Adaptive LQ Control of a Shell and Tube Heat Exchanger

P. Dostal, J. Vojtesek, and V. Bobal

***Abstract*—** The paper deals with adaptive control of a shell and tube heat exchanger. Two different control inputs are considered. The procedure of the control design is based on approximation of a nonlinear model of the process by continuous-time external linear models with parameters recursively estimated via corresponding external delta models. The structures of approximating linear models are chosen on the basis of primary steady-state and dynamic analysis of the process. The control system configuration with two feedback controllers is used. The control laws for both control inputs are derived using the polynomial approach and the LQ control technique. The proposed method is verified by control simulations.

***Keywords*—** Delta model, external linear model, heat exchanger, LQ control, polynomial approach.

I. Introduction

Heat exchangers are equipments widely used in space heating, refrigeration, air conditioning, power plants, chemical plants, polymer manufacturing, petrochemical plants, petroleum refineries, natural gas processing, and some others. There various types of heat exchangers exist. Among them, shell and tube heat exchangers (STHEs) are most common types of heat exchangers.

From the system engineering point of view, shell and tube exchangers belong to a class of nonlinear distributed parameter systems. Their mathematical models are described by nonlinear partial differential equations (PDEs). Procedures of modelling of distributed parameter systems are described e.g. in [1] – [4]. Nonlinearities in STHEs models are caused by products among their state and input variables.

It is well known that these systems often are hardly controllable by conventional control methods, and, its effective control requires application some of advanced methods as in [5].
Obviously, the design of the control must be based on a preliminary static and dynamic analysis of the controlled process by simulation methods. Some methods of numerical mathematics used to build simulation models can be found e.g. in [6] – [8].

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One possible approach to achieve efficient control algorithm is using adaptive strategies based on an appropriate choice of a continuous-time external linear model (CT ELM) with recursively estimated parameters. These parameters are consequently used for parallel updating of controller‘s parameters.

For the CT ELM parameters estimation, either the direct method, see e.g. [9] and [10] or application of an external delta
model with the same structure as the CT model can be used. The basics of delta models have been described e.g. in [11] - [13]. Although delta models belong into discrete models, they do not have such disadvantageous properties connected with shortening of a sampling period as discrete z-models. In addition, parameters of delta models can directly be estimated from sampled signals. It can be easily proved that these parameters converge to parameters of CT models for a sufficiently small sampling period (compared to the dynamics of the controlled process). Complete description and experimental verification can be found in [13].

The paper deals with continuous-time adaptive control of the STHE. Two different control inputs and one controlled output are considered. The parameters of corresponding CT ELMs are obtained via corresponding delta models parameter estimation. The control system structure with two feedback controllers is used, see, e.g. [14] and [15]. The resulting controllers are derived using polynomial method, eg. [16] and [17], and the LQ control technique, as in [18] and [19]. The approach is tested on a nonlinear model of the STHE.

II. Model of the STHE

Consider an ideal plug-flow shell and tube heat exchanger in the fluid phase and with the counterflow cooling. The fluid flowing in tubes is cooled by the fluid flowing in the shell as shown in Fig. 1.

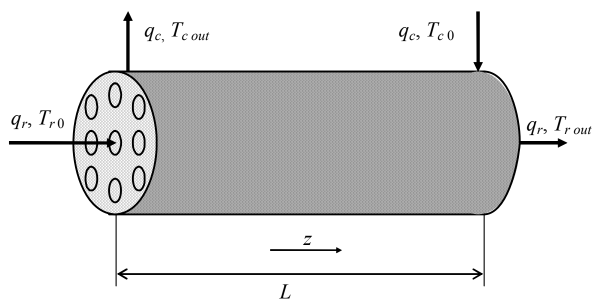

Fig. 1 Shell and tube heat exchanger.

Heat losses and heat conduction along the metal walls of tubes are assumed to be negligible, but dynamics of the metal walls of tubes are significant. All densities, heat capacities, and heat transfer coefficients are assumed to be constant. Under above assumptions, the STHE model can be described by three partial differential equations (PDEs) in the form

$$\frac{\partial T_r}{\partial t} + v_r \frac{\partial T_r}{\partial z} = c_1 (T_s - T_r) \tag{1}$$

$$\frac{\partial T_w}{\partial t} = c_2 (T_r - T_w) + c_3 (T_c - T_w) \tag{2}$$

$$\frac{\partial T_c}{\partial t} - v_c \frac{\partial T_c}{\partial z} = c_4 (T_s - T_c) \tag{3}$$

with initial conditions

$T_r(z,0) = T_r^s(z)$, $T_w(z,0) = T_w^s(z)$, $T_c(z,0) = T_c^s(z)$

and boundary conditions

$T_r(0,t) = T_{r0}(t)$, $T_c(L,t) = T_{cL}(t)$.

