Monte Carlo Approach Applied in the Design of Machine Parts and Structures

Karel Frydrýšek

***Abstract*—** This article focuses on the probabilistic (i.e. direct Monte Carlo) numerical solution of the problems in mining, machine industry and biomechanics. Theory and applications of the Simulation-Based Reliability Assessment (SBRA) method (i.e. Monte Carlo approach) are presented in the solution of the hard rock (ore) disintegration process (i.e. the bit moves into the ore and subsequently disintegrates it, the results are compared with experiments, new design of excavation tool is proposed), in the proposal design of machine for fatigue testing of railway axles and in the solution of designing of the external fixators applied in traumatology and orthopaedics (application for the treatment of open and unstable fractures or for lengthening human or animal bones etc.). Applications of the SBRA method connected with FEM in these areas are new and innovative trends in mechanics and designing.

***Keywords*—** Probability, SBRA Method, FEM, design, rock mechanics, fatigue testing, railway axles, biomechanics, traumatology

I. Introduction

LET us consider the Simulation-Based Reliability Assessment (SBRA) Method, see [2], [6], [8], [10], [11] and [17], a probabilistic direct Monte Carlo approach, in which all inputs are given by bounded (truncated) histograms. Bounded histograms include the real variability of the variables. Development of SBRA Method is well presented in books [10] and [11].

Using SBRA method, the probability of failure (i.e. the probability of undesirable situation) is obtained mainly by analyzing the reliability function:

$$RF = RV - S, \qquad (1)$$

see Fig.1. Where in general terms, RV is the reference (allowable) value and S is a variable representing the load effect combination.

The probability of failure is the probability that S exceeds RV, i.e. $P(\mathrm{RF} \le 0)$). The probability of failure is a relative value depending on the definition of RV and it usually does not reflect an absolute value of the risk of failure.

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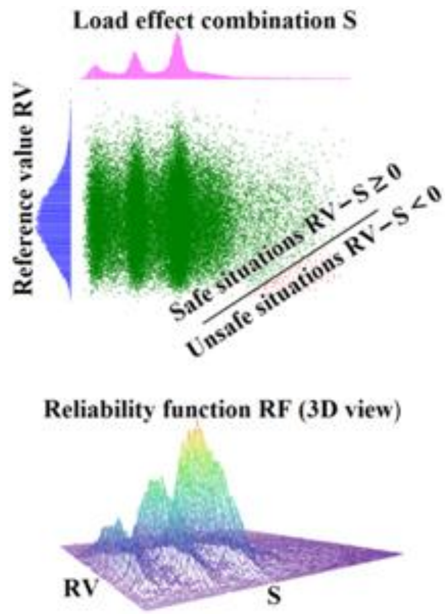

Fig. 1 Reliability function RF (SBRA Method)

This paper focuses on the SBRA Method applied in the quite large area of designing (mining, see chapter II, machinery industry, see chapter III, and biomechanics, see chapter IV).

II. Mining Industry - Solution of the Hard Rock Disintegration Process

The provision of sufficient quantities of raw materials is the main limiting factor of further industry development.

It is therefore very important to understand the ore disintegration process, including an analysis of the bit (i.e. excavation tool) used in mining operations.

This chapter is focused on the modelling of the mechanical contact between the bit and the platinum ore and its evaluation (i.e. practical application in the mining technology), see Fig.2.

However, material properties of the ore have a large stochastic variability. Hence, the stochastic approach (i.e. SBRA Method in combination with FEM is applied). MSC.Marc/Mentat software was used in modelling this problem, see Fig.3 and references [2], [3] and [5].

Fig.2 Typical example of mechanical interaction between bits and hard rock (ore disintegration process)

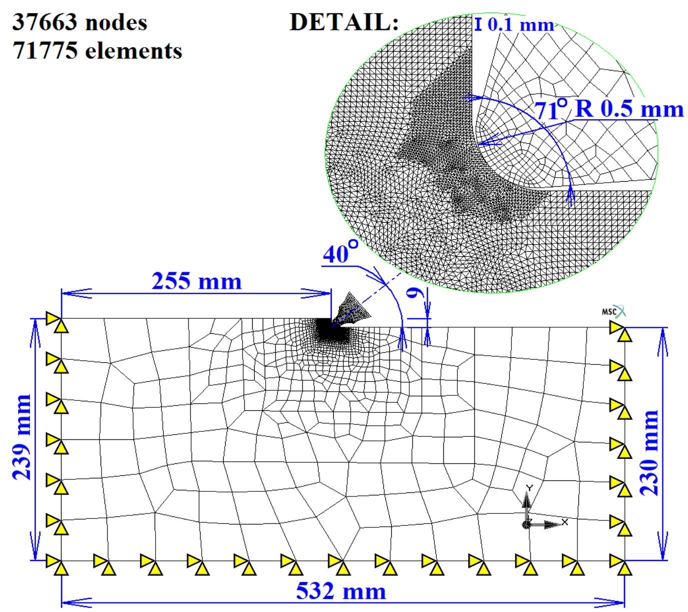

Fig.3 Ore disintegration process (solution via FEM)

