Parallel lines: Application for a multiphase flow

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***Abstract*—**Computation of parallel lines for different geometric shapes varieties is of mayor importance for the development of models of physical problems such as cavitation bubbles.

The use of parallel lines and Göebner basis to find solutions for complex problems as multiphase flow allows us to track the evolutions of a surface over time.

***Keywords*—**Gröebner basis, Multiphase Flow, Parallel lines.

I. Introduction

THE application of a mathematical tool such as parallel lines coupled with the Gröebner basis method allows us to calculate approximations to physical problems such as cavitation bubbles.

In this paper we try to give an approximation to the multiphase flow problem of a cavitation bubble, in which case we must first describe how the shape of the bubble changes under different kind of fields.

The approximation used in this paper is the application of parallel lines to an elliptical shape, in which case we find a single polynomial that implicitly generates an affine variety that contains the parallel lines.

II. PARALLEL SURFACE

The conditions we need to describe a surface on a flow are for a 2d case five equations, fist we describe the initial shape of the bubble using its variety.

$$f1 := b^2(y_0 - y_c)^2 + a^2(x_0 - x_c)^2 - a^2 b^2 = 0 \quad (1)$$

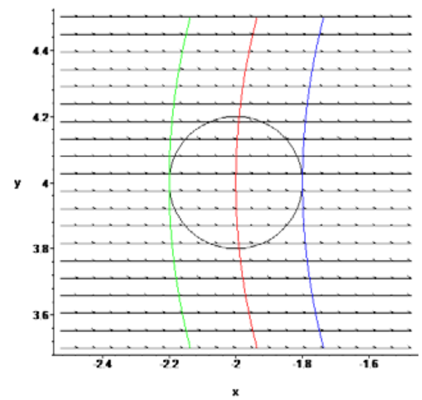
Where (a,b) and (x_0,y_0) denote parameters that define an elliptical shape which we can be set for different sizes and different placements.

For the second condition we must define small circles of radius r within centered on the bubble surface (1). The size of the radius must be set depending of the magnitude of the flow at each point.

$$f2 := (y - y_0)^2 + (x - x)^2 - r^2 = 0 \quad (2)$$

$$f4 := \|vf1\| - r = 0 \quad (3)$$

The last condition for the system sets the growth direction of the parallel line, using a line with the same direction of the flow at (x_0,y_0).



III. Field Examples

A. Constant Field

Let us set a constant field *vf1*.

$$vf1 := [4,4] \quad (4)$$

Now we must find the associated variety for the polynomials (1) (2) (3) and (4).

$$V := V(b^2(y_0 - y_c)^2 + a^2(x_0 - x_c)^2 - a^2b^2,\ y^2 - 2yy_0 + y_0^2 + x^2 - 2xx_0 + x_0^2 - r^2,\ -x + x_0 + y - y_0,\ r^2 - 32) \quad (5)$$

Once we fix this conditions, and using the Gröebner basis resolutions tools from Maple Software, we get solutions for the system that doesn't contain the factors (x0,y0,r).

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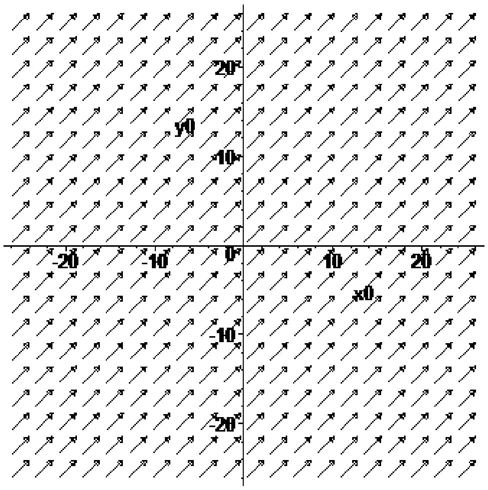

Fig2. Constant Field [4,4]

So in the general case using the elimination teory and the lexicographic order we can find a minimal basis wich contains the parallel lines polynomial without dependence of (x_0,y_0).

$$g = \left(8b_c^{2y} + 2xa_c^{2x} - b^{2y_c^2} + a^{2b^2} - a^{2x_c^2} - 16a^2 - 16b^2 + 8a_c^{2x} - 8xa^2 - 8yb^2 - x^{2a^2} - y^{2b^2} + 2yb_c^{2y}\right)\left(-8b_c^{2y} + 2xa_c^{2x} - b^{2y_c^2} + a^{2b^2} - a^{2x_c^2} - 16a^2 - 16b^2 - 8a_c^{2x} + 8xa^2 + 8yb^2 - x^{2a^4} - y^{2b^4} + 2yb_c^{2y}\right)$$

(6)

Now that we have the solution for the general case we fix a geometric shape setting the parameters for an ellipse for a=3 and b=2 , and an initial position x_c=0 and y_c=4.