The parameters in (1) – (3) are

$$\begin{aligned} v_r &= \frac{4q_r}{n_1 \pi d_1^2}, \quad v_c = \frac{4q_c}{\pi (d_3^2 - n_1 d_2^2)} \\ c_1 &= \frac{4\alpha_1}{d_1 \rho_r c_{pr}}, \quad c_2 = \frac{4 d_1 \alpha_1}{(d_2^2 - d_1^2)\rho_w c_{pw}} \\ c_3 &= \frac{4 d_2 \alpha_2}{(d_2^2 - d_1^2)\rho_w c_{pw}}, \quad c_4 = \frac{4 n_1 d_2 \alpha_2}{(d_3^2 - n_1 d_2^2)\rho_c c_{pc}} \end{aligned} \tag{4}$$

where t stands for the time, z for the axial space variable, T for temperatures, v for fluid flow velocities, d_1 for inner diameter of the tube, d_2 for outer diameter of the tube, d_3 for diameter of the shell, ρ for densities, c_p for specific heat capacities, α for heat transfer coefficients, n_1 is the number of tubes and L is the length of tubes. Subscripts denoted r describe the refrigerated fluid, w the metal walls of tubes, c the cooling fluid, and the superscript s steady-state values. The parameter values with their correspondent units are given in Table 1.

TABLE I
PARAMETER VALUES

$L = 8$ m	$n_1 = 1100$
$d_1 = 0.022$ m	$d_2 = 0.024$ m
$d_3 = 1$ m	
$\rho_r = 985$ kg/m^3	$c_{pr} = 4.05$ kJ/kg K
$\rho_w = 7800$ kg/m^3	$c_{pw} = 0.71$ kJ/kg K
$\rho_c = 998$ kg/m^3	$c_{pc} = 4.18$ kJ/kg K
$\alpha_1 = 5.8$ kJ/m^{2}s K	$\alpha_2 = 3.6$ kJ/m^{2}s K

From the system engineering point of view, $T_r(L,t) = T_{r\,\text{out}}$ and $T_c(0,t) = T_{c\,\text{out}}$ are the output variables, and, $q_r(t)$, $q_c(t)$, $T_{r0}(t)$ and $T_{cL}(t)$ are the input variables. Among them, for the control purposes, mostly $q_c(t)$ and $q_r(t)$ can be taken into account as the control variables, whereas other inputs can enter into the process as disturbances. In this paper, the output temperature of the refrigerated fluid $T_r(L,t) = T_{rout}(t)$ is considered as the controlled output.

III. Computation Models

For computation of both steady-state and dynamic characteristics, the finite diferences method is employed. The procedure is based on substitution of the space interval $z \in <0, L>$ by a set of discrete node points $\{z_i\}$ for $i = 1, \ldots, n$, and, subsequently, by approximation of derivatives with respect to the space variable in each node point by finite differences. Two types of finite differences are applied, either the backward finite difference

$$\left.\frac{\partial y(z,t)}{\partial z}\right|_{z=z_i} \approx \frac{y(z_i,t) - y(z_{i-1},t)}{h} = \frac{y(i,t) - y(i-1,t)}{h} \tag{5}$$

or the forward finite difference

$$\left.\frac{\partial y(z,t)}{\partial z}\right|_{z=z_i} \approx \frac{y(z_{i+1},t) - y(z_i,t)}{h} = \frac{y(i+1,t) - y(i,t)}{h} . \tag{6}$$

Here, the function $y(z,t)$ is continuously differentiable in $<0, L>$, and, $h = L/n$ is the diskretization step.

A. Dynamic model

Applying the substitutions (5), (6) in (1) – (3) and, omitting the argument t in parenthesis, PDEs (1) – (3) are approximated by a set of ODEs in the form

$$\frac{dT_r(i)}{dt} = -\left(\frac{v_r}{h} + c_1\right) T_r(i) + \frac{v_r}{h} T_r(i-1) + c_1 T_w(i) \tag{7}$$

$$\frac{dT_w(i)}{dt} = c_2 \left[T_r(i) - T_w(i)\right] + c_3 \left[T_c(i) - T_w(i)\right] \tag{8}$$

$$\frac{dT_c(j)}{dt} = -\left(\frac{v_c}{h} + c_4\right) T_c(j) + \frac{v_c}{h} T_c(j+1) + c_4 T_w(j) \tag{9}$$

for $i = 1, \ldots, n$, $j = n - i + 1$, and, with initial conditions

$T_r(i,0) = T_r^s(i)$, $T_w(i,0) = T_w^s(i)$ and $T_c(i,0) = T_c^s(i)$

for $i = 1, \ldots, n$.

Boundary conditions enter into Eqs. (7) – (9) for $i = 1$.

