Optimal determination of partial ratios of three-step helical gearboxes with first and third step double gear-sets for getting minimal gearbox length

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***Abstract*—**This paper introduces a new study on the applications of optimization and regression analysis techniques for optimal determination of partial ratios of three-step helical gearboxes with first and third step double gear-sets for getting minimal gearbox length. In the study, from the condition of the moment equilibrium of a mechanic system including gear units and their regular resistance condition, an optimization program for determining the partial ratios of the gearboxes are performed. From the results of the optimization program, explicit models for determining the partial ratios are found by using regression analysis technique. Using these models, the prediction of the partial ratios of the gearboxes is accurate and simple.

***Keywords*—** Gearbox design; optimal design; helical gearbox; transmission ratio.

I. Introduction

In gearbox design, the optimal determination of partial transmission ratios of the gearbox has a decisive role. This is because the size, the mass, and the cost of the gearbox depend mainly on the partial ratios. For this reason, many researches have been done in order to find the optimal partial ratios of gearboxes.

For three-step helical gearboxes, there have been several ways for determination of the partial transmission ratios. V.N. Kudreavtev et al. [1] introduced a graph method for determining the partial ratios for getting the minimal gearbox mass (see Figure 1). The graph method was also carried out by A.N. Petrovski et al. [2] for getting the minimal volume of gears. Romhild I. et al. [3] proposed models for prediction of the partial ratios in order to get the minimal gearbox mass. Recently, Vu Ngoc Pi et al. [4] introduced models for determining the partial ratios of the gearboxes for getting the minimal mass of the gears.

From previous studies, it is obvious that there have been many studies on the prediction of the partial ratios for three-step helical gearboxes. However, for three-step helical gearboxes with first and third step double gear-sets (see Figure 2), the optimal partial ratios of the gearboxes are still not investigated. This paper presents a new study on the optimal calculation of partial ratios in order to get the minimal length of the gearboxes.

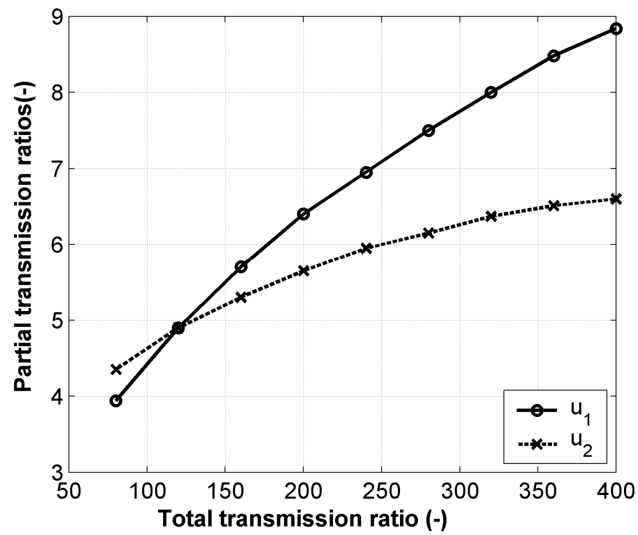

Fig. 1 Determining partial ratios of three-step helical gearbox [1]

II. Determination of the length of the gearbox

The length of a three-step helical gearbox with first and third step double gear-sets is decided by the dimension of L which is determined by the following equation (see Fig. 2):

$$L = \frac{d_{w11}}{2} + a_{w1} + a_{w2} + a_{w3} + a_{w4} + \frac{d_{w24}}{2} \quad (1)$$

The center distance of the first step is calculated as follows:

$$a_{w1} = \frac{d_{w11}}{2} + \frac{d_{w21}}{2} = \frac{d_{w21}}{2} \cdot \left(\frac{d_{w11}}{d_{w21}} + 1 \right) \quad (2)$$

Or:

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$$a_{w1} = \frac{d_{w11}}{2} + \frac{d_{w21}}{2} = \frac{d_{w21}}{2} \cdot \left(\frac{d_{w11}}{d_{w21}} + 1 \right) \quad (3)$$

