APPLICATION OF SBRA METHOD IN MECHANICS OF CONTINENTAL PLATES

Karel Frydrýšek, Ivo Wandrol and Pavel Kalenda

Abstract— This paper shows how the probabilistic SBRA (Simulation-Based Reliability Assessment) Method (i.e. Monte Carlo approach) is applied to the model of the behaviour of the lithosphere of the Earth. The method extends our initial work where we created the geomechanical model of the lithosphere. The basic idea was about the genesis of thermoelastic waves due to thermal expansion of the rock mass and the ratcheting mechanisms. SBRA method applied in this problem is a new and innovative trend for modelling in mechanics.

Keywords— Continental plate, creep, field of temperature, heat conductivity factor, lithosphere, ratcheting, SBRA method, strain tensor, stress tensor, thermoelastic wave.

I. Introduction

THIS work is focused on the geomechanics of lithosphere of our planet (for example see Fig. 1.1 and references [8], [9], [10, [11], [12], [13] and [16]) and extends the works of M. Hvozdara et al. and J. Berger, see references [3] and [7], who detected and described the behaviour of the thermoelastic wave, and the work of J.Croll, see reference [8], who described the ratcheting mechanism.

Application of the Simulation-Based Reliability (SBRA) Method (i.e. fully probabilistic direct Monte Carlo approach), see references [2], [4] [5] and [6], in the solution of this problem of geomechanics is a new and innovative trend.

In the first step, we created the geomechanical model of the lithosphere, where we tested the directions of the relative expansion of the lithosphere plate in two places. This model assumes that the main part of the deformation depends on the solar irradiation. We used the simplified mathematical model, which consists of linear differential equation, applied to the Eurasian continental plate. The probabilistic SBRA method showed the possibility to simulate some physical quantities and the limits of the linear model where the non-linear behaviour and the ratcheting begins.

Assoc. Prof. M.Sc. Karel FRYDRÝŠEK, Ph.D., ING-PAED IGIP – Department of Mechanics of Materials, Faculty of Mechanical Engineering, VŠB – Technical University of Ostrava, 17. listopadu 15/2172, 708 33 Ostrava, Czech Republic (e-mail: karel.frydrysek@vsb.cz, phone: +420 597323495).

RNDr. Ivo WANDROL – Ph.D. student in the branch of Applied Mechanics at the Department of Mechanics of Materials, Faculty of Mechanical Engineering, VŠB – Technical University of Ostrava, 17. listopadu 15/2172, 708 33 Ostrava, Czech Republic (e-mail: ivo.wandro@slu.cz).

RNDr. Pavel KALENDA, Ph.D. – IRSM CAS CZ, Praha, Czech Republic (e-mail: pkalenda@irsm.cas.cz).

