(\omega) = \dfrac{I_1(\omega)+I_2(\omega)}{\omega((R_1(\omega)+R_2(\omega))^2+(I_1(\omega)+I_2(\omega))^2)} \end{cases} \tag{35}$$

Substituting (32) and (33) into (35) leads to obtain K_p and K_d values.

As seen previously, the stability region which is shaded in Fig. 5 is bounded by the curve defined in (K_p, K_d) plane.

Closed loop responses of this system for different K_p and K_d values are represented by the figures Fig. 6, Fig. 7.

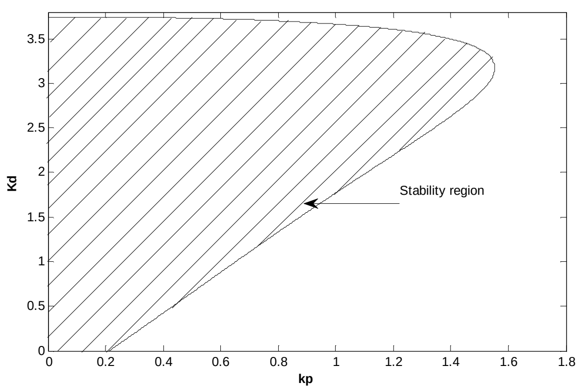

Figure 5: Stability region with PD Controller

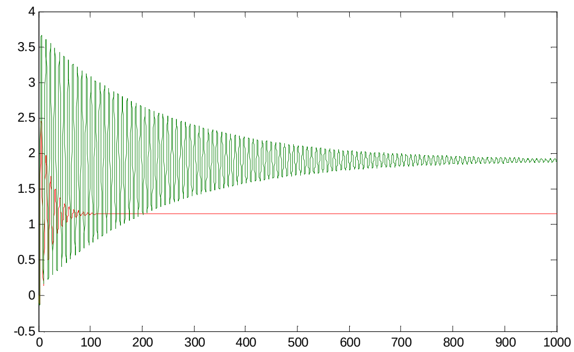

Figure 6: Closed loop step response with PD Controller (Kp= 0.2, Kd=0.5: red response; Kp= 0.41, Kd=0.5: green response): stable case

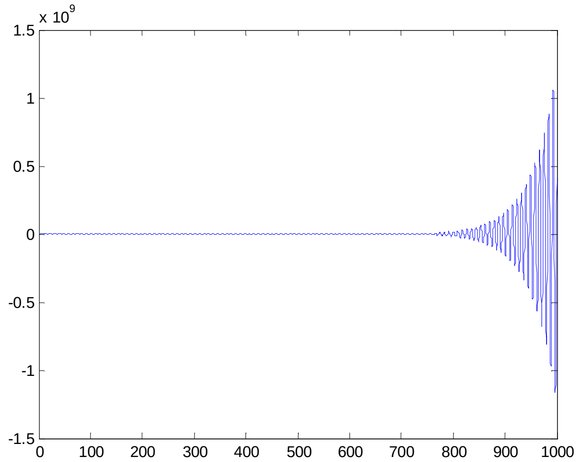

Figure 7: Closed loop response step with PD Controller: (Kp= 0.5, Kd=0. 4): unstable case

For K_p = 0.2, K_d =0.5, the closed loop system is stable.

In the case of K_p = 0.41, K_d =0.5, the system is on the limit of the stability in the closed loop.

Finally, for K_p = 0.5, K_d =0.4, the system is unstable in the closed loop.

The obtained results show the effectiveness of the proposed approach.

C. *Stabilization with PID controller*

Considering the previous approach, two cases are presented.

In the first step, the stability region can be determined in the (K_p, K_i) plane with fixed K_d. The results are as follows

DOI: 10.46300/9102.2021.15.23 Volume 15, 2021

$$\begin{cases} K_p(\omega) = -\dfrac{R_1(\omega)+R_2(\omega)}{(R_1(\omega)+R_2(\omega))^2+(I_1(\omega)+I_2(\omega))^2} \\ K_i(\omega) = \omega^2 K_d - \omega\dfrac{I_1(\omega)+I_2(\omega)}{(R_1(\omega)+R_2(\omega))^2+(I_1(\omega)+I_2(\omega))^2} \end{cases} \quad (36)$$

In the second step, the stability region can be determined in the (K_p, K_d) plane with fixed K_i . The results obtained are as follow:

$$\begin{cases} K_p(\omega) = -\dfrac{R_1(\omega)+R_2(\omega)}{(R_1(\omega)+R_2(\omega))^2+(I_1(\omega)+I_2(\omega))^2} \\ K_d(\omega) = \dfrac{K_i}{\omega^2} + \dfrac{I_1(\omega)+I_2(\omega)}{\omega((R_1(\omega)+R_2(\omega))^2+(I_1(\omega)+I_2(\omega))^2)} \end{cases} \quad (37)$$

Substituting (32) and (33) into (36) and (37) leads to obtain K_p, K_i and K_d values.