Here, the controlled output is computed as

$$T_{rout}(t) = T_r(n,t) . \tag{10}$$

B. Steady-state model

Computation of the steady-state characteristics is necessary not only for a steady-state analysis but the steady state values also constitute initial conditions in ODEs (7) – (9). The steady-state model can simply be derived equating the time derivatives in (7) – (9) to zero. Then, after some algebraic manipulations, the steady-state model takes the form of difference equations

$$T_r^s(i) = \frac{1}{v_r + c_1 h}\left[v_r T_r^s(i-1) + c_1 h T_w^s(i) \right] \tag{11}$$

$$T_w^s(i) = \frac{1}{c_2 + c_3}\left[c_2 T_r^s(i) + c_3 T_c^s(i) \right] \tag{12}$$

$$T_c^s(j) = \frac{1}{v_c + c_4 h}\left[v_c T_c^s(j+1) + c_4 h T_w^s(j) \right] \tag{13}$$

for $i = 1, \ldots, n$ and $j = n - i + 1$.
The controlled output is computed as

$$T_{r\,out}^s = T_r^s(n)\,. \tag{14}$$

C. Steady-state characteristics

Some basic steady-state characteristics are shown in Figs. 2 – 6.

The course of temperatures along the tubes of the heat exchanger for $q_r^s = 0.1$ and $q_c^s = 0.12$ is shown in Fig. 2. This shape is typical for considered type of the exchanger.

The courses of the refrigerated fluid (RF) temperature along the tubes for various values q_c^s and q_r^s are in Figs. 3 and 4.

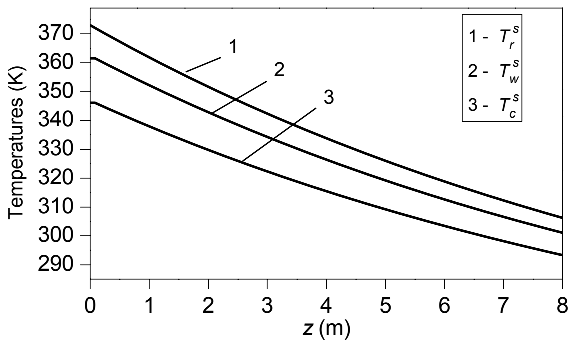

Fig. 2 Temperatures along the heat exchanger.

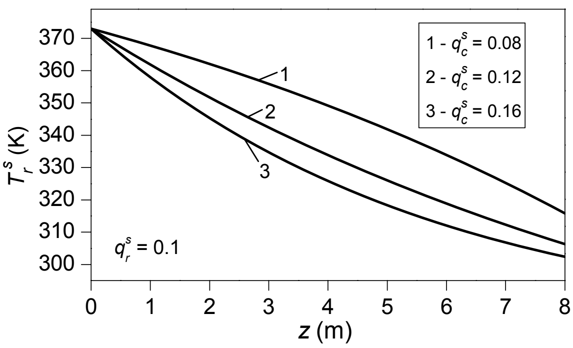

Fig. 3 RF temperature along the reactor for various q_c^s.

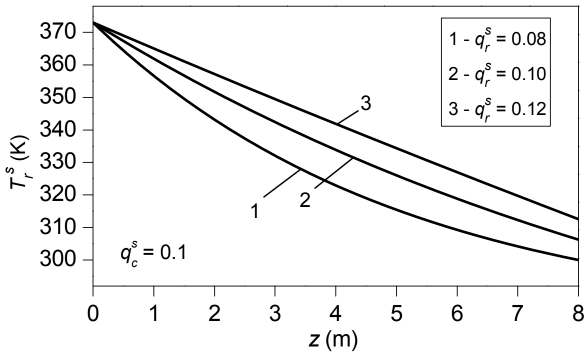

Fig. 4 RF temperature along the reactor for various q_r^s.

Dependences of the output refrigerated fluid temperature $T_{r\,out}^s$ on the cooling fluid (CF) flow rate and on the refrigerated fluid flow rate are shown in Fig. 5 and 6. A nonlinearity of both curves is evident. There are also shown values defining the operating point and interval later used for the process control.

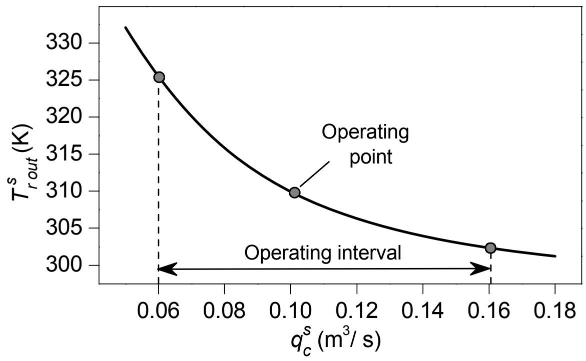

Fig. 5. Dependence of the controlled output on the cooling fluid flow rate ($q_r^s = 0.1$).

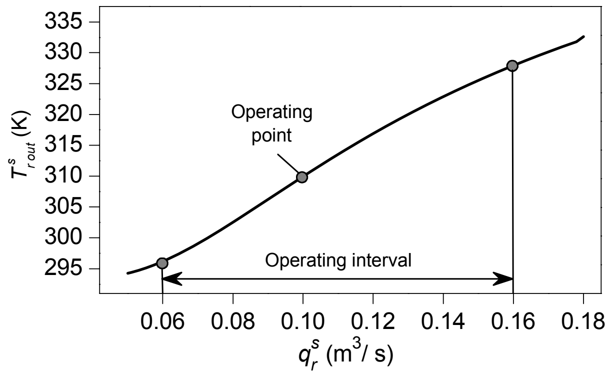

Fig. 6. Dependence of the controlled output on the refrigerated fluid flow rate ($q_c^s = 0.1$).

