A study on optimum calculation of the total transmission ratio of four-step helical gearboxes with first and third step double gear-sets

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***Abstract*—**This paper presents a new study on the applications of optimization and regression analysis techniques for optimum determination of the partial transmission ratios of four-step helical gearboxes with first and third step double gear-sets. The objectives of the calculations are minimum mass of gears and minimum gearbox cross section dimension. Based on the moment equilibrium condition of a mechanic system including gear units and their regular resistance condition, optimization programs for predicting the partial ratios of the gearboxes is conducted. Regression analyses were carried out based on the results of the optimization programs and explicit models for the determinations of the partial ratios are proposed. Using these models, the calculation of the partial ratios is accurate and simple.

***Keywords*—** Gearbox design; optimum design; helical gearbox; transmission ratio.

I. Introduction

In optimum gearbox design, the optimum determination of partial transmission ratios of the gearbox plays the most important role. This is because size, the dimension, the weight, and the cost of the gearbox depend mainly on the partial ratios. Therefore, optimum prediction of the partial ratios has been subjected to many researches.

For helical gearboxes, up to now, many studies have been done on the determination of the partial ratios. To determine the partial ratios of the gearboxes, the following methods have been used:

-By graph method: in this method, the partial ratios are predicted graphically. This method was used in [1], [2], [3] for two, three and four-step helical gearboxes. Fig. 1 shows an example of the graph method.

-By "practical method": this method was proposed by G. Milou et al. [4] for determining the partial ratios of two step helical gearboxes. The method is call "practical" because it was based on practical data.

-By models: in this method, from the results of optimization problems, models for calculation of the partial ratios have been found for different targets, such as for getting the minimum cross section dimension of two-step gearboxes [5], for getting the minimum gearbox mass of two and three-step gearboxes [6], or for minimum mass of gears of three-step gearboxes [7].

It is clear that the modeling method is more accurate and easier to determine the partial transmission ratios than the other methods. Particularly, the modeling method can be usefully applied in computer programming or other applications. Also, it is allowed to have different objectives when using the modeling method for determining the partial transmission ratios.

From previous studies it can be noted that there have been many researches on the prediction of the partial transmission ratios of two, three and four-step helical gearboxes. However, there still have not been studies on the prediction of the partial ratio of four-step helical gearboxes with first and third step double gear-sets. This paper introduces a new study on optimum calculation of the partial transmission ratio for the gearbox in order to get the minimum gearbox mass and the minimum gearbox cross section dimension.

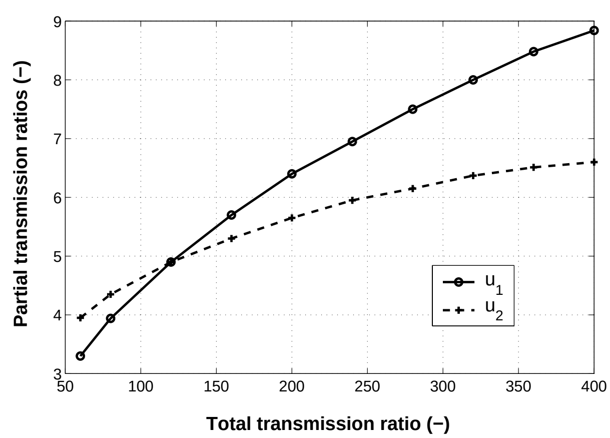

Fig. 1 Determination of the partial ratios of three-step helical gearboxes [1]

II. Determination of mass of gears

In practice, the mass of gears of a four-step helical gearbox with first and third-step double gear-sets can be determined as follows:

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$$G = 2 \cdot G_{1a} + G_2 + 2 \cdot G_{3a} + G_4 \quad (1)$$

Where, G_{1a}, G_2, G_{3a} and G_4 are the mass of gears of gear unit 1a, unit 2, unit 3a and unit 4, respectively (see Fig. 2). G_{1a} can be calculated as follows:

$$G_{1a} = \frac{\pi \cdot \rho \cdot d_{w11}^2 \cdot b_{w1} \cdot (e_1 + e_2 \cdot u_1^2)}{4} \quad (2)$$