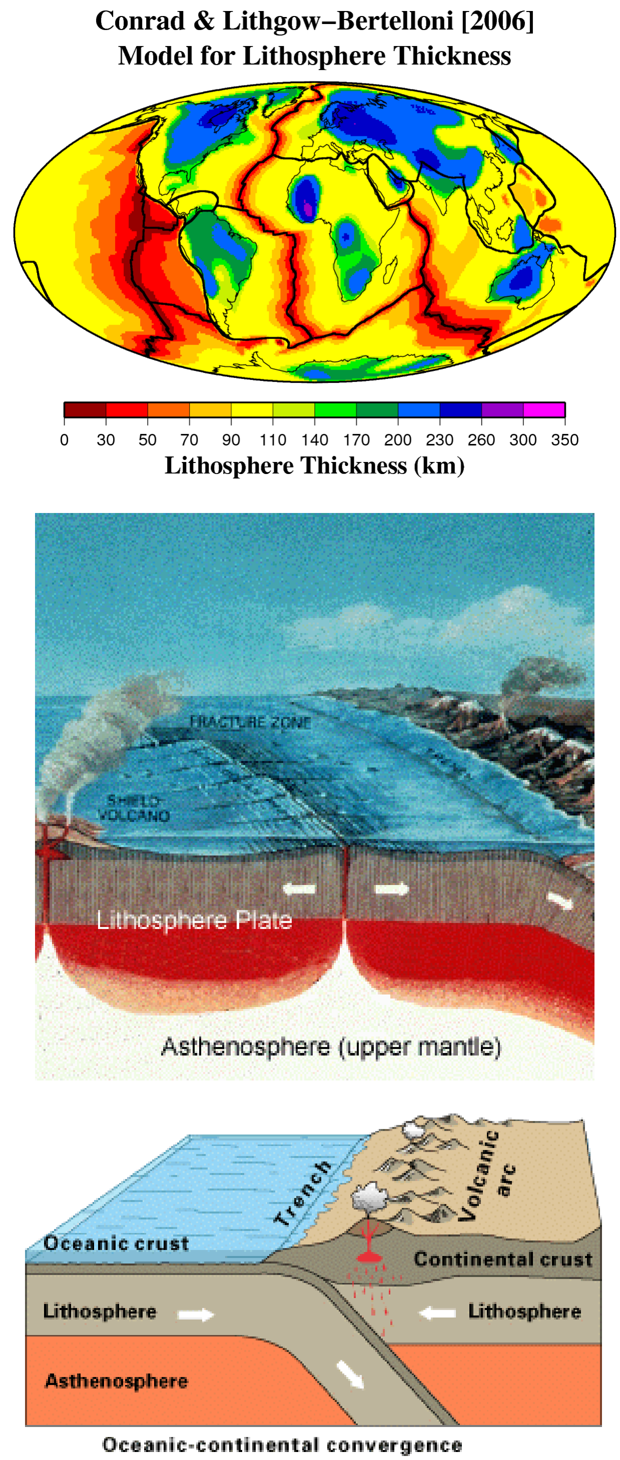

Fig. 1.1 Lithosphere thickness variations and its problems, see reference [9] and [10]

II. NUMERICAL MODEL, PROCESS OF CALCULATION OF STRAIN

The first theoretical analysis and the first mathematical model show, how thermal variations on the Earth´s surface penetrate the depths (see Fig. 2.1) and how the diurnal and annual thermoelastic waves are generated and how big the mechanical strain is.