D. Dynamic Characteristics

Here, both inputs and the controlled output were considered as deviations from their steady values defined as

$q_c^s = 0.1\,\mathrm{m}^3/\mathrm{s}$, $q_r^s = 0.1\,\mathrm{m}^3/\mathrm{s}$ and $T_{rout}^s = 309.94\,\mathrm{K}$. This form is mostly used in the control. The deviations are denoted as follows:

$$u_1(t) = \Delta q_c(t) = q_c(t) - q_c^s \tag{15}$$

$$u_2(t) = \Delta q_r(t) = q_r(t) - q_r^s \tag{16}$$

$$y(t) = T_{rout}(t) - T_{rout}^s \tag{17}$$

The controlled output y step responses to u_1 and u_2 are shown in Figs. 7 and 8.

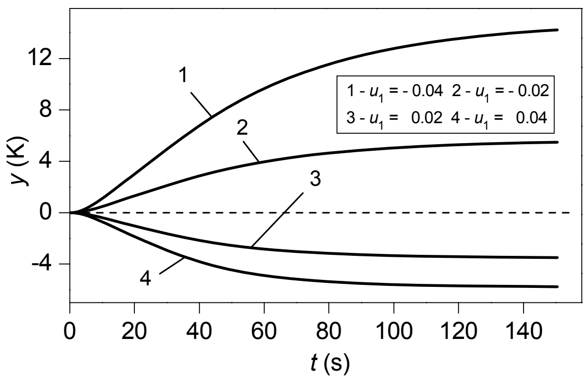

Fig. 7. Controlled output step responses to u_1.

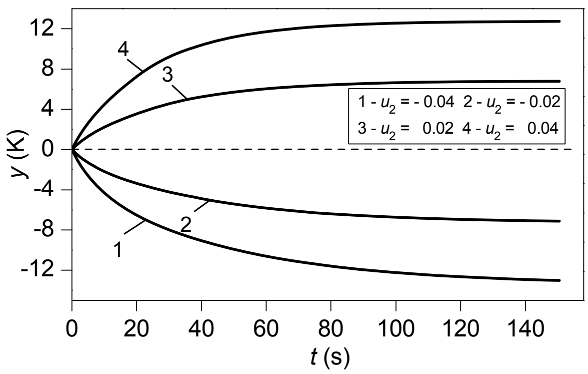

Fig. 8. Controlled output step responses to u_2.

IV. Control Design

A. CT external linear model

Taking into account profiles of curves in Figs. 7 and 8, the second order CT ELM has been chosen for the input u_1 and the first order CT ELM for the input u_2. Then, both CT ELMs are described in the time domain by differential equations

$$\ddot{y}(t) + a_1 \dot{y}(t) + a_0 y(t) = b_0 u_1(t) \tag{18}$$

$$\dot{y}(t) + a_0 y(t) = b_0 u_2(t) \tag{19}$$

or, in the transfer function representation as

$$G_1(s) = \frac{Y(s)}{U_1(s)} = \frac{b_0}{s^2 + a_1 s + a_0} \tag{20}$$

$$G_2(s) = \frac{Y(s)}{U_2(s)} = \frac{b_0}{s + a_0} \tag{21}$$

B. Delta external linear model

Establishing the δ operator

$$\delta = \frac{q-1}{T_0} \tag{22}$$

where q is the forward shift operator and T_0 is the sampling period, the delta ELMs corresponding to (18) and (19) take forms

$$\delta^2 y(t') + a_1' \delta y(t') + a_0' y(t') = b_0' u_1(t') \tag{23}$$

$$\delta y(t') + a_0' y(t') = b_0' u_2(t') \tag{24}$$

where t' is the discrete time.

When the sampling period is shortened, the delta operator approaches the derivative operator, and, the estimated parameters a', b' in (23) and (24) reach the parameters a, b of the CT models (18) and (19) as proved e.g. in [13].

Substituting $t' = k-2$ in (23) and $t' = k-1$ in (24), both equations may be rewriten to forms

$$\delta^2 y(k-2) + a_1' \delta y(k-2) + a_0' y(k-2) = b_0' u_1(k-2) \tag{25}$$

$$\delta y(k-1) + a_0' y(k-1) = b_0' u_2(k-1) \tag{26}$$

where

$$\begin{aligned} \delta^2 y(k-2) &= \frac{y(k) - 2y(k-1) + y(k-2)}{T_0^2} \\ \delta y(k-2) &= \frac{y(k-1) - y(k-2)}{T_0} \\ \delta y(k-1) &= \frac{y(k) - y(k-1)}{T_0} \end{aligned} \tag{27}$$