For each value of K_i, a stability region is defined in the (K_p, K_d) plane. A three dimensional curve is then obtained by varying K_i as shown in Fig. 8.

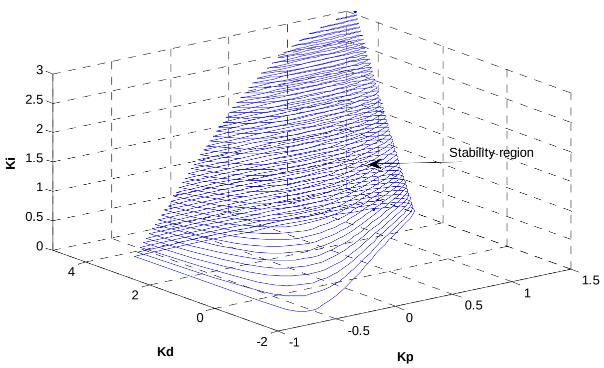

Figure 8: Stability region with PID Controller

The figures Fig. 9, Fig. 10 and Fig. 11 show the closed loop responses of the system for different chosen PID parameters.

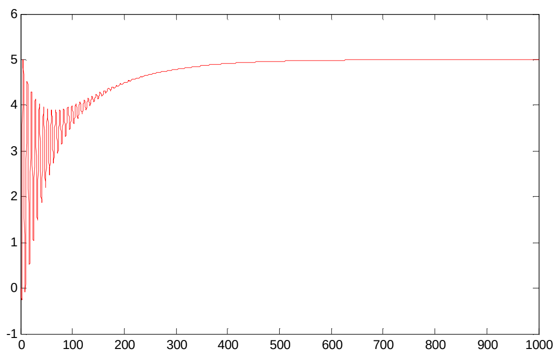

Figure 9: Closed loop step response with PID Controller (Kp= 1, Ki=2.085, Kd=4.2): stable case

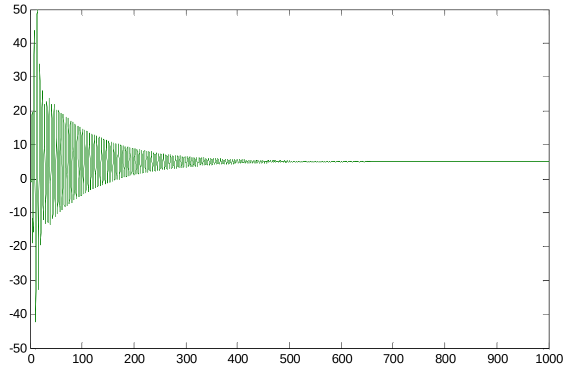

Figure 10: Closed loop step response with PID Controller (Kp= 0.6, Ki=2.085, Kd=4.2): stable case

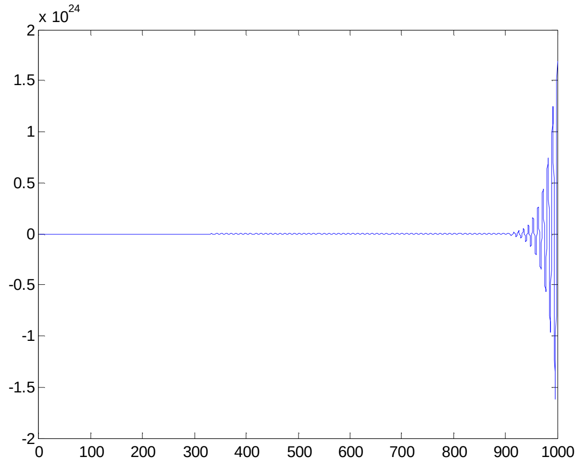

Figure 11: Closed loop step response with PID Controller (Kp= 0.5, Ki=0.05, Kd=0.1): unstable case

For K_p =1, K_i = 2.085 and K_d =4.2, the closed loop system is stable.

In the case of K_p =0.6, K_i = 2.085 and K_d =4.2, the system is on the limit of the stability in the closed loop.

Finally, for K_p =0.5, K_i = 0.05 and K_d =0.1, the system is unstable in the closed loop.

VI. Conclusions

The main contribution of this paper concerns the stabilization of continuous linear time invariant system of any order and which presents several delays using a PID controller. The proposed approach is based on mathematical calculation of the proportional, derivative and integral gains by extracting the real and imaginary parts of the system transfer function. This not complicated method leads to the determination of the stability regions in (K_p , K_i) plane for PI controller, (K_p, K_d) plane for PD controller and (K_p, K_i, K_d) space for PID controller.

The simulation results point out the correspondence between the time domain responses and the obtained stability regions. This proposed method is then efficient to stabilize system with any order and several time delays.

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