In which, b_{w1} is the face width (m) of gear unit 1, d_{w11} is pitch diameter (m), ρ is the density of the gear material (kg/m^3), e_1, e_2 are volume coefficients of gear 1 and 2, respectively. The volume coefficient of a gear is ratio of the actual volume to the theoretical volume of the gear. For a helical gear unit, we can have e_1=1; e_2=0.6 [2].

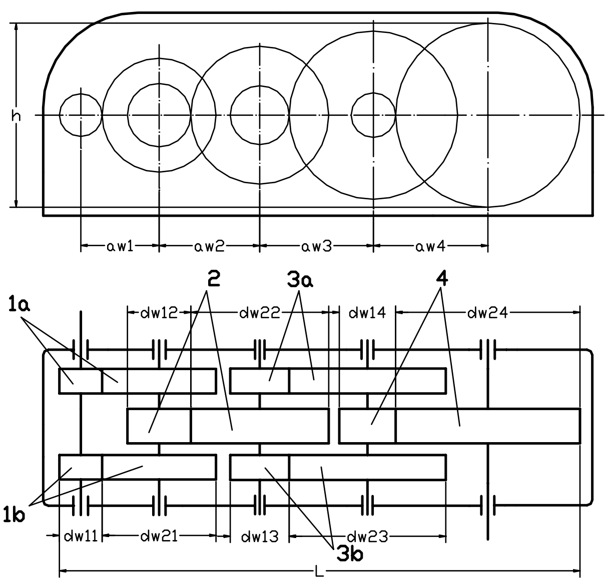

Fig 2: Calculating schema for four-step helical gearbox with first and third step double gear-sets

With $\psi_{bd1} = b_{w1} / d_{w11}$ is diameter coefficient of the first helical gear step, Equation 2 can be rewritten as follows:

$$G_{1a} = \frac{\pi \cdot \rho \cdot \psi_{bd1} \cdot d_{w11}^3 \cdot (1 + 0.6 \cdot u_1^2)}{4} \quad (3)$$

The allowable torque on the driving shaft of the first helical gear unit can be determined by the following equation [8]:

$$\left[T_{11}\right] = \frac{\psi_{bd1} \cdot d_{w11}^3 \cdot u_1 \cdot [K_{01}]}{2 \cdot (u_1 + 1)} \quad (4)$$

Where

$$[K_{01}] = \frac{[\sigma_{H1}]^2}{K_{H1} \cdot \left(Z_{M1} \cdot Z_{H1} \cdot Z_{\varepsilon 1}\right)^2}$$

In the above equations, Z_{M1}, Z_{H1}, $Z_{\varepsilon 1}$ are coefficients which consider the effects of the gear material, contact surface shape, and contact ratio of the first gear unit when calculate the pitting resistance; $\left[\sigma_{H1}\right]$ is allowable contact stresses of the first helical gear unit.

It follows Equation 4 that:

$$\psi_{bd1} \cdot d_{w11}^3 = \frac{2 \cdot (u_1 + 1) \cdot [T_{11}]}{u_1 \cdot [K_{01}]} \quad (5)$$

Substituting (5) into (3) we get

$$G_{1a} = \frac{2 \cdot \pi \cdot \rho \cdot [T_{11}] \cdot (u_1 + 1) \cdot (1 + 0.6 \cdot u_1^2)}{4 \cdot [K_{01}] \cdot u_1} \quad (6)$$