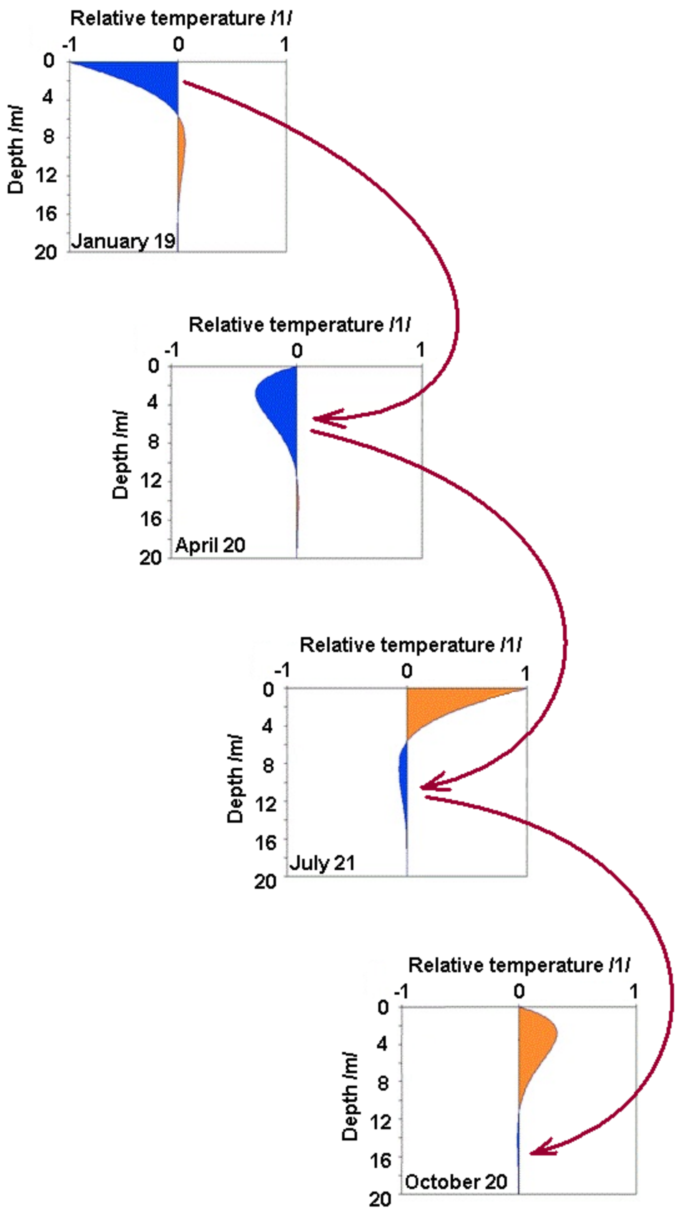

Fig. 2.1 The penetrating of annual thermal wave into depths (temperature profile in borehole)

The first (simple) model of thermoelastic wave generation was made for Eurasian lithosphere plate. On the basis of annual and diurnal thermal variations on the surface in equidistant net on the Eurasian plate the thermal history was calculated in the depths. With respect to the thermal expansion/dilatation the strain and stress were calculated. The results show that the diurnal thermoelastic waves can be measured in the depths much greater than 100 m, which is incomparable with depth of the 0.15 m which can be penetrate diurnal thermal waves. The orientation of principal stress component varies in time during one year (see Fig. 2.2-a, b).

