Effects of Induction Motors Inductances Modification on Stability Analyzed with Numerical Methods

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***Abstract*—**The effects of the induction motors parameters modification on stability of driving systems operating at variable frequency are analyzed in this paper with the help of the numerical methods and of the computer simulation. In this purpose, the used numerical method is presented. The acquisition program necessary for performing the experimental verifications is also presented. Finally the results and the conclusions of the study are presented.

***Keywords*—**Numerical methods, induction motors, stability, computer simulations, experimental verifications.

I. Introduction

The computer aided resolution of scientific and technical problems involves to identify a numerical method and to implement it on a computation system. Such a numerical method will be used further on for solving a stability problem.

The stability is a qualitative feature of the systems associated with the dynamic regime behaviour.

This problem is a very present one, especially in the field of the driving systems [3], [6] and [7]. The systems having induction motors operating at variable frequency do not make an exception, too.

This paper aims to analyze this problem, first of all from mathematical point of view. In the second part a new analysis method will be detailed, method conceived by authors.

II. Numerical Method

Further on we aim that, by using known representations of the transfer locus and of the amplitude-phase characteristics, to analyze the influences of the machine windings resistances on stability. In this purpose, it is imposed, at the beginning, to establish an adequate mathematical model of the system which is referred to.

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Further on the induction motor is supposed to be supplied by means of a sinusoidal voltage source having variable frequency. In order to obtain the relations necessary to carry out the proposed study the following matrix equation, written in accordance with [5], will be used:

$$s\begin{vmatrix} I_{ds}(s) \\ I_{qs}(s) \\ I_{df}(s) \\ I_{qf}(s) \end{vmatrix} =$$

$$= \begin{vmatrix} -\left(\frac{1}{T_s}-\frac{1}{T_r}\right)\cdot\frac{1}{\sigma} & \omega_r & \frac{1}{T_r\sigma} & \frac{\omega}{\sigma} \\ -\omega_r & -\left(\frac{1}{T_s}-\frac{1}{T_r}\right)\cdot\frac{1}{\sigma} & -\frac{\omega}{\sigma} & \frac{1}{T_r\sigma} \\ -\frac{1}{T_s} & 0 & 0 & \omega_s \\ 0 & -\frac{1}{T_s} & -\omega_s & 0 \end{vmatrix} \cdot \qquad (1)$$

$$\cdot\begin{vmatrix} I_{ds}(s) \\ I_{qs}(s) \\ I_{df}(s) \\ I_{qf}(s) \end{vmatrix} + \begin{vmatrix} \frac{1}{L_s\sigma} \\ 0 \\ \frac{1}{L_s} \\ 0 \end{vmatrix} \cdot U_{ds}(s),$$

where the following notations have been used:

$T_s = \frac{L_s}{R_s}$ - time constant of the stator winding;

$T_r^{/} = \frac{L_r^{/}}{R_r^{/}}$ - time constant of the rotor winding;

$\sigma = 1 - \frac{L_{sh}^2}{L_s L_r^{/}}$ - leakage coefficient of the machine windings;

$$I_{df}(s) = I_{ds}(s) + \frac{L_{sh}}{L_s} I_{dr}^{/}(s) \qquad (2)$$

$$I_{qf}(s) = I_{qs}(s) + \frac{L_{sh}}{L_s} I_{qr}^{/}(s)$$

The motion equation is added to (1):

$$\frac{J}{p}\frac{d\omega}{dt}=\frac{3}{2}pL_{sh}(i_{qs}i'_{dr}-i_{ds}i'_{qr})-m_r \tag{3}$$

If the electrical transient process is considered to be much faster than the mechanical one (J is very great), it results that the mathematical model which will be used further on, is limited to the matrix equation.

This one may be also written as:

$$s[Y(s)]=[A]\cdot[Y(s)]+[B]\cdot[U(s)] \tag{4}$$

or, equivalently:

$$[Y(s)]=(s[I]-[A])^{-1}\cdot[B]\cdot[U(s)] \tag{5}$$

which is the input-output operational equation for the analyzed multivariable system.

The transfer matrix is obtained from this equation:

$$[H(s)]=(s[I]-[A])^{-1}\cdot[B]=\frac{adj(s[I]-[A])}{\det(s[I]-[A])}\cdot[B] \tag{6}$$

where I is the unit matrix.