Doing the same way for the second and the third step we get:

$$a_{w2} = \frac{d_{w22}}{2} \left(\frac{1}{u_2} + 1 \right) \quad (4)$$

$$a_{w3} = \frac{d_{w23}}{2} \left(\frac{1}{u_3} + 1 \right) \quad (5)$$

Substituting (3), (4) and (5) into (1) with the note that $d_{w11} = d_{w21} / u_1$ we have:

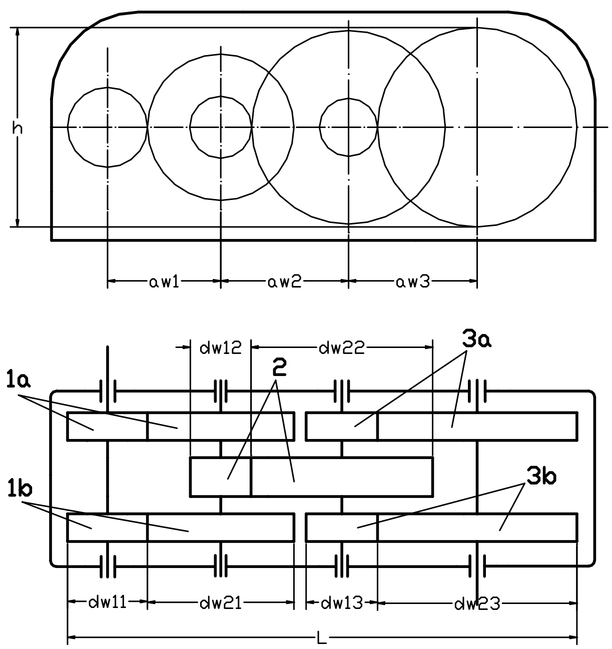

Fig 2: Calculating schema for three-step helical gearbox with first and third step double gear-sets

$$L = \frac{d_{w21}}{2} \cdot \left(\frac{2}{u_1} + 1 \right) + \frac{d_{w22}}{2} \cdot \left(\frac{1}{u_2} + 1 \right) + \frac{d_{w23}}{2} \cdot \left(\frac{1}{u_3} + 2 \right) \quad (6)$$

In the above equations, u_1, u_2, u_3 are partial ratios, d_{w11}, d_{w12}, d_{w21}, d_{w22}, d_{w23} are pitch diameters (mm) and a_{w1}, a_{w2}, a_{w3} are center distances (mm) of helical gear units 1, 2 and 3, respectively.

For the first step (a helical unit), the design equation for the pitting resistance can be written as follows [5]:

$$\sigma_{H1} = Z_{M1} Z_{H1} Z_{\varepsilon 1} \sqrt{\frac{2 T_{11} K_{H1} \sqrt{u_1 + 1}}{b_{w1} d_{w11}^2 u_1}} \le [\sigma_{H1}] \quad (7)$$

From (7) we have:

$$[T_{11}] = \frac{b_{w1} \cdot d_{w11}^2 \cdot u_1}{2 \cdot (u_1 + 1)} \frac{[\sigma_{H1}]^2}{K_{H1} \cdot (Z_{M1} \cdot Z_{H1} \cdot Z_{\varepsilon 1})^2} \quad (8)$$

Where, b_{w1} and d_{w11} are determined by the following equations:

$$b_{w1} = \psi_{ba1} \cdot a_{w1} = \frac{\psi_{ba1} \cdot d_{w11} \cdot (u_1 + 1)}{2} \quad (9)$$

$$d_{w11} = \frac{d_{w21}}{u_1} \quad (10)$$

Substituting (9) and (10) into (8) we have:

$$[T_{11}] = \frac{\psi_{ba1} \cdot d_{w21}^3 \cdot [K_{01}]}{4 \cdot u_1^2} \quad (11)$$