The bit moves into the ore with the prescribed time dependent function and subsequently disintegrates it. When the bit moves into the ore (i.e. a mechanical contact occurs between the bit and the ore) the stresses increase. When the equivalent stress is greater than the tensile strength in some elements of the ore, then these elements break off. Hence, a part of the ore disintegrates. This is done by deactivating the elements, see Fig.4.

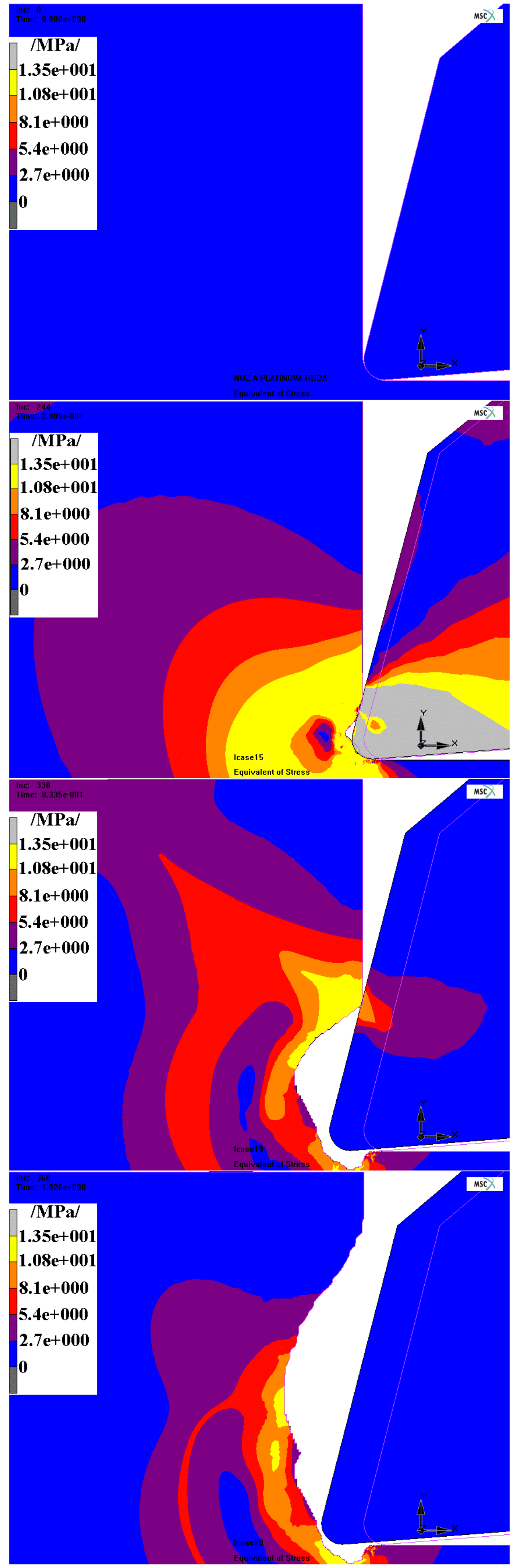

Fig.4 Disintegration of the ore and movement of the bit (equivalent von Mises stresses distributions, result of one Monte Carlo simulation)

The ore material is elasto-plastic with isotropic hardening rule. The probabilistic inputs, i.e. elastic properties (Modulus of elasticity E and Poisson's ratio μ) and plastic properties (yield stress R_p and fracture stress R_m) are described by bounded histograms, see Fig.5 and 6.

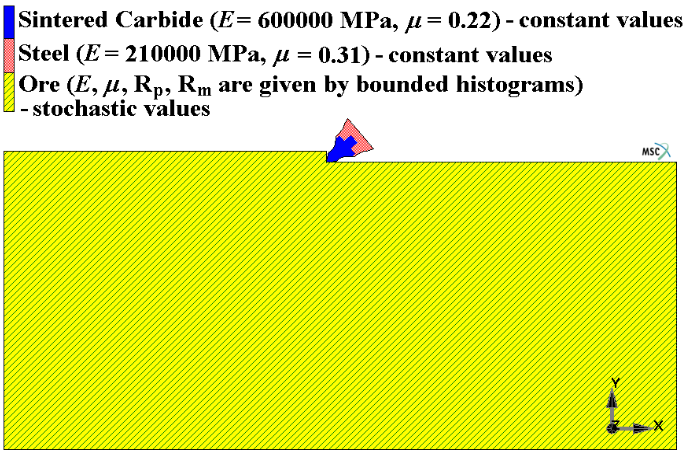

Fig.5 Material properties (deterministic and probabilistic inputs)