The transfer function denominator will have the form:

$$n(s)=a_0+a_1s+a_2s^2+a_3s^3+a_4s^4 \tag{7}$$

with

$$\begin{aligned}
a_0 &= \frac{1}{T_s^2T_r^2\sigma^2}+\frac{\omega_s^2}{T_r^2\sigma^2}+\frac{\omega_r^2}{T_s^2\sigma^2}+2\frac{\omega_s\omega_r}{T_sT_r\sigma^2}(1-\sigma)\\
a_1 &= \frac{2}{T_sT_r\sigma^2}\cdot\left(\frac{1}{T_s}+\frac{1}{T_r}\right)+\frac{2\omega_r}{T_s\sigma}+\frac{2\omega_s}{T_r\sigma}\\
a_2 &= \omega_s^2+\omega_r^2+2\frac{\sigma+1}{T_sT_r\sigma^2}+\frac{1}{T_s^2\sigma^2}+\frac{1}{T_r^2\sigma^2}\\
a_3 &= \frac{2}{\sigma}\left(\frac{1}{T_s}+\frac{1}{T_r}\right)\\
a_4 &= 1
\end{aligned} \tag{8}$$

Further on the following approximation will be used:

$$\mu=\frac{1}{2\sigma}\left(\frac{1}{T_s}+\frac{1}{T_r}\right)\cong\frac{1}{T_s\sigma}\cong\frac{1}{T_r\sigma} \tag{9}$$

For a concrete case of an induction motor (R_s =7,5 Ω, R'_r =5,5 Ω, $L_s=0{,}529$ H, $L'_r=0{,}528$ H, $L_{sh}=0{,}498$ H) the following data are obtained:

$$\sigma=0{,}112\,,\quad \mu=110 \tag{10}$$

In order to determine the transfer function poles for the studied system, its denominator will be made equal to zero:

$$n(s)=0 \tag{11}$$

with n(s) given by the (7) and (8).

The following poles have been obtained by solving this equation:

a) If $\mu\ge\frac{\omega}{2\sqrt{1-\sigma}}$,

$$s_{1,2,3,4}=-\mu\pm\sqrt{\mu^2(1-\sigma)-\frac{\omega^2}{4}}\pm j(\omega+2\omega_r)/2 \tag{12}$$

b) If $\mu<\frac{\omega}{2\sqrt{1-\sigma}}$,

$$s_{1,2,3,4}=-\mu\pm j\left[\sqrt{\frac{\omega^2}{4}-\mu^2(1-\sigma)}\pm(\omega+2\omega_r)/2\right] \tag{13}$$

As one can observe, in Eq. (12), $\mu>\sqrt{\mu^2(1-\sigma)-\frac{\omega^2}{4}}$, irrespective of the machine parameters or the operation point. So, the real part of the transfer function poles is always negative. In conclusion, the studied system is always stable.

Since the inductances L_s and L'_r are inverse proportional with μ, these ones have a non-stabilizing effect on the induction motor operating at variable frequency.

III. A New Method for Stability Analysis

This method, detailed in [2], has as starting point the following relations, written in per unit values:

$$hs\cdot\Delta\omega^*=-k\Delta i'^*_{dr} \tag{14}$$

$$\Delta i'_{dr}=\frac{s+j\omega_s^*+\varepsilon}{s^2+(s_{ks}+s_{kr}+j\omega_s^*)s+s_{kr}(\varepsilon+j\omega_s^*)}\cdot k(\Delta\omega^*-\Delta\omega_s^*) \tag{15}$$

where

$$\varepsilon=(1-k^2)s_{ks}=\frac{r_s^*}{x_s^*}=\frac{r_s^*}{x_r^{*\prime}} \tag{16}$$

At the same time it must be mentioned that the per unit quantities used in the previous equations have been noted with "*", the quantities depicted here having the significance from [2].

The equation (15) may also be written in the following form:

$$\Delta\omega^* = -\frac{k}{hs} \cdot \Delta i_{dr}^{/*} \tag{17}$$

or, equivalently:

$$\Delta\omega^* = G_1(s) \cdot \Delta i_{dr}^{/*}, \text{ with } G_1(s) = -\frac{k}{hs} \tag{18}$$

Similarly, the equation (16) becomes:

$$\Delta i_{dr}^{/*} = G_2(s) \cdot (\Delta\omega_s^* - \Delta\omega^*) \tag{19}$$

where

$$G_2(s) = -\frac{s + j\omega_s^* + \varepsilon}{s^2 + (s_{ks} + s_{kr} + j\omega_s^*)s + s_{kr}(\varepsilon + j\omega_s^*)} \cdot k \tag{20}$$

The previous relations lead to the equivalent scheme of the induction machine operating at variable frequency.