Calculating in same way, the following equations were found for the second, the third and the fourth steps:

$$G_2 = \frac{2 \cdot \pi \cdot \rho \cdot [T_{12}] \cdot (u_2 + 1) \cdot (1 + 0.6 \cdot u_2^2)}{4 \cdot [K_{02}] \cdot u_2} \quad (7)$$

$$G_{3a} = \frac{2 \cdot \pi \cdot \rho \cdot [T_{13}] \cdot (u_3 + 1) \cdot (1 + 0.6 \cdot u_3^2)}{4 \cdot [K_{03}] \cdot u_3} \quad (8)$$

$$G_4 = \frac{2 \cdot \pi \cdot \rho \cdot [T_{14}] \cdot (u_4 + 1) \cdot (1 + 0.6 \cdot u_4^2)}{4 \cdot [K_{04}] \cdot u_4} \quad (9)$$

From the condition of moment equilibrium of the mechanic system including the gear units and the regular resistance condition of the system we have:

$$\frac{T_r}{2 \cdot T_{11}} = \frac{\left[T_r\right]}{2 \cdot \left[T_{11}\right]} = u_1 \cdot u_2 \cdot u_3 \cdot u_4 \cdot \eta_{brt}^4 \cdot \eta_o^4 \quad (10)$$

In the above equation, T_r and $\left[T_r\right]$ are torque and allowable torque on the output shaft (N/m), T_{11} and $\left[T_{11}\right]$ are torques and allowable torque of the and driving shaft of gear unit 1 (N/m), η_{brt} is helical gear transmission efficiency (η_{brt} is from 0.96 to 0.98 [7]), η_o is transmission efficiency of a pair of rolling bearing (η_o is from 0.99 to 0.995 [7]).

By choosing $\eta_{brt} = 0.97$, $\eta_o = 0.992$ and substituting them into Equation 10 we get:

$$\left[T_{11}\right]=\frac{\left[T_r\right]}{1.7146\cdot u_1\cdot u_2\cdot u_3\cdot u_4} \quad (11)$$

Substituting (11) into (6) we have:

$$G_{1a}=\frac{2\cdot\pi\cdot\rho\cdot[T_r]}{4\cdot[K_{01}]}\cdot\frac{(u_1+1)\cdot(1+0.6\cdot u_1^2)}{1.7146\cdot u_1^2\cdot u_2\cdot u_3\cdot u_4} \quad (12)$$

For the second gear unit we also have:

$$\frac{T_r}{T_{12}}=\frac{\left[T_r\right]}{\left[T_{12}\right]}=u_2\cdot u_3\cdot u_4\cdot\eta_{brt}^3\cdot\eta_o^3 \quad (13)$$

Choosing $\eta_{brt}=0.97$ and $\eta_o=0.992$ we can get $\left[T_{12}\right]$ from Equation 13:

$$\left[T_{12}\right]=\frac{\left[T_r\right]}{0.8909\cdot u_2\cdot u_3\cdot u_4} \quad (14)$$

From Equations 7 and 14 we have the following equation:

$$G_2=\frac{2\cdot\pi\cdot\rho\cdot[T_r]}{4\cdot[K_{02}]}\cdot\frac{(u_2+1)\cdot(1+0.6\cdot u_2^2)}{0.8909\cdot u_2^2\cdot u_3\cdot u_4} \quad (15)$$

By using the same way, the mass of gears of unit 3a and unit 4 can be found as follows:

$$G_{3a}=\frac{2\cdot\pi\cdot\rho\cdot[T_r]}{4\cdot[K_{03}]}\cdot\frac{(u_3+1)\cdot(1+0.6\cdot u_3^2)}{1.8518\cdot u_3^2\cdot u_4} \quad (16)$$

$$G_4=\frac{2\cdot\pi\cdot\rho\cdot[T_r]}{4\cdot[K_{04}]}\cdot\frac{(u_4+1)\cdot(1+0.6\cdot u_4^2)}{0.9622\cdot u_4^2} \quad (17)$$