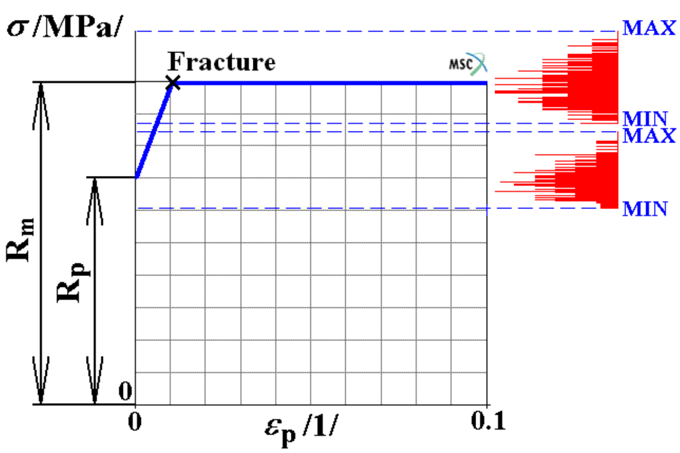

Fig.6 Material properties (probabilistic inputs, stress vs. plastic strain)

The results (acquired by SBRA Method in combination with FEM) were subsequently statistically evaluated (Anthill, MSC.Marc/Mentat and Mathcad software were used). Because of the material non-linearities, the mechanical contacts with friction, the large number of elements, many iteration steps, and the choice of 500 Monte Carlo simulations, four parallel computers (with 26 CPU) were used to handle the large computational requirements for this problem.

From the results, the total reaction forces Rv /N/ can be calculated. These forces act in the bit, see Fig.7 (distribution of the forces acquired from 1 Monte Carlo simulation) and Fig.8 (distribution of the total reaction forces acquired from 500 Monte Carlo simulations - stochastic result, i.e. print of 500 curves). The calculated maximum forces (i.e. SBRA-FEM solutions) can be compared with the experimental measurements. However, the experimental results also have large variability due to the anisotropic and stochastic properties of the material and due to the large variability of the reaction forces.

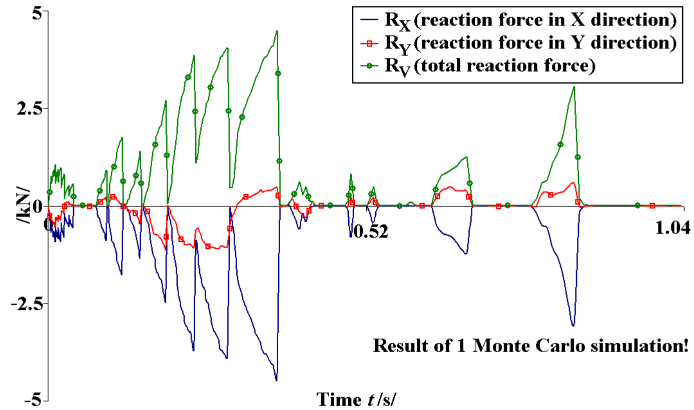

Fig.7 Reaction forces in the bit (one simulation)

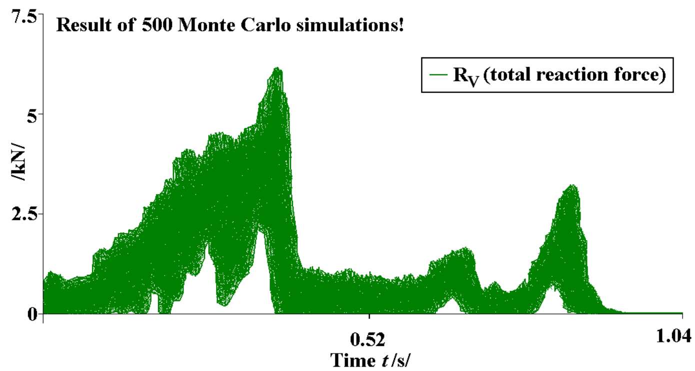

Fig.8 Reaction forces in the bit (probabilistic outputs – 500 simulations)

Reliability function RF, see equation (1) in chapter I and references [2], [6], [10], [11] and [17], can be defined by:

$$\mathrm{RF} = \mathrm{R}_{\mathrm{V\ MAX}_{\mathrm{ALLOWABLE}}} - \mathrm{R}_{\mathrm{V\ MAX}_{\mathrm{SBRA,\ FEM}}}, \tag{2}$$

where $\mathrm{R}_{\mathrm{V\ MAX}_{\mathrm{ALLOWABLE}}}$ is the allowable reaction force in the cutting bit, which can be acquired from the real capacity of the whole cutter-loader system in the mine and $\mathrm{R}_{\mathrm{V\ MAX}_{\mathrm{SBRA,\ FEM}}} = 5068^{+1098}_{-984}$ N is the maximum total reaction force (acquired from 500 Monte Carlo simulations and solved by FEM).