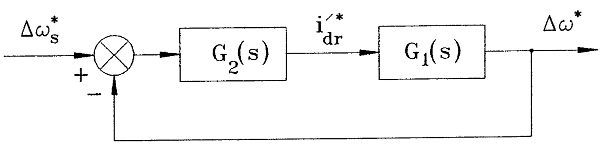


Fig. 1 Machine block scheme in the mentioned situation

IV. SIMULATIONS

A Matlab program for the stability analysis has been conceived, by using the previous scheme. The representations from figures 2, 3 and 4 have been obtained by running this program, for the concrete case of a motor rated at 1,1 kW.

There must be also mentioned the importance of the introduced method resulting from the possibility to emphasize the machine parameters influence and especially the inertia moment influence, on stability when operating at variable frequency, fact that provides originality to this method.

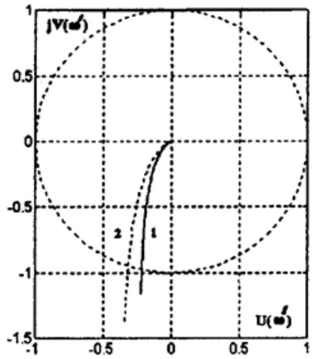
In order to catch quantitatively these interdependences the following table has been filled.

TABLE I
ABSOLUTE VALUES AND PHASE MARGINS

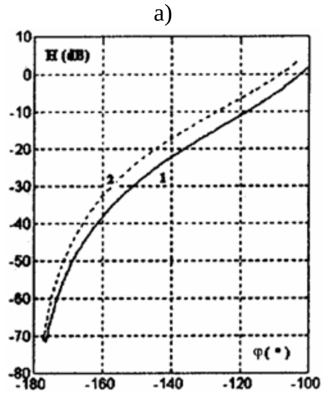
Param.	Absolute Value	Per unit param.	Per unit value	Phase margin [^{0}]
L_s	0,549	x_s^*	2,2735	69,13
$L_r^/$	0,548	$x_r^{/*}$	2,2694	75,31
L_{sh}	0, 558	x_{1m}^*	2,3104	71,32

These results help us to emphasize a few important conclusions regarding the resistances influence on the studied system stability:

- the increase of the inductance L_s leads to the stability decrease;
- at the same time with the rotor inductance increase the system stability decreases;
- the increase of the main inductance has a stabilizing effect.

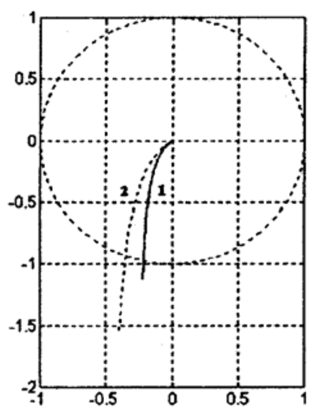


a)

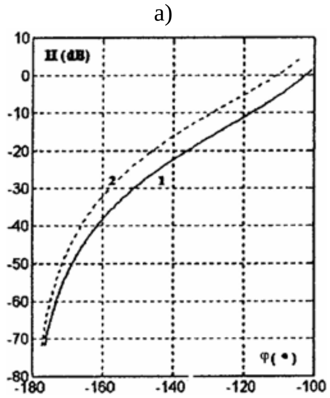


b)

Fig. 2 Transfer locus (a) and amplitude-phase characteristics (b) obtained in the case of the inductances modification: $L_S = 0{,}529$ H (1) and $L_S = 0{,}549$ H (2)

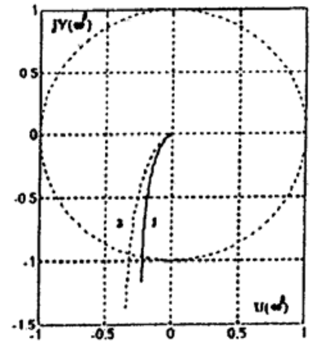


a)

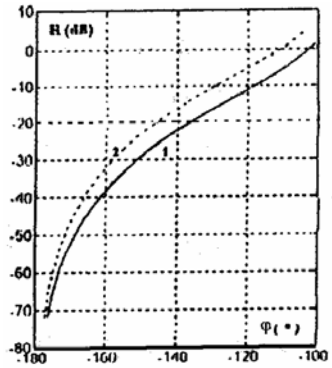


b)

Fig. 3 Transfer locus (a) and amplitude-phase characteristics (b) obtained in the case of the inductances modification: $L_r^/ = 0{,}528$ H (1) and $L_r^/ = 0{,}548$ H (2)



a)



b)

Fig. 4 Transfer locus (a) and amplitude-phase characteristics (b) obtained in the case of the inductances modification: $L_{sh} = 0{,}498$ H (1) and $L_{sh} = 0{,}558$ H (2)

V. Program of Experimental Verifications

In order to verify the conclusions emphasized before, the experimental circuit detailed in [1] has been carried out.