Substituting (12), (15), (16) and (17) into (1) with the note that $u_1=u_h/\left(u_2\cdot u_3\cdot u_4\right)$, we have the following equation for the calculation of the mass of gears:

$$G=\frac{2\cdot\pi\cdot\rho\cdot[T_r]}{4\cdot[K_{01}]}\cdot \cdot\left\{\frac{(u_h+u_2\cdot u_3\cdot u_4)\cdot\left[1+0.6\cdot u_h^2/\left(u_2^2\cdot u_3^2\cdot u_4^2\right)\right]}{1.7146\cdot u_h^2}+ +\frac{(u_2+1)\cdot(1+0.6\cdot u_2^2)}{0.8909\cdot k_{C2}\cdot u_2^2\cdot u_3\cdot u_4}+ +\frac{(u_3+1)\cdot(1+0.6\cdot u_3^2)}{1.8518\cdot k_{C3}\cdot u_3^2\cdot u_4}+\frac{(u_4+1)\cdot(1+0.6\cdot u_4^2)}{0.9622\cdot k_{C4}\cdot u_4^2}\right\} \quad (18)$$

In which, $k_{C2}=[K_{02}]/[K_{01}]$, $k_{C3}=[K_{03}]/[K_{01}]$ and $k_{C4}=[K_{04}]/[K_{01}]$.

III. Determination of gearbox cross section dimention

The cross section dimension of a four-step helical gearbox with first and third step double gear-sets is decided by the dimension of A which is determined as follows (see Fig.2):

$$A=L\cdot h \quad (19)$$

In which, h and L are determined by the following equations:

$$h=\max(d_{w21},d_{w22},d_{w23},d_{w24}) \quad (20)$$

$$L=\frac{d_{w11}}{2}+a_{w1}+a_{w2}+a_{w3}+a_{w4}+\frac{d_{w24}}{2} \quad (21)$$

The center distance of the first helical gear unit is calculated by the following equation:

$$a_{w1}=\frac{d_{w11}}{2}+\frac{d_{w21}}{2}=\frac{d_{w21}}{2}\cdot\left(\frac{d_{w11}}{d_{w21}}+1\right) \quad (22)$$

Or

$$a_{w1}=\frac{d_{w21}}{2}\left(\frac{1}{u_1}+1\right) \quad (23)$$

Using the same way for the second, the third and the fourth step we get:

$$a_{w2}=\frac{d_{w22}}{2}\left(\frac{1}{u_2}+1\right) \quad (24)$$

$$a_{w3}=\frac{d_{w23}}{2}\left(\frac{1}{u_3}+1\right) \quad (25)$$

$$a_{w4}=\frac{d_{w24}}{2}\left(\frac{1}{u_4}+1\right) \quad (26)$$

From (23), (24), (25) and (26) and note that $d_{w11}=d_{w21}/u_1$, Equation (21) can be expressed by the following equation:

$$L=\frac{d_{w21}}{2}\cdot\left(\frac{2}{u_1}+1\right)+\frac{d_{w22}}{2}\cdot\left(\frac{1}{u_2}+1\right)+ +\frac{d_{w23}}{2}\cdot\left(\frac{1}{u_3}+1\right)+\frac{d_{w24}}{2}\cdot\left(\frac{1}{u_4}+2\right) \quad (27)$$

In the above equations, u_1, u_2, u_3, u_4 are partial ratios, d_{w11}, d_{w12}, d_{w21}, d_{w22}, d_{w23}, d_{w24} are pitch diameters (mm) and a_{w1}, a_{w2}, a_{w3}, a_{w4} are center distances (mm) of helical gear units 1, 2, 3 and 4, respectively.