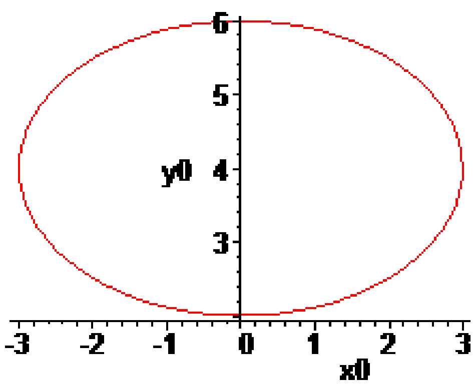

Fig 3 Initial shape

When we superpose the constant field and the geometric shape we find a set of solutions for our system that only depend of polynomials within the real domain that contains the parallel lines.

$$g1 := 9y^2 + 28 + 32x + 4x^2$$
$$g2 := 9y^2 - 144y + 604 - 32x + 4x^2 \qquad (7)$$

The geometric representation of this two equations is the evolving shape of every point around the bubble, subject to a flow in the same direction and with the same strenght.

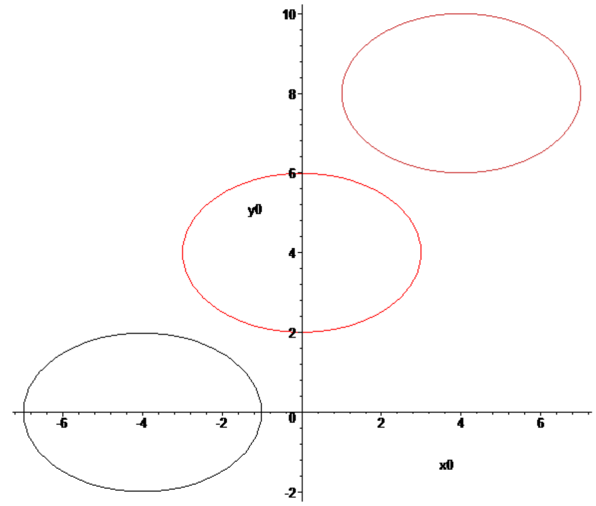

Fig.4 Displacement over a constant field.

B. Speed Field

If now we set a speed field that depends on position, it will simulate the flow against a solid boundary. Different speeds

$$vf1 = [Ux_0, -Uy_0] \qquad (8)$$

can be set changing the U values.

Let U=1/10 so we can appreciate the effect over the surface when the flow gets close to a solid boundary. Later on we can superpose the geometric shape.

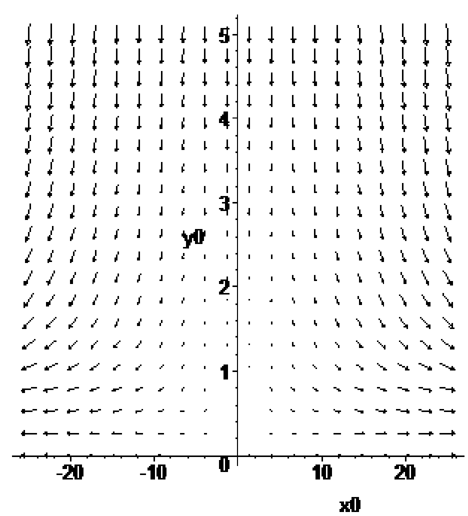

Fig 5. Speed field against a solid boundary

Applying the theory of parallel lines we must now find the associated variety to this polynomials in which case we find four polynomials, but only the following two are on the real domain.

$$
\begin{aligned}
g1 &:= 16y^2 - 120y + 225 + 16x^2 \\
g2 &:= 16x^6 + 9x^4 - 24x^4y + 48x^4y^2 + 18x^2y^2 \\
&48x^2y^4 - 48x^2y^3 + 81968x^2y - 7992x^2 - 106 \\
&24y^5 + 1774224 + 9y^4 + 16y^6 + 7992y^2
\end{aligned}
\tag{9}
$$

Using now this polynomials we can track the evolution of

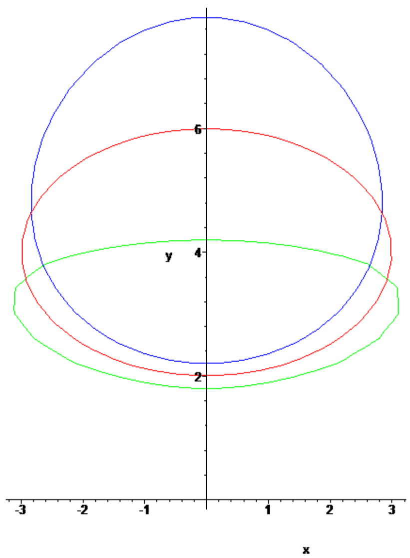

Fig6. Bubble in a speed field

the geometric shape subject to flow.