An original program has been conceived in Visual Basic for acquiring and processing the obtained data.

This program can be run by a double click applied on the pictogram placed on the desktop (figure 5).

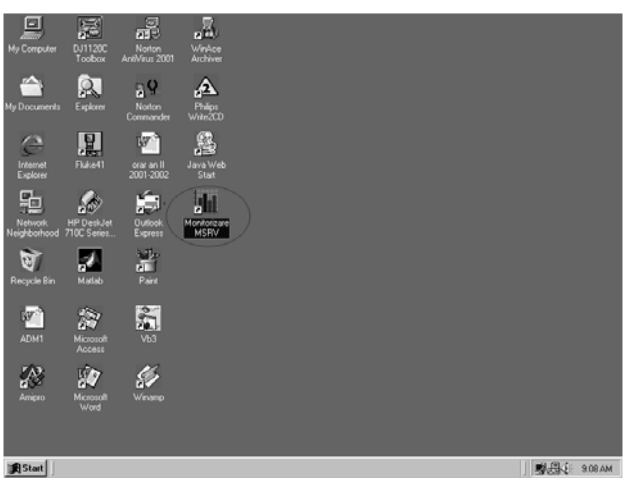


Fig. 5 Desktop

Fig. 6 Window “Coperta”

When running the program, the presentation cover of the program appears on the display, figure 6 (the texts corresponding to the following windows are written in the Romanian language).

The main windows of the program are detailed in figures 7-14.

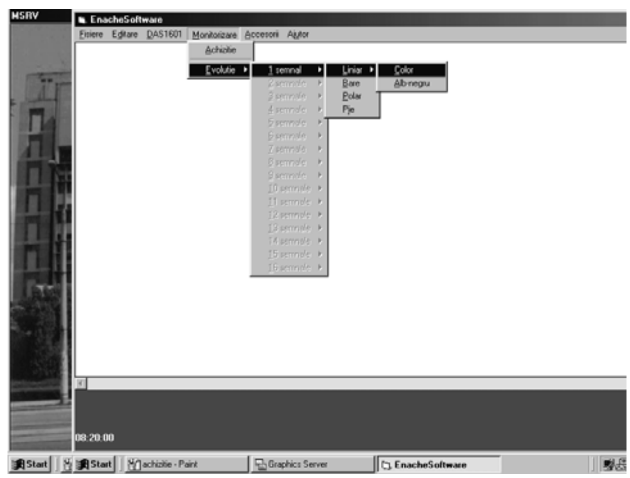


Fig. 7 Window “Meniu”

Fig. 8 Detail of menu “Fisiere”

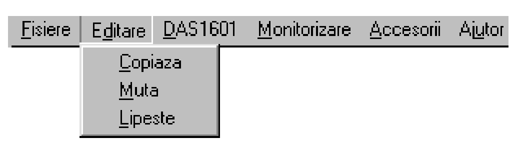


Fig. 9 Detail of menu “Editare”

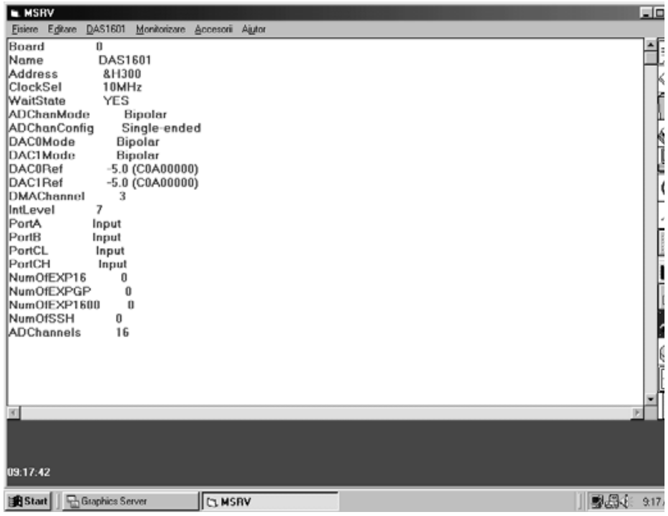


Fig. 10 Window “Configurare”