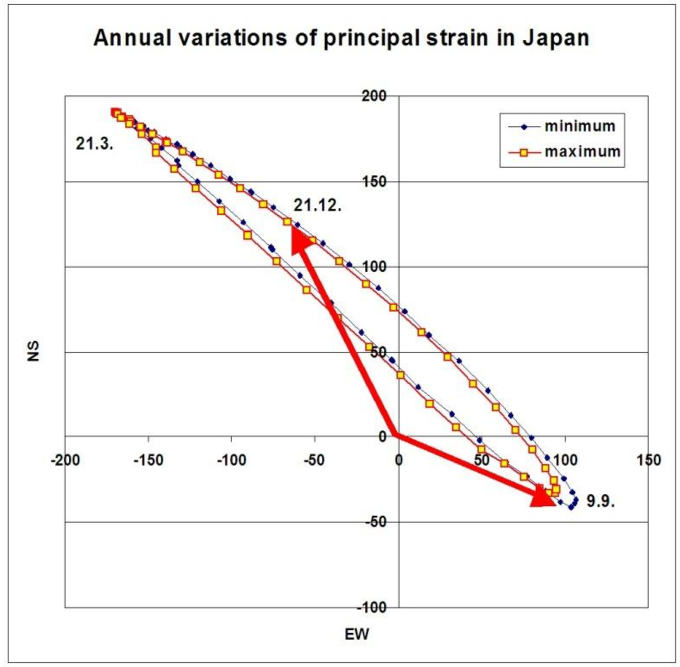

Fig. 2.2-a The annual variations of principal strain in Japan

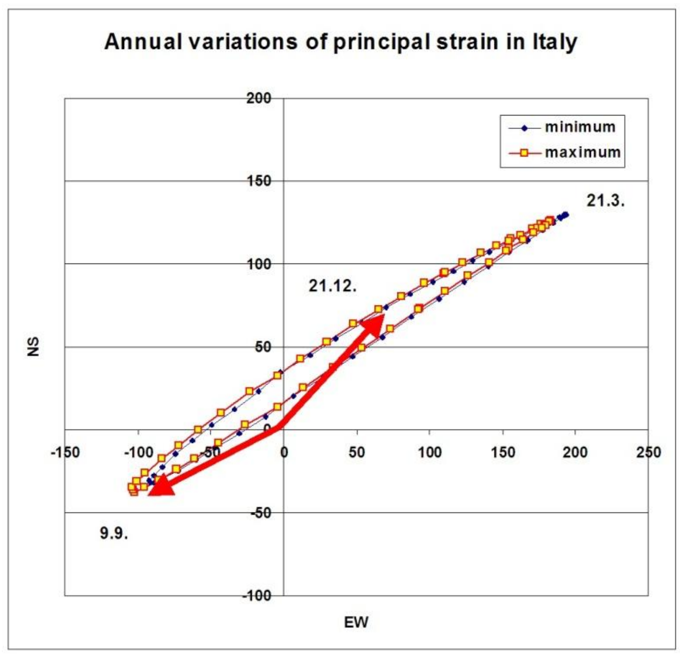

Fig. 2.2-b The annual variations of principal strain in Italy

The results are in accordance with observed tilt in Grotta Gigante (Rossi & Zadro 1996, Braitenberg et al. 2006) and with direction of subduction of Philippine and Pacific plates under Japan (DeMets et al. 1990).

As a first step, the numerical model of thermoelastic strain was evaluated. We used the measured curves of temperature development in various latitudes with step of 10 deg during the whole year. As an example the temperature development in Helsinki in January and July is on the Fig. 2.3-a,b.

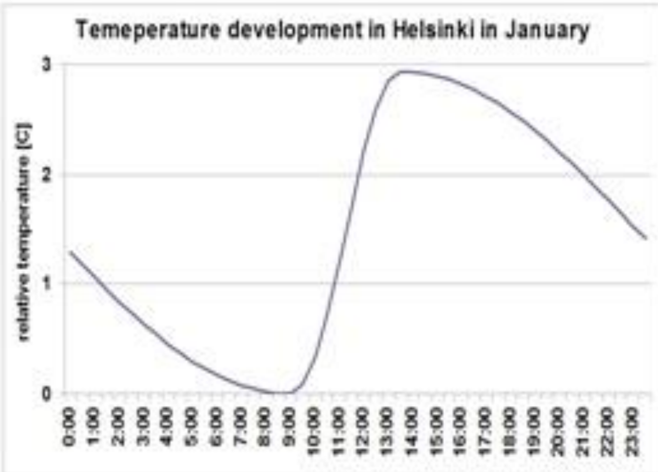

Fig. 2.3-a Relative temperature development in Helsinki (Finland) in January

Fig. 2.3-b Relative temperature development in Helsinki (Finland) in July

The temperature profile in continental rocks was calculated in one-day steps and in one-year cycles for each latitude. The same temperature profile was calculated in 30-minutes steps in one-day cycles. Both cycles were superimposed and the relative temperature development was calculated in 30-minutes steps during one year.

Than the relative strains ε_{xx} , ε_{yy} and ε_{zz} /1/ in the far field under a surface can be evaluated as an integral (or they are directly proportional to this integral) of temperature (depth) profile multiplied by linear thermal expansion coefficient α /K^{-1}/.

Because the attenuation of thermal wave with the depth is high (due to low thermal expansion of rock), the far field is supposed to be in order of one kilometre outside the expanded block of lithosphere. We are able to evaluate the equivalent (normalised) relative strains of each block and to evaluate the principal component of stress tensor and its relative development in time. It depends mainly on the geometry and geographical position of the continents. We calculated the relative values of the principal component of the stress tensor.

The maximum and minimum strain in diurnal period was evaluated and the annual strain development was calculated for the points on the border of continents. Japan (140.625 E, 50 N) and Italy (16.47 E, 40 N) were chosen as examples.

The principal component of relative strain changes its direction in time in both cases. At the end of March the direction is towards the continent. It is the result of the contraction of the Eurasian lithosphere plate after winter. In the case of Italy, it is in the direction of the NE and, in the case of Japan, it is in the direction of the NW. The opposite direction can be seen in September. This is the result of the expansion of the Eurasian lithosphere plate after summer. The results are in accordance with the field GPS measurement of continental deformations at both places.