Where

$$[K_{01}] = \frac{[\sigma_{H1}]^2}{K_{H1} \cdot (Z_{M1} \cdot Z_{H1} \cdot Z_{\varepsilon 1})^2} \quad (12)$$

From (11), d_{w21} can be calculated by:

$$d_{w21} = \left(\frac{4 [T_{11}] u_1^2}{\psi_{ba1} [K_{01}]} \right)^{1/3} \quad (13)$$

Calculating in the same way, the following equations were found:

$$d_{w22} = \left(\frac{4 [T_{12}] u_2^2}{\psi_{ba2} [K_{02}]} \right)^{1/3} \quad (14)$$

$$d_{w23} = \left(\frac{4 [T_{13}] u_3^2}{\psi_{ba3} [K_{03}]} \right)^{1/3} \quad (15)$$

In the above equations, Z_{M1}, Z_{H1}, $Z_{\varepsilon 1}$ are coefficients which consider the effects of the gear material, contact surface shape, and contact ratio of the first gear unit when calculate the pitting resistance; $[\sigma_{H1}]$ is allowable contact stresses of the first helical gear unit; ψ_{ba1}, ψ_{ba2} and ψ_{ba3} are coefficients of helical gear face width of steps 1, 2 and 3, respectively.

Based on the moment equilibrium condition of the mechanic system including three gear steps and the regular resistance condition of the system we have:

$$\frac{T_r}{2 \cdot T_{11}} = \frac{[T_r]}{2 \cdot [T_{11}]} = u_1 \cdot u_2 \cdot u_3 \cdot \eta_{brt}^3 \cdot \eta_o^3 \tag{16}$$

In the above equation, η_{brt} is helical gear transmission efficiency (η_{brt} is from 0.96 to 0.98 [5]); η_o is transmission efficiency of a pair of rolling bearing (η_o is from 0.99 to 0.995 [5]).

Choosing the above equation, η_{brt} is helical gear transmission efficiency (η_{brt} is from 0.96 to 0.98 [5]); η_o is transmission efficiency of a pair of rolling bearing (η_o is from 0.99 to 0.995 [5]).

Substituting (17) into (13) with the note that $u_1 = u_h / u_2$ we have:

$$[T_{11}] = \frac{0.5612 \cdot [T_r]}{u_1 \cdot u_2 \cdot u_3} \tag{17}$$

Substituting (17) into (13) with the note that $u_1 = u_h / u_2$ we have:

$$d_{w21} = \left(\frac{2.2448 \cdot [T_r] \cdot u_h}{\psi_{ba1} \cdot [K_{01}] \cdot u_2^2 \cdot u_3^2} \right)^{1/3} \tag{18}$$

For the second step we have:

$$\frac{T_r}{T_{12}} = \frac{[T_r]}{[T_{12}]} = u_2 \cdot u_3 \cdot \eta_{brt}^2 \cdot \eta_o^2 \tag{19}$$

With $\eta_{brt} = 0.97$ and $\eta_o = 0.992$ (see above), Equation 19 becomes:

$$[T_{12}] = \frac{[T_r]}{0.9259 \cdot u_2 \cdot u_3} \tag{20}$$

Substituting (20) into (14) we have:

$$d_{w22} = \left(\frac{4.3201 \cdot [T_r] \cdot u_2}{\psi_{ba2} \cdot [K_{02}] \cdot u_3} \right)^{1/3} \tag{21}$$

For the third step we also have:

$$\frac{T_r}{2 \cdot T_{13}} = \frac{[T_r]}{2 \cdot [T_{12}]} = u_3 \cdot \eta_{brt} \cdot \eta_o \tag{22}$$

As it was done for the first and the second steps, with $\eta_{brt} = 0.97$ and $\eta_o = 0.992$, the following equation was found:

$$[T_{13}] = \frac{0.5196 \cdot [T_r]}{u_3} \tag{23}$$

From (23) and (15) we have:

$$d_{w23} = \left(\frac{2.0784 \cdot [T_r] \cdot u_3}{\psi_{ba3} \cdot [K_{03}]} \right)^{1/3} \tag{24}$$