C. Delta model parameter estimation

Establishing regression vectors

$$\boldsymbol{\Phi}_{1\delta}^T(k-1) = \left(\delta y(k-2)\ y(k-2)\ u_1(k-2)\right) \tag{28}$$

$$\boldsymbol{\Phi}_{2\delta}^T(k-1) = \left(y(k-1)\ u_2(k-1)\right) \tag{29}$$

then, vectors of delta models parameters

$$\boldsymbol{\Theta}_{1\delta}^T(k) = \left(a_1'\ a_0'\ b_0'\right) \tag{30}$$

$$\boldsymbol{\Theta}_{2\delta}^T(k) = \left(a_0'\ b_0'\right) \tag{31}$$

are recursively estimated from the ARX models

$$\delta^2 y(k-2) = \boldsymbol{\Theta}_{1\delta}^T(k)\boldsymbol{\Phi}_{1\delta}(k-1) + \varepsilon(k) \tag{32}$$

$$\delta y(k-1) = \boldsymbol{\Theta}_{2\delta}^T(k)\boldsymbol{\Phi}_{2\delta}(k-1) + \varepsilon(k) \tag{33}$$

The recursive parameter estimations were performed with the sampling interval T_0 = 1 s. Here, the recursive identification method with exponential and directional forgetting was used according to [20] and [21].

D. Control System Description

In this paper, the control system with two feedback controllers is considered according to Fig. 9.

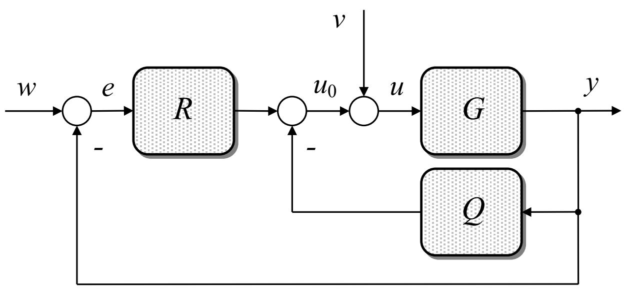

Fig. 9. Control system with two feedback controllers.

In the scheme, w is the reference signal, v denotes the load disturbance, e is the tracking error, u_0 is the output of the controller, y is the controlled output and u is the control input.

In general therms, G represents the ELM with the transfer function

$$G(s) = \frac{b(s)}{a(s)} \tag{34}$$

and Q and R are feedback controllers with transfer functions

$$Q(s) = \frac{\tilde{q}(s)}{\tilde{p}(s)}, \quad R(s) = \frac{r(s)}{\tilde{p}(s)} \tag{35}$$

where $\tilde{q}, r$ and $\tilde{p}$ are polynomials in s.

Both w and v are considered to be step functions with transforms

$$W(s) = \frac{w_0}{s},\ V(s) = \frac{v_0}{s}. \tag{36}$$

E. Application of Polynomial Method

The controller design described in this section appears from the polynomial approach. General conditions required to govern the control system properties are formulated as strong stability (in addition to the control system stability, also the stability of controllers is required), internal properness, asymptotic tracking of the reference and load disturbance attenuation.

The procedure to obtain admissible controllers can be briefly described as follows:

Establish the polynomial t as

$$t(s) = r(s) + \tilde{q}(s). \tag{37}$$

Then, the control system stability is ensured when polynomials $\tilde{p}$ and t are given by a solution of the polynomial equation

$$a(s)\tilde{p}(s) + b(s)t(s) = d(s) \tag{38}$$

with a stable polynomial d on the right side. Evidently, the roots of d determine poles of the closed-loop.

Further, the asymptotic tracking and load disturbance attenuation are provided by polynomials $\tilde{p}$ and $\tilde{q}$ having forms

$$\tilde{p}(s) = s\,p(s),\quad \tilde{q}(s) = s\,q(s). \tag{39}$$

Subsequently, the transfer functions of controllers take forms

$$Q(s) = \frac{q(s)}{p(s)},\quad R(s) = \frac{r(s)}{s\,p(s)}. \tag{40}$$

A stable polynomial $p(s)$ in denominators of (40) ensures the stability of controllers.

The control system satisfies the condition of internal properness when the transfer functions of all its components are proper. Consequently, the degrees of polynomials q and r must fulfill inequalities

$$\deg q \le \deg p,\quad \deg r \le \deg p + 1. \tag{41}$$

Now, the polynomial t can be rewritten into the form

$$t(s) = r(s) + s\,q(s). \tag{42}$$

Taking into account solvability of (38) and conditions (41), the degrees of polynomials in (38) and (41) can be easily derived as

$$\begin{array}{l} \deg t = \deg r = \deg a, \quad \deg q = \deg a - 1 \\ \deg p \ge \deg a - 1, \quad \deg d \ge 2\deg a \end{array}. \tag{43}$$

.