Cyclic variations of temperature $\vartheta_{(h,t)}$ /K/ close to Earth‘s surface can be evaluated for homogeneous isotropic environment by equation:

$$\vartheta_{(h,t)} = \vartheta_0 e^{-h\sqrt{\omega/2a}} \cos\left(\omega t - h\sqrt{\frac{\omega}{2a}}\right) \ , \qquad (2.1)$$

where ϑ_0 /K/ is an amplitude of temperature variations on the Earth’s surface as a function of time t /s/, h /m/ is depth, parameter $\omega = 2\pi/\tau$ /s^{-1}/, τ /s/ is period and parameter $a \in < 7.10^{-7};\ 22.10^{-7} >$ /m^2s^{-1}/ is temperature conductance coefficient.

The character of the stress tensor behaviour of rock is entirely dependent on temperature variations in layers close to the surface due to the small velocity of the temperature penetration into depth and its large attenuation. The equivalent (normalised) relative strain at point is given:

$$\varepsilon \approx \int_0^H \alpha\ \Delta\vartheta_{(t,h)}\ dh \ , \qquad (2.2)$$

where H /m/ is depth, which the heat penetrates during five periods with $a = 13.10^{-7} m^2 s^{-1}$ (i.e. its mean value).

To estimate the ratio between the lowest and the highest strain and their direction at the border of the continent it will be sufficient to simplify the function $\alpha\,\Delta\vartheta_{(t,h)}$ inside the integral (2.2) in this way:

$$\alpha\,\Delta\vartheta_{(t,h)} \approx \Delta\bar{\vartheta}_{(t,h)} = \frac{T_{(t,h)} - T_s}{T_s}\ e^{-h\sqrt{\omega/2a}} \ , \qquad (2.3)$$

where T_s /K/ is the mean temperature on the surface and $T_{(t,h)}$ /K/ is the relative temperature curve on the surface:

$$T_{(t,h)} = T_s \cos\left(\omega t - h\sqrt{\frac{\omega}{2a}}\right) , \qquad (2.4)$$

where the term $-h\sqrt{\omega/2a}$ represents the delay of the temperature variations at the depth h with the respect to the variations on the surface. When we substitute (2.4) into the relationship (2.3), we obtain:

$$\Delta\bar{\vartheta}_{(t,h)} = e^{-h.\sqrt{\omega/2a}} \cos\left(\omega t - h\sqrt{\frac{\omega}{2a}}\right). \qquad (2.5)$$

When we substitute eq. (2.5) into the integral (2.2), we obtain:

$$\varepsilon \approx \int_0^H e^{-h.\sqrt{\omega/2a}} \cos\left(\omega t - h\sqrt{\frac{\omega}{2a}}\right) dh. \qquad (2.6)$$

An primitive (antiderivative) function of the integral (2.6) can be expressed as:

$$F(x) =$$

$$= \frac{e^{-h.\sqrt{\omega/2a}}}{2\sqrt{\omega/2a}} \left(\sin(h\sqrt{\omega/2a} - \omega t) - \cos(h\sqrt{\omega/2a} - \omega t)\right) + C \qquad (2.7)$$

Due to a fast attenuation the enumeration was only done for five annual periods, which means $H = 113.48799\ \text{m}$.

III. THE PROBABILISTIC SBRA METHOD (THEORY)

Let us consider the Simulation-Based Reliability Assessment (SBRA) Method, see [2], [4], [5], and [6], a probabilistic direct Monte Carlo approach, in which all inputs are given by bounded (truncated) histograms. Bounded histograms include the real variability of the variables. Development of SBRA Method is well presented in books [2], [4] and [5].