If situations when $\mathrm{RF} \leq 0$ occur, then the cutter-loader system is overloaded. Else if $\mathrm{RF} > 0$, then safe situations of loading occurs, see also definition in Fig.1.

Hence, fully probabilistic assessment can be calculated by comparing of probabilities:

$$P(\mathrm{RF} \leq 0) \leq P_{\mathrm{ALLOWABLE}}, \tag{3}$$

where, $P_{ALLOWABLE}$ is the acceptable probability of overloading of the cutting-loader system. This overloading sometimes really occurs in the mine. Value of $P_{ALLOWABLE}$ can be given by chosen performance requirements of the client (i.e. investor), see Fig.9.

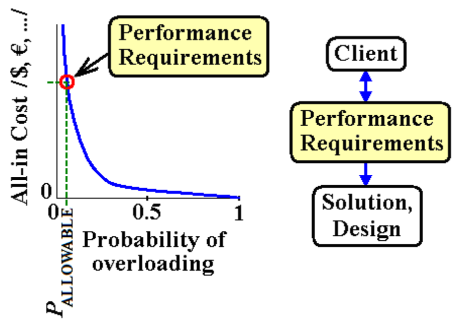

Fig.9 Definition of the acceptable probability of overloading

All the results presented here were applied for optimizing and redesigning of the cutting bit (excavation tool for platinum ore), see Fig.10 and 11.

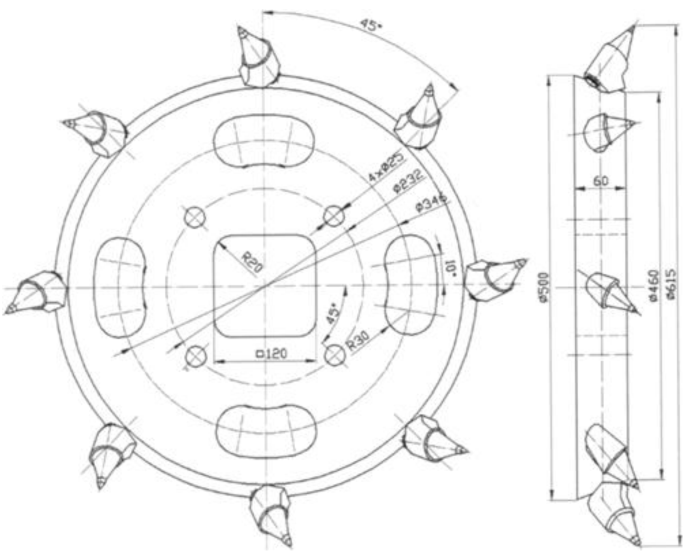

Fig.10 Final shape of excavation tool for platinum ore disintegration process

For more information see references [2], [3] and [5].

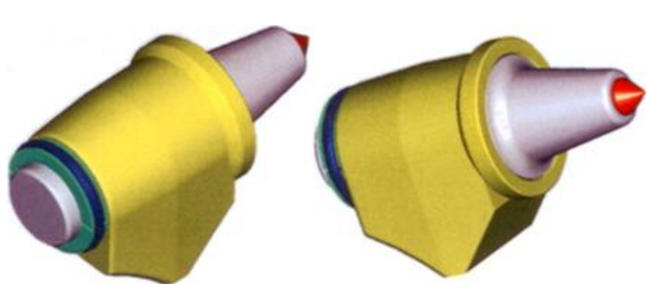

Fig.11 Final shape of cutting bit for platinum ore disintegration process

III. Machinery Industry - Machine for Fatigue Testing of Railway Axles

There were done some proposal calculations for a new testing machine. This new testing machine is determined for a fatigue testing of railway axles, see Fig.12. The railway axles are subjected to bending and rotation (centrifugal effects). The fatigue tests for railway axles which are made in actual size are very important for verifications of all calculations.