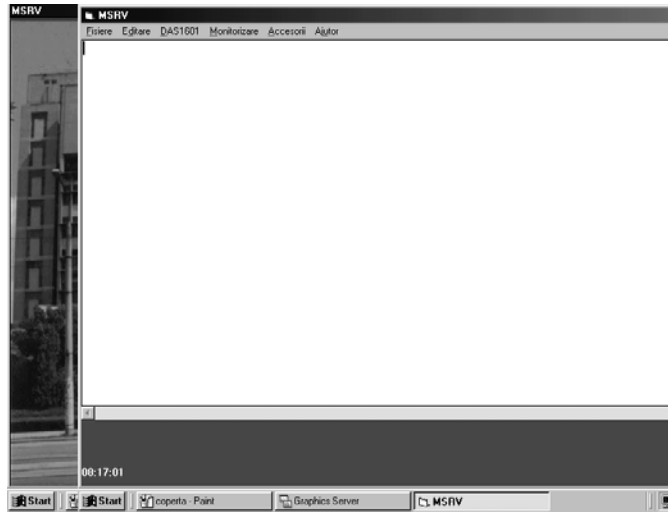


Fig. 13 Window “Editare”

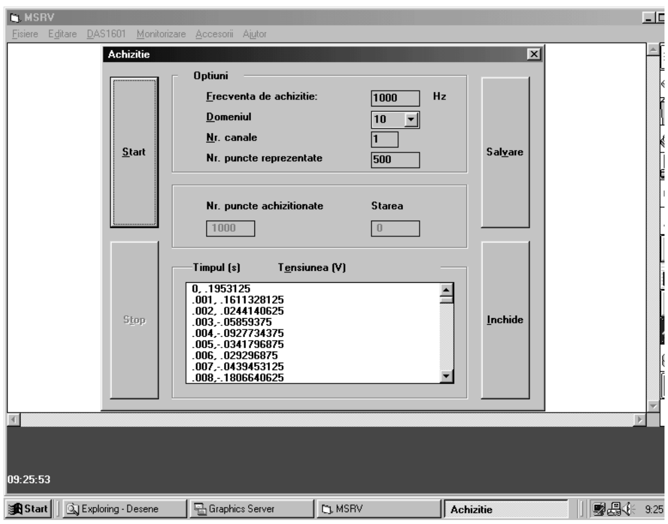


Fig. 11 Window “Achizitie”

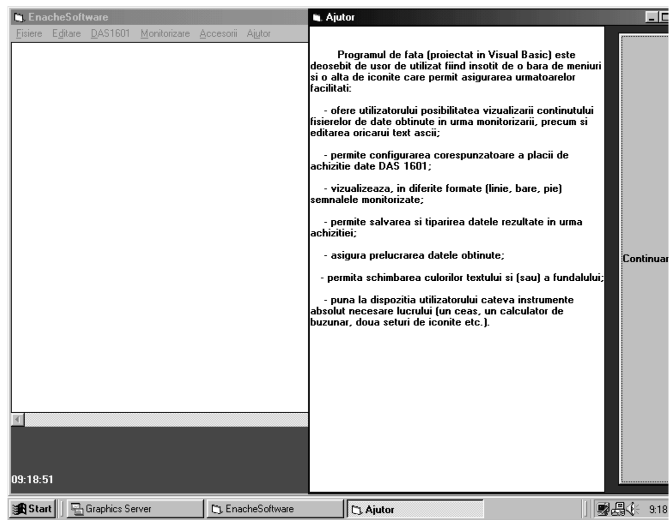


Fig. 14 Window “Ajutor”