For the first helical gear unit, the following equation for the pitting resistance is written [8]:

$$\sigma_{H1} = Z_{M1} Z_{H1} Z_{\varepsilon 1} \sqrt{\frac{2T_{11} K_{H1} \sqrt{u_1 + 1}}{b_{w1} d_{w11}^2 u_1}} \le \left[\sigma_{H1}\right] \tag{28}$$

From Equation 28 we have

$$\left[T_{11}\right] = \frac{b_{w1} \cdot d_{w11}^2 \cdot u_1}{2 \cdot (u_1 + 1)} \frac{\left[\sigma_{H1}\right]^2}{K_{H1} \cdot \left(Z_{M1} \cdot Z_{H1} \cdot Z_{\varepsilon 1}\right)^2} \tag{29}$$

Where, b_{w1} and d_{w11} are calculated by the following equations:

$$b_{w1} = \psi_{ba1} \cdot a_{w1} = \frac{\psi_{ba1} \cdot d_{w11} \cdot (u_1 + 1)}{2} \tag{30}$$

$$d_{w11} = \frac{d_{w21}}{u_1} \tag{31}$$

Substituting (30) and (31) into (29) we have:

$$\left[T_{11}\right] = \frac{\psi_{ba1} \cdot d_{w21}^3 \cdot \left[K_{01}\right]}{4 \cdot u_1^2} \tag{32}$$

Where

$$\left[K_{01}\right] = \frac{\left[\sigma_{H1}\right]^2}{K_{H1} \cdot \left(Z_{M1} \cdot Z_{H1} \cdot Z_{\varepsilon 1}\right)^2} \tag{33}$$

From Equation 32 d_{w21} can be calculated by:

$$d_{w21} = \left(\frac{4\left[T_{11}\right] u_1^2}{\psi_{ba1}\left[K_{01}\right]}\right)^{1/3} \tag{34}$$

Calculating in the same way, the following equations were found:

$$d_{w22} = \left(\frac{4\left[T_{12}\right] u_2^2}{\psi_{ba2}\left[K_{02}\right]}\right)^{1/3} \tag{35}$$

$$d_{w23} = \left(\frac{4\left[T_{13}\right] u_3^2}{\psi_{ba3}\left[K_{03}\right]}\right)^{1/3} \tag{36}$$

$$d_{w24} = \left(\frac{4\left[T_{14}\right] u_4^2}{\psi_{ba4}\left[K_{04}\right]}\right)^{1/3} \tag{37}$$

In the above equations, Z_{M1}, Z_{H1}, $Z_{\varepsilon 1}$ are coefficients considering the effects of the gear material, contact surface shape, and contact ratio of the first gear unit when calculate the pitting resistance; $\left[\sigma_{H1}\right]$ is allowable contact stresses of the first helical gear unit; ψ_{ba1}, ψ_{ba2}, ψ_{ba3} and ψ_{ba4} are coefficients of helical gear face width of steps 1, 2, 3 and 4, respectively.

Based on the moment equilibrium condition of the mechanic system including gear units and the regular resistance condition of the system we get:

$$\frac{T_r}{2 \cdot T_{11}} = \frac{\left[T_r\right]}{2 \cdot \left[T_{11}\right]} = u_1 \cdot u_2 \cdot u_3 \cdot u_4 \cdot \eta_{brt}^4 \cdot \eta_o^4 \tag{38}$$

In which, η_{brt} is helical gear transmission efficiency (η_{brt} is from 0.96 to 0.98 [8]), η_o is transmission efficiency of a pair of rolling bearing (η_o is from 0.99 to 0.995 [8]).

Choosing $\eta_{brt} = 0.97$, $\eta_o = 0.992$ and substituting them into Equation 20 we have:

$$\left[T_{11}\right] = \frac{\left[T_r\right]}{1.7146 \cdot u_1 \cdot u_2 \cdot u_3 \cdot u_4} \tag{39}$$

Substituting (39) into (34) with the note that $u_1 = u_h / u_2$ we have:

$$d_{w21} = \left(\frac{2.3329 \cdot \left[T_r\right] \cdot u_h}{\psi_{ba1} \cdot \left[K_{01}\right] \cdot u_2^2 \cdot u_3^2 \cdot u_4^2}\right)^{1/3} \tag{40}$$

The following equation can be written for the second helical gear unit:

$$\frac{T_r}{T_{12}} = \frac{\left[T_r\right]}{\left[T_{12}\right]} = u_2 \cdot u_3 \cdot u_4 \cdot \eta_{brt}^3 \cdot \eta_o^3 \tag{41}$$