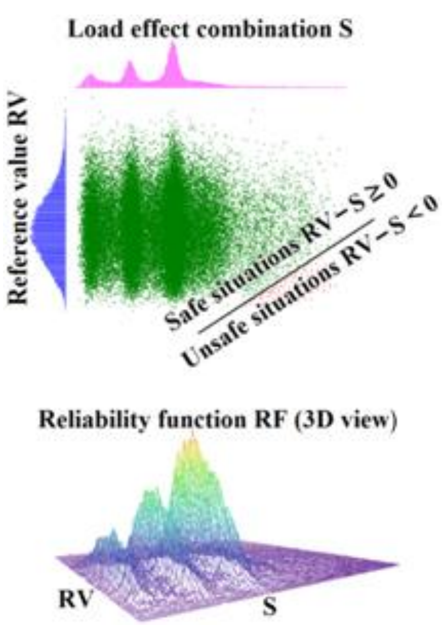

Fig. 3.1 Reliability function RF (SBRA Method)

Using SBRA method, the probability of failure (i.e. the probability of undesirable situation) is obtained mainly by analyzing the reliability function:

$$RF = RV - S, \qquad (3.1)$$

see Fig. 3.1. Where in general terms, RV is the reference (allowable) value and S is a variable representing the load effect combination.

The probability of failure is the probability that S exceeds RV, i.e. $P(\mathrm{RF} \le 0)$. The probability of failure is a relative value depending on the definition of RV and it usually does not reflect an absolute value of the risk of failure, see Fig. 3.2.

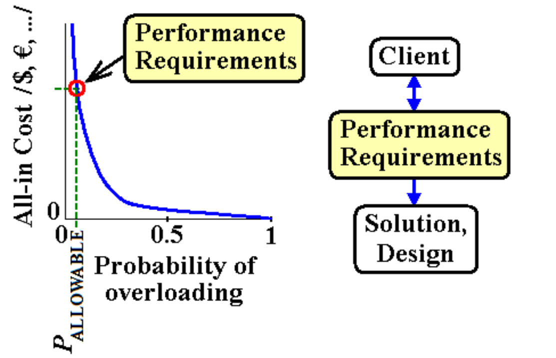

Fig. 3.2 Definition of the acceptable probability of overloading

IV. THE PROBABILISTIC SBRA METHOD APPLICATION (ANTHILL SOFTWARE)

The first approximate calculation was modified in this way. We added a random variable, which simulated a temperature conductance coefficient a /m^{2}s^{-1}/, see Fig. 4.1.

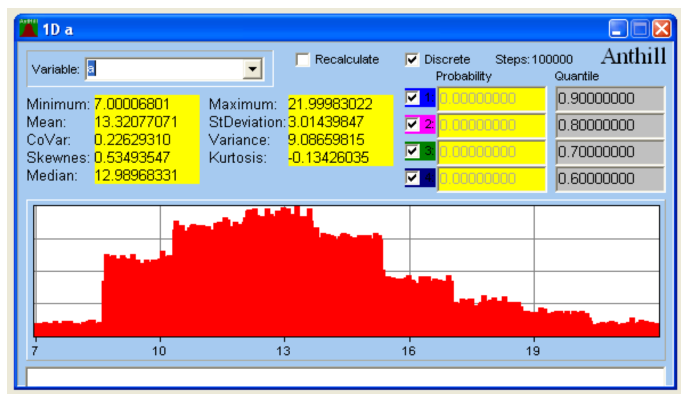

Fig. 4.1 The temperature conductance coefficient a

The relationship (2.6) describes enumeration of the variable ε - the equivalent (normalised) relative strain**.** For this very reason, it is necessary to determine an initial condition – the actual time. The temperature field close to the surface is dependent on the temperature variations at the surface. It may be assumed that in the case of constant surface temperature, the strain and the evaluated value ε will tend towards zero. We made a simulation of the behaviour of this variable for the diurnal period. For the relationship (2.7), the random variable t was used with uniform distribution in the range <0; 86400> /s/. Therefore, according to the relationships (2.7) we obtain:

$$\varepsilon_{vyp} = F(H) - F(0), \qquad (4.1)$$

where H /m/ is the depth, to which the heat penetrates during five periods. The value of the parameter H depends on the temperature conductance coefficient a; the example of generated values are in Fig. 4.2. and Fig. 4.3 shows the relationships of the calculation of the reliability (SW AntHill). (3.1).