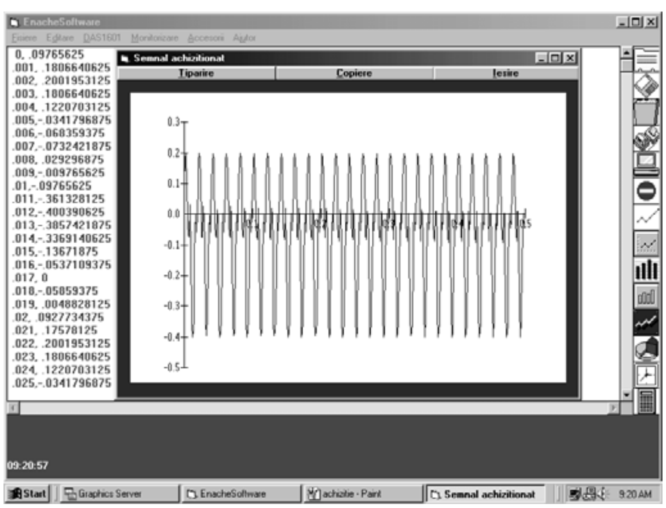


Fig. 12 Window “Semnal”

This program has many facilities:

- allows the configuration of the data acquisition board;
- ensures the work aided by a help window.
- ensures the acquisition corresponding to the dynamic signal we want;
- allows the visualization in different forms (line, bars, pie) for the acquired signal;
- allows to edit the files ASCII of the obtained data;
- allows to save and to type data;
- ensures the access to a series of accessories useful during work (pocket computer, clock);
- allows the configuration corresponding to the work interface (background and text colour, text dimensions, icons on the desktop).

VI. Experimental Results

A series of graphic results have been obtained with the help of the acquisition program detailed before; the following figures are presented further on.

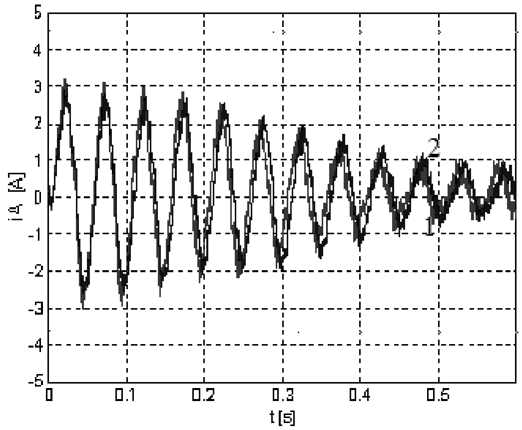


Fig. 15 Graphics dependences corresponding to the cases $L_S = 0{,}529$ H (1) and $L_S = 0{,}549$ H (2)

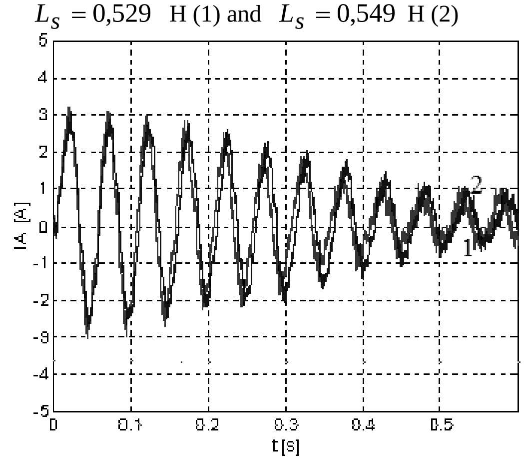


Fig. 16 Graphics dependences corresponding to the cases $L_r^{/}$ =0,528 H (1) and $L_r^{/}$ =0,548 H (2)

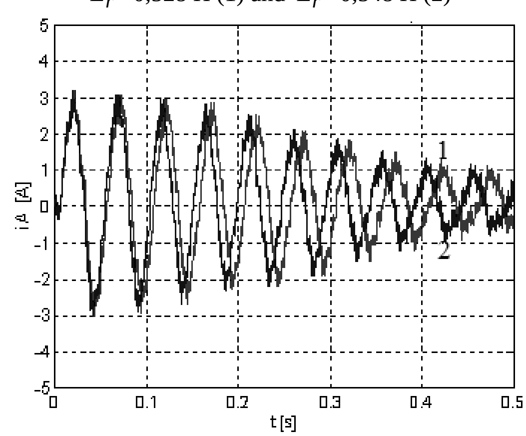


Fig. 17 Graphics dependences corresponding to the cases $L_{sh} = 0{,}498$ H (1) and $L_{sh} = 0{,}558$ H (2)

VII. Conclusions

These graphics lead to the following conclusions:

- when the value of the stator inductance increases the transient process duration increases (the stability decrease);
- the increase of the rotor inductance also involves the increase of the transient process duration (the stability decreases);
- the decrease of the main inductance value determines a faster stabilization of the process (stability increase).

These conclusions confirm the theoretical analysis performed before.

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