Denoting deg $a = n$, polynomials t, r and q have forms

$$t(s) = \sum_{i=0}^{n} t_i s^i,\ r(s) = \sum_{i=0}^{n} r_i s^i,\ q(s) = \sum_{i=1}^{n} q_i s^{i-1} \tag{44}$$

where their coefficients fulfill equalities

$$r_0 = t_0\,,\ \ r_i + q_i = t_i \ \text{ for } \ i = 1, \dots, n \tag{45}$$

Then, unknown coefficients r_i and q_i can be obtained by a choice of selectable coefficients $\beta_i \in \langle 0,1 \rangle$ such that

$$r_i = \beta_i t_i\,,\ \ q_i = (1-\beta_i)\,t_i \ \text{ for } \ i = 1, \dots, n\,. \tag{46}$$

The coefficients β_i distribute weights between numerators of transfer functions Q and R. With respect to (40) and (46), it may be expected that higher values of β_i accelerate control responses to step references.

Remark: If $\beta_i = 1$ for all i, the control system in Fig. 9 simplifies to the 1DOF control configuration. If $\beta_i = 0$ for all i and both reference and load disturbance are step functions, the control system corresponds to the 2DOF control configuration.

Now, for the second order model (20) with $\deg a = 2$, the numerators in (40) take forms

$$q(s) = q_2 s + q_1\,,\ \ r(s) = r_2 s^2 + r_1 s + r_0 \tag{47}$$

and, for the first order model (21) with $\deg a = 1$

$$q(s) = q_1\,,\ \ r(s) = r_1 s + r_0 \tag{48}$$

where

$$r_1 = \beta_1 t_1\,,\ \ r_2 = \beta_2 t_2\,,\ \ q_1 = (1-\beta_1)\,t_1\,,\ \ q_2 = (1-\beta_2)\,t_2\,. \tag{49}$$

The controller parameters then follow from solution of the polynomial equation (38) and depend upon coefficients of the polynomial d.

F. Pole Placement

A suitable control quality can be achieved by a determination of the closed-loop poles given by roots of the polynomial d on the right side of (38). In this paper, the procedures are based on the LQ control theory.

The polynomial d is chosen as a product of two stable factors

$$d(s) = g(s)\,n(s) \tag{50}$$

where the polynomial g is a monic form of the polynomial g' obtained by spectral factorization

$$\left[s\,a(s)\right]^* \varphi \left[s\,a(s)\right] + b^*(s)b(s) = g'^*(s)\,g'(s) \tag{51}$$

where $\varphi > 0$ is the weighting coefficient.

In the LQ optimal control theory, spectral factorization (51) results from the minimization of the quadratic cost function

$$J = \int_0^\infty \left\{ e^2(t) + \varphi \dot{u}^2(t) \right\} dt \tag{52}$$

where $e(t)$ is the tracking error and $\dot{u}(t)$ is the control input derivative.

The polynomials g' and derived formulas for their parameters calculation have forms

$$g'(s) = g_3' s^3 + g_2' s^2 + g_1' s + g_0' \tag{53}$$

for the second order system (20) where

$$\begin{aligned} &g_0' = |b_0|,\ g_3' = \sqrt{\varphi},\ g_1' = \sqrt{\varphi a_0^2 + 2 g_0' g_2'} \\ &g_2' = \sqrt{\varphi\,(a_1^2 - 2\,a_0) + 2 g_1' g_3'} \end{aligned} \tag{54}$$

and,

$$g'(s) = g_2' s^2 + g_1' s + g_0' \tag{55}$$

for the first order system (21) where

$$g_0' = |b_0|\,,\ \ g_2' = \sqrt{\varphi}\ ,\ \ g_1' = \sqrt{\varphi a_0^2 + b_1^2 + 2 g_0' g_2'} \tag{56}$$

For calculation of d given by (50), polynomials (53) and (55) are arranged to monic forms $g(s)$ (with unit coefficients by the highest power of s) such that

$$g_j = g_j' / g_n' \quad j = 0, 1, \dots, n \tag{57}$$

where $n = \deg g'$.

The stable polynomial n in (50) is a result of spectral factorization

$$n^*(s)\,n(s) = a^*(s)\,a(s) \tag{58}$$

having forms

$$n(s) = s^2 + n_1 s + n_0 \tag{59}$$

for the second order system (20), and,

$$n(s) = s + n_0 \tag{60}$$

for the first order system (21) with coefficients

$$n_0 = \sqrt{a_0^2}\ ,\ \ n_1 = \sqrt{a_1^2 + 2 n_0 - 2 a_0}\ . \tag{61}$$

Polynomials d then have forms

$$d(s) = s^5 + d_4 s^4 + d_3 s^3 + d_2 s^2 + d_1 s + d_0 \tag{62}$$

for the second order system with coefficients

$$\begin{aligned} &d_4 = g_2 + n_1,\, d_3 = g_2 n_1 + g_1 + n_0,\, d_2 = g_2 n_0 + g_1 n_1 + g_0, \\ &d_1 = g_1 n_0 + g_0 n_1,\, d_0 = g_0 n_0 \end{aligned} \tag{63}$$

and,

$$d(s) = s^3 + d_2 s^2 + d_1 s + d_0 \tag{64}$$

for the first order system with coefficients

$$d_2 = g_1 + n_0,\, d_1 = g_1 n_0 + g_0,\, d_0 = g_0 n_0 \,. \tag{65}$$

The procedure leads to strictly proper controllers with transfer functions

$$Q(s) = \frac{q_2 s + q_1}{s^2 + p_1 s + p_0}, \quad R(s) = \frac{r_2 s^2 + r_1 s + r_0}{s(s^2 + p_1 s + p_0)} \tag{66}$$