C. *Pressure Field*

Now we setup a gradient field which representing the pressure field around a spherical bubble is defined by the equation.

$$P := p_0 - 2\rho U^2\left(x_0^2 + (y_0 + 3 + \epsilon)^2\right) \tag{10}$$

Where the ϵ is a arbitrary small distance between the bubble and the solid boundary. We can now fix the growth direction

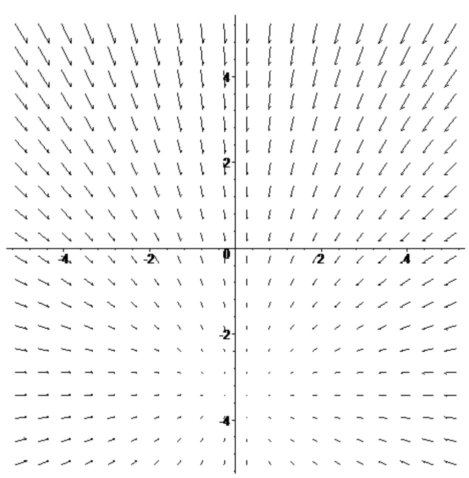

Fig 7. Pressure Field

for the polynomials in the same direction as the pressure field.

If we superpose the elliptical surface using a lexicographic order we obtain that the minimal Gröebner basis that contains twelve polynomials but only one independent of x_0 and y_0, this

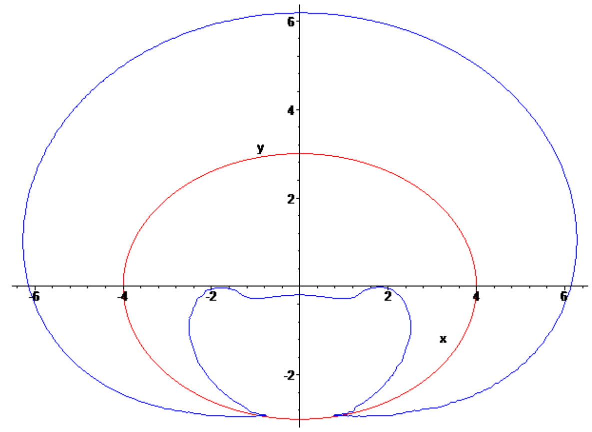

Fig 8. Bubble in gradient field

polynomial show the evolution of the surface through the pressure field.

We can also try other shapes like circular ones for which case we find eight polynomial for the Gröebner basis, which evolve differently.

$$
\begin{aligned}
g1 &:= 3240000x^4 - 68079001x^2 - 10368000x^2y + \\
&6480000x^2y^2 - 59784601y^2 - 10368000y^3 + \\
&3240000y^4 \\
g2 &:= 3240000x^4 - 6497401x^2 + 10368000x^2y + \\
&6480000x^2y^2 + 1796999y^2 + 10368000y^3 + 3240000y^4
\end{aligned}
\tag{11}
$$

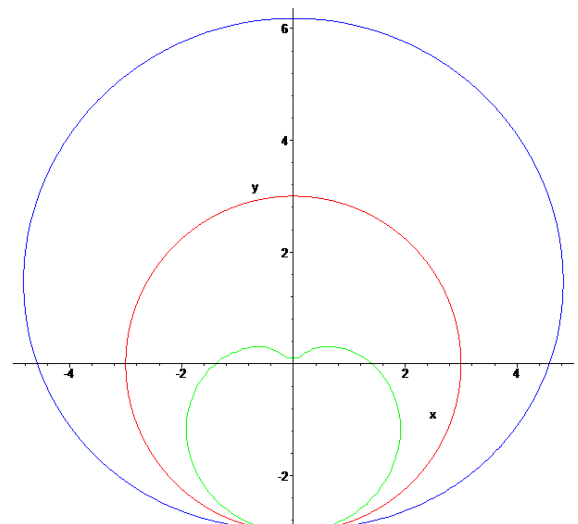

Fig 9. Circular shape in a pressure field

IV. Conclusions

The application of Gröbner basis to find solutions for elliptical shapes in a flow allows us to observe a representation of the cavitation bubble in a field, in which case the bubble shape is affected only by external forces that generate the flow around it when it gets close to a solid boundary.

The use of this method with the aid of only a few simple rules like the growth direction of the field and the geometric description of the initial shape allowed us to describe the evolution of a system. This kind of descriptions will also let us simulate more complex problems in a multiphase flow .

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