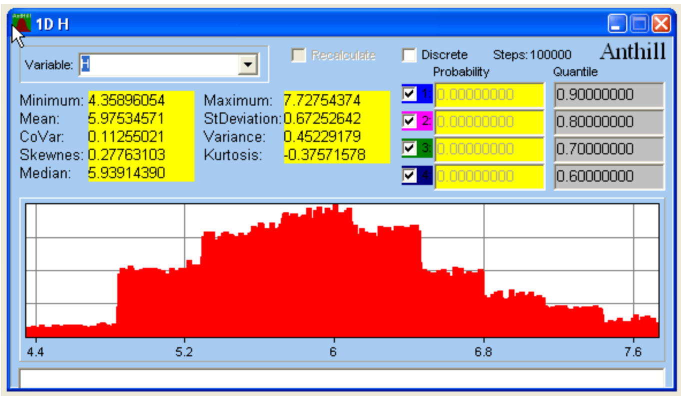

Fig. 4.2 Histogram for values of H /m/ - depth of temperature field (AntHill software)

Equations

```
FS=epsdov-epsvyp
epsvyp=abs(epsIntegral)
epsIntegral=FH-F0
FH=0.5*EXP*(-cos(-Omega*t+H*V1odm) + sin(-Omega*t+H*V1odm))/V1odm
F0=0.5*(-cos(-Omega*t) + sin(-Omega*t))/V1odm
EXP=Euler^(-H*V1odm)
H=5*2*PI/V1odm
V1odm = (omega/(2*a*10^(-7)))^0.5
Omega = 2*PI/86400
epsdov=epsdov0*epsdov0var
a=a0var
t=t0var
```

Fig. 4.3 Calculation description (programming, AntHill software)

The behaviour of the variable ε_{vyp} /1/ is shown in Fig. 4.4 (result of AntHill sw), which shows that the strain inside the rocks varies during the diurnal period. Although the relative values are shown they respect the dynamics of the model well (maximum and minimum). We added the new variable “epsvyp0“ to show the behaviour of strain, which corresponds to the relationship

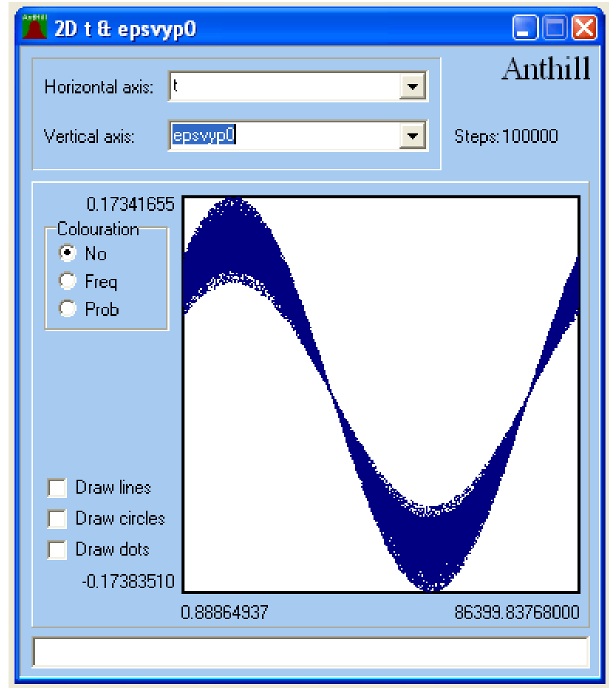

Fig. 4.4 Progression ε_{vyp} /1/ during diurnal period (AntHill software)

There are two extremes included in Fig. 4.5. Both of them show the situation, when the model is approaching its limits when the linear law ceases to apply and the non-linear behaviour starts. One extreme describes the situation when the stress is approaching the ultimate compressive strength of the massif and the second describes the situation when the stress is approaching the ultimate tensile strength. The first of them, in a long perspective, leads to the creep or seismic events. The second extreme leads to the opening of the micro-cracks, cracks or faults, when ratcheting can occur.