for the second order system (20), and,

$$Q(s) = \frac{q_1}{s + p_0}, \quad R(s) = \frac{r_1 s + r_0}{s(s + p_0)} \tag{67}$$

for the first order system (21). Their parameters are computed from equations

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ a_1 & 1 & 0 & 0 & 0 \\ a_0 & a_1 & b_0 & 0 & 0 \\ 0 & a_0 & 0 & b_0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} p_1 \\ p_0 \\ t_2 \\ t_1 \\ t_0 \end{pmatrix} = \begin{pmatrix} d_4 - a_1 \\ d_3 - a_0 \\ d_2 \\ d_1 \\ d_0 \end{pmatrix} \tag{68}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ a_0 & b_0 & 0 \\ 0 & 0 & b_0 \end{pmatrix} \cdot \begin{pmatrix} p_0 \\ t_1 \\ t_0 \end{pmatrix} = \begin{pmatrix} d_2 - a_0 \\ d_1 \\ d_0 \end{pmatrix} \tag{69}$$

and, subsequently, using equations (46).

Now, it follows from the above introduced procedure that the parameters of both controllers depend upon coefficients β as well as upon the parameter φ which affects the closed-loop poles. Consequently, tuning of the controllers can be performed by a suitable choice of selectable parameters β and φ.

V. Simulation Results

The simulations were performed around the operating point $q_c^s = 0.1\,\mathrm{m}^3/\mathrm{s}$, $q_r^s = 0.1\,\mathrm{m}^3/\mathrm{s}$ and $T_{rout}^s = 309.94\,\mathrm{K}$. For the start (adaptation phase), a P controller was used in all simulations.

As the control inputs, either the CF flow or the RF flow were used.

An effect of the parameter φ on the controlled outputs and the control input responses is presented in Figs. 10, 11 and 12. Evidently, a decreasing φ results in controlled outputs with higher overshoots. Moreover, a smaller φ can lead to oscillations of the control input. Therefore, the selection of an appropriate value φ is important especially in control of a real process.

Overshoots (undershoots) can be reduced using a suitable selection of parameters β as shown in Figs. 13 and 14. However, it should be noted that very small β can result to a slow control.

The control results by the CF and RF flow as the control inputs with the same parameters are compared in Fig. 15. Both curves show that the use of the RF flow provides a faster control. However, selecting the appropriate control input always depends on specific technological conditions.

The controlled output in control by the RF flow in the presence of random disturbance v loaded on the CF flow is shown in Fig. 16.

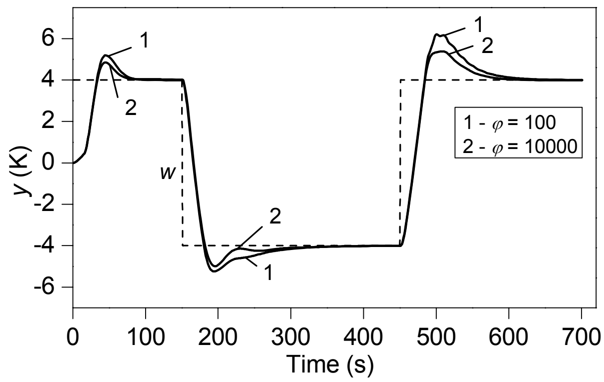

Fig. 10. Effect of φ on controlled output in control by CF flow ($\beta_1 = \beta_2 = 1$).

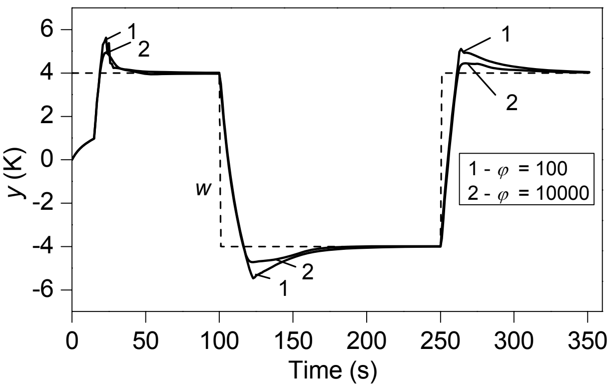

Fig. 11. Effect of φ on controlled output in control by RF flow ($\beta_1 = \beta_2 = 1$).

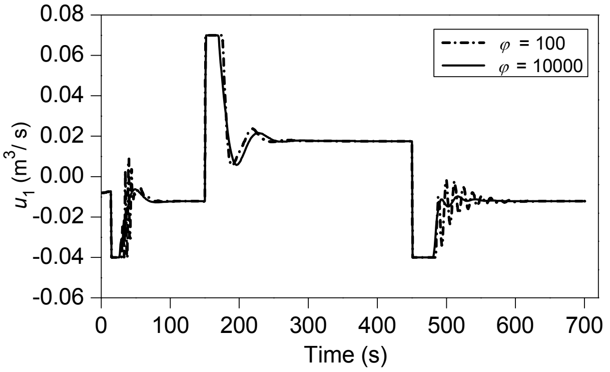

Fig. 12. Effect of φ on control input u_1 in control by CF flow ($\beta_1 = \beta_2 = 1$).