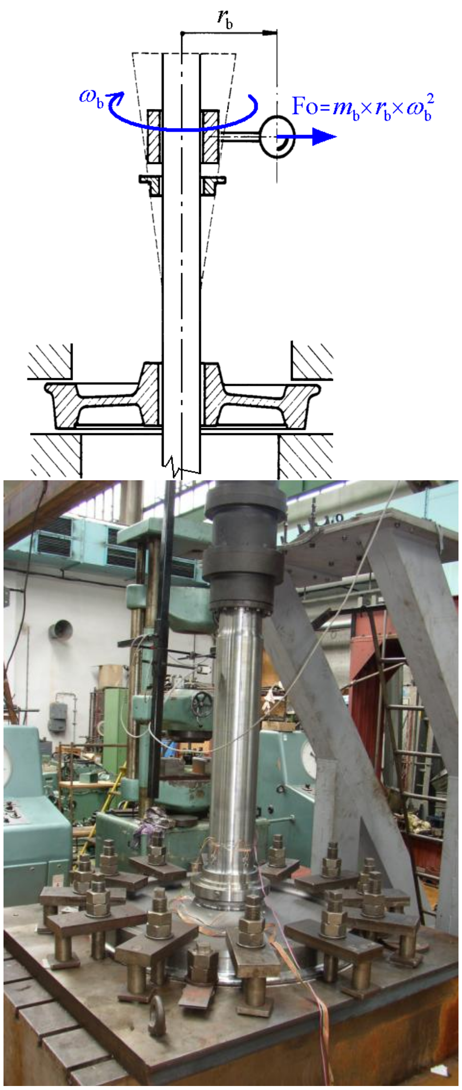

Fig.12 Fatigue testing of railway axles (principles and measurements)

For the right proposition of a new machine for fatigue testing of railway axles (resonator) is very important to know the basic dynamic characteristics of whole system. These dynamic characteristics are solved via FEM (MSC.Marc/Mentat software, see Fig.13) in combination with SBRA method (Anthill and Python software). The base (bottom part) of the testing machine in made of concrete and the upper part is made of steel. Two versions of testing machines with different dimensions and with 12 or 16 springs (non-linear stiffness) were solved. Damping properties of concrete and steel (elastic materials) was described by

Rayleigh material damping.

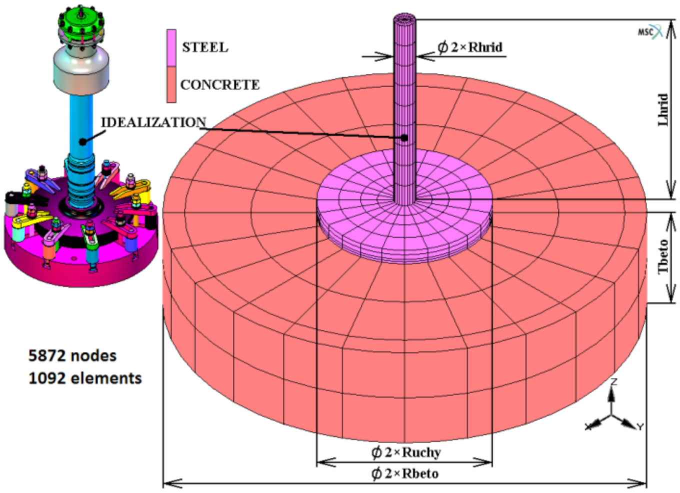

Fig.13 Fatigue testing of railway axles (Finite Element Method)

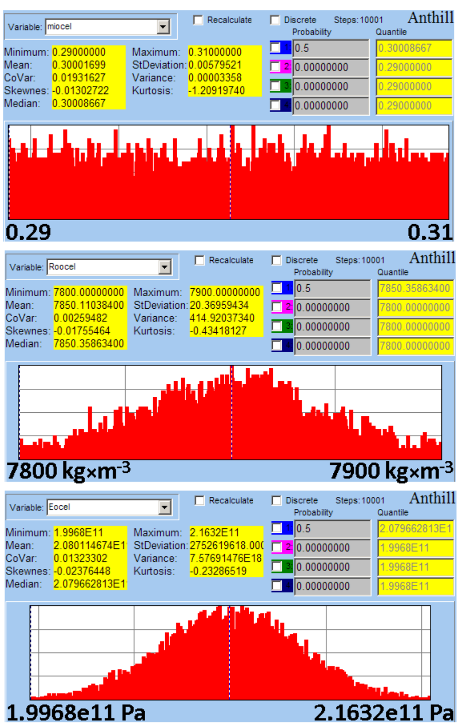

Fig.14 Histograms of steel properties (Poisson's ratio, modulus of elasticity, density)

Examples of some probabilistic inputs (i.e. material properties) are shown in Fig.14 and some results are presented in Fig.15 and 16.