From (41) and with $\eta_{brt} = 0.97$ and $\eta_o = 0.992$ (see above) we have:

$$\left[T_{12}\right] = \frac{\left[T_r\right]}{0.8909 \cdot u_2 \cdot u_3 \cdot u_4} \tag{42}$$

Substituting (42) into (35) we have:

$$d_{w22} = \left(\frac{4.4898 \cdot \left[T_r\right] \cdot u_2}{\psi_{ba2} \cdot \left[K_{02}\right] \cdot u_3 \cdot u_4} \right)^{1/3} \tag{43}$$

As it was done for the first step, for the third and the fourth steps the following equations were found:

$$d_{w23} = \left(\frac{2.1601 \cdot \left[T_r\right] \cdot u_3}{\psi_{ba3} \cdot \left[K_{03}\right] \cdot u_4} \right)^{1/3} \tag{44}$$

$$d_{w24} = \left(\frac{4.1571 \cdot \left[T_r\right] \cdot u_4}{\psi_{ba4} \cdot \left[K_{04}\right]} \right)^{1/3} \tag{45}$$

Substituting (40), (43), (44) and (45) into (9) we get:

$$\begin{aligned} L = \frac{1}{2}\left(\frac{\left[T_r\right]}{\left[K_{01}\right]}\right)^{1/3} \cdot \Bigg[& \left(\frac{2.3329 \cdot u_h}{\psi_{ba1} \cdot u_2^2 \cdot u_3^2 \cdot u_4^2} \right)^{1/3} \cdot \left(\frac{2}{u_1} + 1 \right) + \\ & + \left(\frac{4.4898 \cdot u_2}{\psi_{ba2} \cdot K_{C2} \cdot u_3 \cdot u_4} \right)^{1/3} + \left(\frac{2.1601 \cdot u_3}{\psi_{ba3} \cdot K_{C3} \cdot u_4} \right)^{1/3} + \\ & + \left(\frac{4.1571 \cdot u_4}{\psi_{ba4} \cdot K_{C4}} \right)^{1/3} \cdot \left(\frac{1}{u_4} + 2 \right) \Bigg] \end{aligned} \tag{46}$$

In which, $K_{C2} = \left[K_{02}\right]/\left[K_{01}\right]$, $K_{C3} = \left[K_{03}\right]/\left[K_{01}\right]$, and $K_{C4} = \left[K_{04}\right]/\left[K_{01}\right]$.

From Equations 22, 25, 26 and 27, Equation 2 can be rewritten as follows:

$$\begin{aligned} h = \max\Bigg(& \frac{2.3329 \cdot u_h}{\psi_{ba1} \cdot u_2^2 \cdot u_3^2 \cdot u_4^2}; \frac{4.4898 \cdot u_2}{\psi_{ba2} \cdot k_{C2} \cdot u_3 \cdot u_4}; \\ & \frac{2.1601 \cdot u_3}{\psi_{ba3} \cdot k_{C3} \cdot u_4}; \frac{4.1571 \cdot u_4}{\psi_{ba4} \cdot k_{C4}} \Bigg) \end{aligned} \tag{47}$$

IV. Optimization Problem and Results

A. For determining the optimum partial ratios for getting the minimum mass of gears

A.1 Optimization problem

From Equation 18, the optimization problem for determining the partial transmission ratio in order to get the minimal mass of gears can be expressed as follows:

The objective function of the problem is:

$$\min G = f(u_h; u_2; u_3; u_4) \tag{48}$$

With the following constraints:

$$\begin{aligned} & u_{h\min} \le u_h \le u_{h\max} \\ & u_{2\min} \le u_2 \le u_{2\max} \\ & u_{3\min} \le u_3 \le u_{3\max} \\ & u_{4\min} \le u_4 \le u_{4\max} \\ & k_{C2\min} \le k_{C2} \le k_{C2\max} \\ & k_{C3\min} \le k_{C3} \le k_{C3\max} \\ & k_{C4\min} \le k_{C4} \le k_{C4\max} \end{aligned} \tag{49}$$

For solving the above optimization problem, a computer program was built. The data used in the program were: u_2, u_3 and u_4 were from 1 to 9 [1], k_{C2}, k_{C3} and k_{C4} were from 1 to 1.3 [8] and u_h was from 50 to 400.