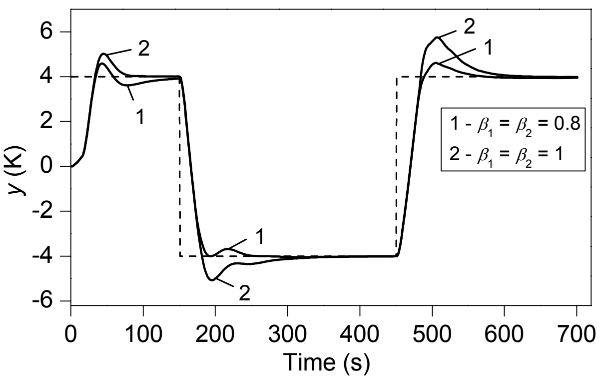

Fig. 13. Effect of β on controlled output in control by CF flow (φ = 2500).

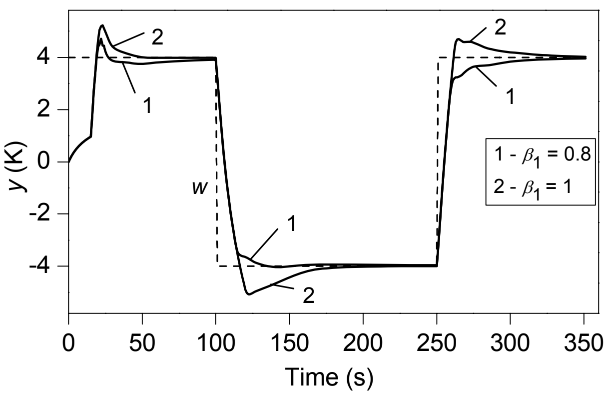

Fig. 14. Effect of β on controlled output in control by RF flow (φ = 2500).

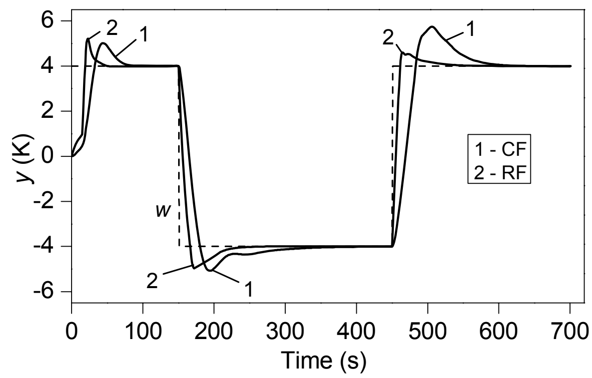

Fig. 15 Comparison of controlled outputs in control by CF and RF flow (φ = 2500, $\beta_1 = \beta_2 = 1$).

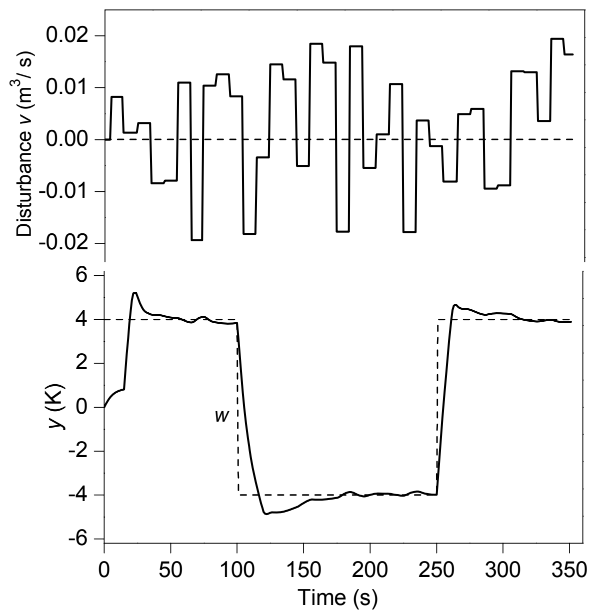

Fig. 16. Controlled output in control by RF flow in presence of random disturbance (φ = 2500, $\beta_1 = \beta_2 = 1$).

VI. Conclusion

The paper deals with design and verification of the adaptive control algorithm for a shell and tube heat exchanger.

As the control inputs, either the cooling fluid flow or the refrigated fluid flow are used. The control system structure with two feedback controllers is considered. The control algorithm is based on an alternative continuous-time external linear model with parameters obtained via recursive parameter estimation of a corresponding delta model. The control law is derived using the polynomial approach and optimal pole assignment resulting from the LQ control theory. Both feedback controllers are obtained by a solution of the polynomial equation. Tuning of their parameters is possible by two selectable parameters

The presented method has been tested by computer simulation on the nonlinear model of the shell and tube heat exchanger.

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