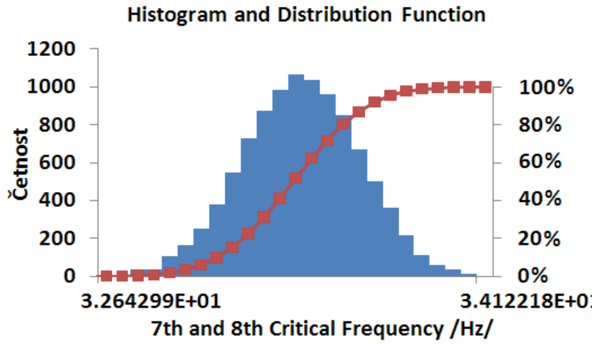

Fig.15 Histogram for 8th. critical frequency

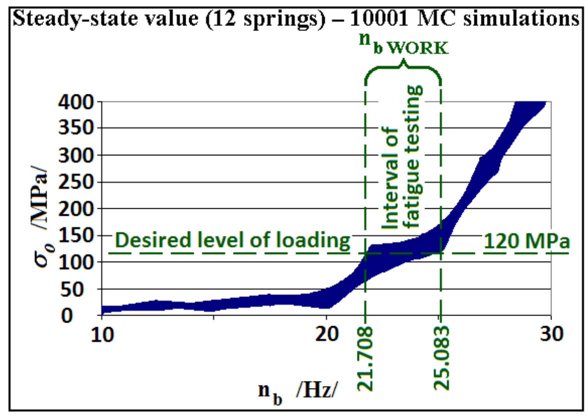

Fig.16 Dependence of bending stresses on excitation frequency

The proposed dimensions and springs of a new machine for fatigue testing of railway axles were used for manufacturing. For more information see references [1] and [2].

IV. Biomechanics - Designing of External Fixators Applied for the Treatment of Open and Unstable Fractures

According to the current studies and research, performed at VŠB – Technical University of Ostrava, Traumatology Centre of the University Hospital of Ostrava, MEDIN a.s. and ProSpon s.r.o., see references [4], [9], [12], [13], [14], [15], [16], [19] and [20], the current design of external fixators (applied in traumatology, surgery and orthopaedics) must be modified, see Fig.17.

Fig.17 Design of external fixators a) based on metals - current design, heavier, expensive, etc. b) based on reinforced polymers - new design, lighter, cheap, more friendly etc.)

There are real needs to make a modern design of fixators which satisfy new trends and demands in medicine. These demands, which are mutually connected, should be solved by:

- Applications of new smart materials (The outer parts of fixators must be x-ray invisible - which leads to shortening the operating time and reducing the radiation exposure of patients and surgeons. Antibacterial protection - application of nanotechnologies on the surface of the outer parts of the fixators to prevent or reduce possible infection. Weight optimalization).
- New design (according to shape, ecological perspective, patient's comfort, reducing the time of the operation, reducing the overall cost, "friendly-looking design").
- Measuring of the real loadings (strain gauges etc.).
- Numerical modelling and experiments (i.e. SBRA application, FEM etc., to avoid the overloading).

It is possible to satisfy all these demands by a new composite materials using proper polymers reinforced by the carbon nanotubes (CNT) or carbon fibres, because some current solutions based on light metals (aluminium, titanium etc.) are heavy and visible in x-ray diagnostic, see Fig.18.

It is possible to satisfy all these mentioned demands by the new composite materials using proper polymers reinforced by the carbon nanotubes (CNT) or carbon fibres.

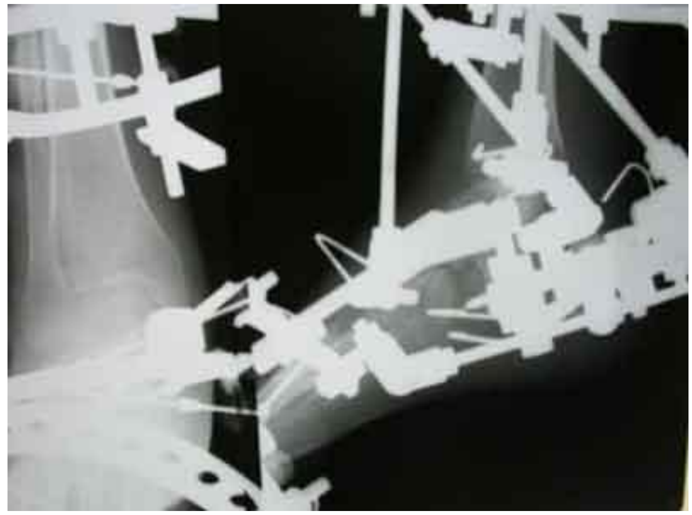

Fig.18 Problems with high x-ray absorption (it is difficult to see broken limbs because there are too many metal parts)

The new types of external fixators for treatment of fractures were designed in the CAD system (Inventor software), tested in the laboratory and modelled by the FEM (Ansys Workbench software), see Fig.19, 20, 21 and 22.