Substituting (18), (21) and (24) into (6), the length of the gearbox is calculated as follows:

$$L = \frac{1}{2} \left(\frac{[T_r]}{[K_{01}]} \right)^{1/3} \cdot \left[\left(\frac{2.2448 \cdot u_h}{\psi_{ba1} \cdot u_2^2 \cdot u_3^2} \right)^{1/3} \cdot \left(\frac{2}{u_1} + 1 \right) + \right. \\ \left. + \left(\frac{4.3201 \cdot u_2}{\psi_{ba2} \cdot K_{C2} \cdot u_3} \right)^{1/3} + \left(\frac{2.0784 \cdot u_3}{\psi_{ba3} \cdot K_{C3}} \right)^{1/3} \cdot \left(\frac{1}{u_3} + 2 \right) \right] \tag{25}$$

Where, $K_{C2} = [K_{02}]/[K_{01}]$ and $K_{C3} = [K_{03}]/[K_{01}]$:

III. Optimization Problem and Results

From Equations 25, the optimal problem for finding the partial ratios in order to get the minimal gearbox length can be expressed as follows:

The objective function is:

$$\min L = f(u_h; u_2; u_3) \tag{26}$$

With the following constraints:

$$\begin{gathered} u_{h\min} \le u_h \le u_{h\max} \\ u_{2\min} \le u_2 \le u_{2\max} \\ u_{3\min} \le u_3 \le u_{3\max} \\ K_{C2\min} \le K_{C2} \le K_{C2\max} \\ K_{C3\min} \le K_{C3} \le K_{C3\max} \\ \psi_{ba1\min} \le \psi_{ba1} \le \psi_{ba1\max} \\ \psi_{ba2\min} \le \psi_{ba2} \le \psi_{ba2\max} \\ \psi_{ba3\min} \le \psi_{ba3} \le \psi_{ba3\max} \end{gathered} \tag{27}$$

To perform the above optimization problem, a computer program was built. The following data were used in the program: K_{C2} and K_{C3} were from 1 to 1.3, ψ_{ba1}, ψ_{ba2} and ψ_{ba3} were from 0.25 to 0.4 [5], u_2 and u_3 were from 1 to 9 [1]; u_h was from 30 to 120.

Regression analysis was carried out based on the results of the program and the following models for the prediction of the optimal partial ratios of the second and the third steps were found:

$$u_2 \approx \frac{0.9824 \cdot \left(K_{C2} \cdot \psi_{ba2}\right)^{0.4412}}{\left(K_{C3} \cdot \psi_{ba3}\right)^{0.1356}} \cdot \frac{u_h^{0.2707}}{\psi_{ba1}^{0.3056}} \tag{28}$$

$$u_2 \approx \frac{1.6644 \cdot \left(K_{C3} \cdot \psi_{ba3}\right)^{0.4413}}{\left(K_{C2} \cdot \psi_{ba2}\right)^{0.3089}} \cdot \frac{u_h^{0.1172}}{\psi_{ba1}^{0.1325}} \tag{29}$$

The above regression models fit very well with the data. The coefficients of determination were $R^2 = 0.9999$ for both Equations 28 and 29.

Equations 28 and 29 are used to calculate the partial ratios u_2 and u_3 of steps 2 and 3, respectively. After determining u_2 and u_3, the partial ratio of the first step u_1 can be predicted as follows:

$$u_1 = \frac{u_h}{u_2 \cdot u_3} \tag{30}$$

IV. Conclusions

It can be concluded that the minimal length of a three-step helical gearbox with first and third step double gear-sets can be obtained by optimal splitting the total transmission ratio of the gearboxes.

Models for prediction of the optimal partial ratios for getting the minimal gearbox length have been proposed.

The partial ratios of the gearboxes can be determined accurately and simply by using explicit models.

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