A2 Results and discussions

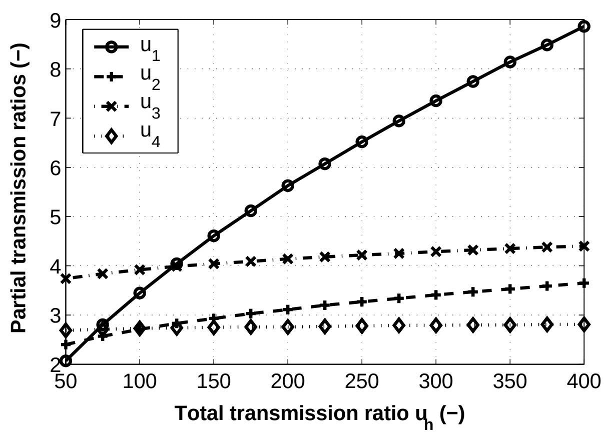

Fig 3: Partial ratios versus the total transmission ratio

Fig.3 shows the relation between the partial transmission ratios and the total transmission ratio when the coefficients k_{C2}, k_{C3} and k_{C4} equal 1.1. It is clear that the larger of the total ratio u_h the larger of the partial ratios. In addition, the increase of u_1 is much larger than that of u_4 when the total ratio increases. The reason of that is with the increase of the total ratio u_h, the torque on the output shaft T_r is much larger than that on the driving shaft of the first gear unit T_{11}. As a result, the increase of the partial ratio u_4 should be much smaller than that of u_1 in order to reduce the mass of the gears.

Based on the results of the optimization program, regression analysis was carried out and the following models were found for determining the optimal values of the partial ratios of the second, the third and the fourth steps:

$$u_2 \approx 0.0969 \cdot \frac{k_{C2}^{0.9547}}{k_{C3}^{0.8039} \cdot k_{C4}^{0.1128}} \cdot u_3^{2.0605} \cdot u_4^{0.3553} \cdot u_h^{0.0334} \tag{50}$$

$$u_3 \approx 2.5041 \cdot \frac{k_{C3}^{0.4274}}{k_{C2}^{0.3962} \cdot k_{C4}^{0.0228}} \cdot u_2^{0.3423} \cdot u_4^{0.0703} \cdot u_h^{0.008} \tag{51}$$

$$u_4 \approx 2.2918 \cdot \frac{k_{C4}^{0.2883}}{k_{C2}^{0.0761} \cdot k_{C3}^{0.2101}} \cdot u_2^{0.0623} \cdot u_3^{0.0743} \cdot u_h^{0.0024} \tag{52}$$

The above regression models fit quite well with the data. The coefficients of determination for the models 50, 51 and 52 were $R^2 = 0.9991$, $R^2 = 0.9994$ and $R^2 = 0.9979$, respectively.

Equations 50, 51 and 52 are used to calculate the transmission partial ratios u_2, u_3 and u_4 of the second, the third and the fourth gear units. After determining u_2, u_3 and u_4, the transmission ratio of the first gear unit u_1 is calculated as follows:

$$u_1 = \frac{u_h}{u_2 \cdot u_3 \cdot u_4} \tag{53}$$

B. For determining the optimum partial ratios for getting the minimum cross section dimension of the gearboxes

B1 Optimization problem

Based on Equations 19, 46 and 47, the optimal problem for determining the partial ratios for getting the minimal cross section dimension of the gearbox can be expressed as follows:

The objective function of the problem is:

$$\min A = f(u_h; u_2; u_3; u_4) \tag{54}$$

The constraints of the problems are:

$$u_{h\min} \le u_h \le u_{h\max}$$

$$u_{2\min} \le u_2 \le u_{2\max}$$

$$u_{3\min} \le u_3 \le u_{3\max}$$

$$u_{4\min} \le u_4 \le u_{4\max}$$

$$K_{C2\min} \le K_{C2} \le K_{C2\max}$$

$$K_{C3\min} \le K_{C3} \le K_{C3\max} \tag{55}$$

$$K_{C4\min} \le K_{C4} \le K_{C4\max}$$

$$\psi_{ba1\min} \le \psi_{ba1} \le \psi_{ba1\max}$$

$$\psi_{ba2\min} \le \psi_{ba2} \le \psi_{ba2\max}$$

$$\psi_{ba3\min} \le \psi_{ba3} \le \psi_{ba3\max}$$

$$\psi_{ba4\min} \le \psi_{ba4} \le \psi_{ba4\max}$$

To solve the above problem a computer program was performed. In the program, the following data were used: K_{C2}, K_{C3}, K_{C4} were from 1 to 1.3, ψ_{ba1}, ψ_{ba2}, ψ_{ba3}, ψ_{ba4} were from 0.25 to 0.4 [8], u_2, u_3, u_4 were from 1 to 9 [1]; u_h was from 50 to 400.

B2 Results and discussions

Regression analysis was carried out based on the results of the optimization program and the following models were found for determining the optimal values of the partial ratios of the second, third and fourth steps:

$$u_2 \approx \frac{0.926 \cdot K_{C2}^{0.4677} \cdot \psi_{ba2}^{0.4706} \cdot u_h^{0.2576}}{K_{C3}^{0.0613} \cdot \psi_{ba3}^{0.0599} K_{C4}^{0.1236} \cdot \psi_{ba4}^{0.1256} \cdot \psi_{ba1}^{0.2735}} \tag{56}$$

$$u_3 \approx \frac{1.6859 \cdot K_{C3}^{0.5712} \cdot \psi_{ba3}^{0.5554} \cdot u_h^{0.1114}}{K_{C2}^{0.149} \cdot \psi_{ba2}^{0.2045} K_{C4}^{0.2392} \cdot \psi_{ba4}^{0.1785} \cdot \psi_{ba1}^{0.1244}} \tag{57}$$

$$u_4 \approx \frac{0.9353 \cdot K_{C4}^{0.3803} \cdot \psi_{ba4}^{0.4107} \cdot u_h^{0.0557}}{K_{C2}^{0.0745} \cdot \psi_{ba2}^{0.1027} K_{C3}^{0.214} \cdot \psi_{ba3}^{0.2226} \cdot \psi_{ba1}^{0.0623}} \tag{58}$$

The above regression models fit quite well with the data (with the coefficients of determination were $R^2 = 0.9912$, $R^2 = 0.9711$, and $R^2 = 0.9746$ for Equations 56, 57 and 58, respectively).

Equations 56, 57 and 58 are used for the prediction of the partial ratios u_2, u_3, and u_4 of steps 2, 3 and 4, respectively. After determining u_2, u_3, and u_4, the partial ratio of the first step u_1 is calculated as follows

$$u_1 = \frac{u_h}{u_2 \cdot u_3 \cdot u_4} \tag{59}$$

V. Conclusions

It can be concluded that the minimal mass of gears, the minimum gearbox cross section dimension of four-step helical gearboxes with first and third-step double gear-sets can be obtained by optimal splitting the total transmission ratio.

Based on the results of the optimization problems, models for predicting the optimal partial ratios of four-step helical gearboxes with first and third-step double gear-sets for getting the minimum mass of gears as well as for getting the minimum gearbox cross section dimension have been proposed.

The partial ratios of the gearboxes can be predicted accurately and simply by using explicit models.

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