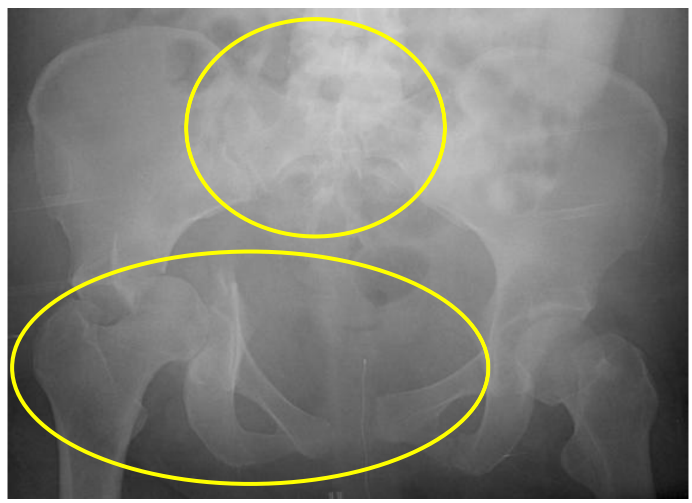

Fig.19 Fracture of pelvis and acetabulum (anteroposterior radiograph - transverse with posterior wall acetabular fracture)

The new proposed designs cannot be more specifically described here, for confidentiality reasons.

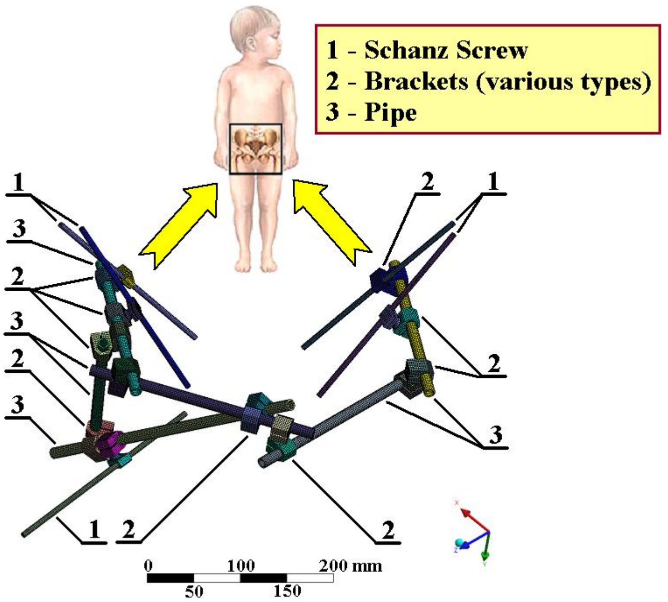

Fig.20 Treatment of fracture of pelvis and acetabulum (numerical modelling of external fixator)

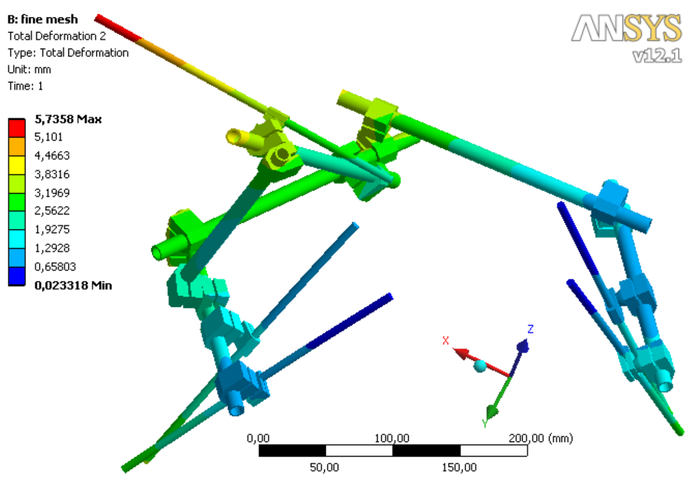

Fig.21 FE modelling of external fixator for pelvis and acetabulum (total displacement)

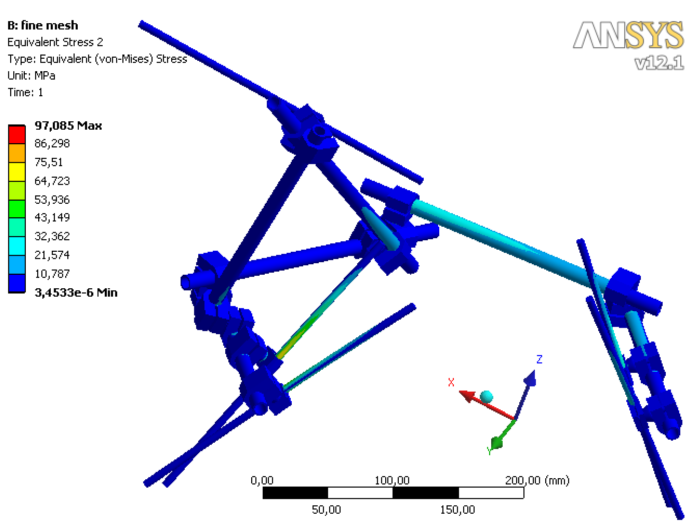

Fig.22 FE modelling of external fixator for pelvis and acetabulum (equivalent stresses)

V. Another Applications of SBRA Method

Another application of SBRA Method are presented in [2] [6], [7], [8], [10], [11], [19], and [21], for example see Fig. 23 (i.e. application in geomechanics).

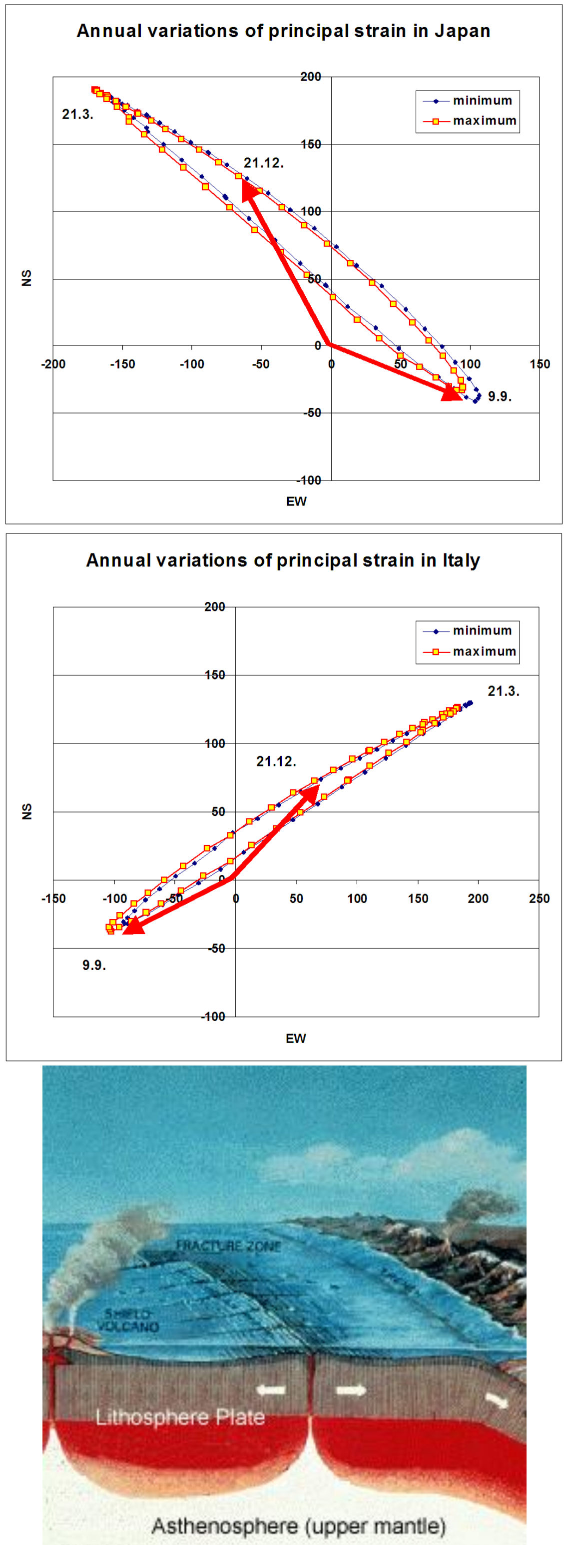

Fig. 23 The annual variations of principal strain in Italy and Japan (application of SBRA Method in geomechanics of continental plates, see reference [19])

VI. Conclusion

Application of the Simulation-Based Reliability Assessment (SBRA) Method in the area of rock mechanics (hard rock disintegration process - design of excavation tool), machine industry (fatigue testing of railway axles) and biomechanics (design of external fixators in traumatology) were reported. Application of the SBRA method is a modern and innovative trend in engineering design. The development of SBRA Method is evident (see also other references).

Acknowledgment

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