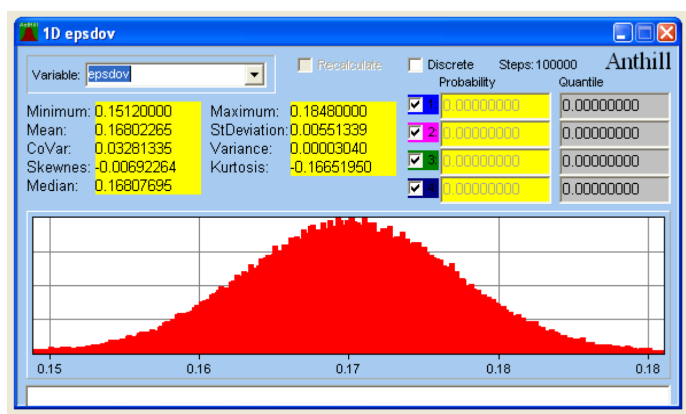

Fig. 4.5 Distribution of critical values ε_{dov} (AntHill software)

To get a more accurate calculation, it will be necessary to take into account the dynamics of the strain growth and the relaxation of rocks for example a non-linearity of the massif, hysteresis or ratcheting. This is the subject of further work.

The distribution of frequencies corresponds to the chosen equation for computation of the temperature field close to the Earth’s surface. The model of the ideal periodic function was used for the simulation of a surface temperature. While, in this paper the ideal temperature function was used (defined by goniometric functions), the real temperature curves must be used in the real approach. These curves have asymmetric forms when the minimum of the diurnal temperature can be determined at the moment of the sunrise and the maximum can be determined approximately one hour after noon or we can use the real temperature development curves. This means that the heating of the Earth's surface is considerably faster than its cooling.

We used the numeric integration method for the evaluation of the temperature. If the surface temperature is constant then the integral will tend towards zero. Therefore, the computation was modified and the new variable $|\varepsilon_{vyp}|$ was set up.

Fig. 4.6 shows the distribution of variable ε_{dov} minus critical value of the variable $|\varepsilon_{vyp}|$.

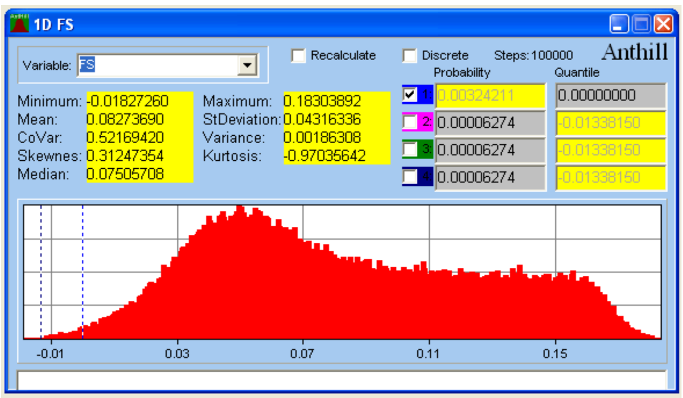

Fig. 4.6 Reliability function F_S

The value ε_{dov} was estimated by probabilistic approach (see Fig. 4.5). In this way the reliability function F_S could be estimated (see figure 4.6):

$$F_S = \varepsilon_{dov} - |\varepsilon_{vyp}| \,. \tag{4.2}$$

We also verified the methodology of the probabilistic SBRA Method in the real geological situation and the number 10^5 Monte Carlo simulations gave a fair result.

The probability of micro-movement P_f (i.e. the probability of the irreversible effect when $F_S < 0$) was evaluated as 0.00324, i.e. 0.324% . The Fig. 4.6, 4.7 and 3.1 show the principles of determining this value.

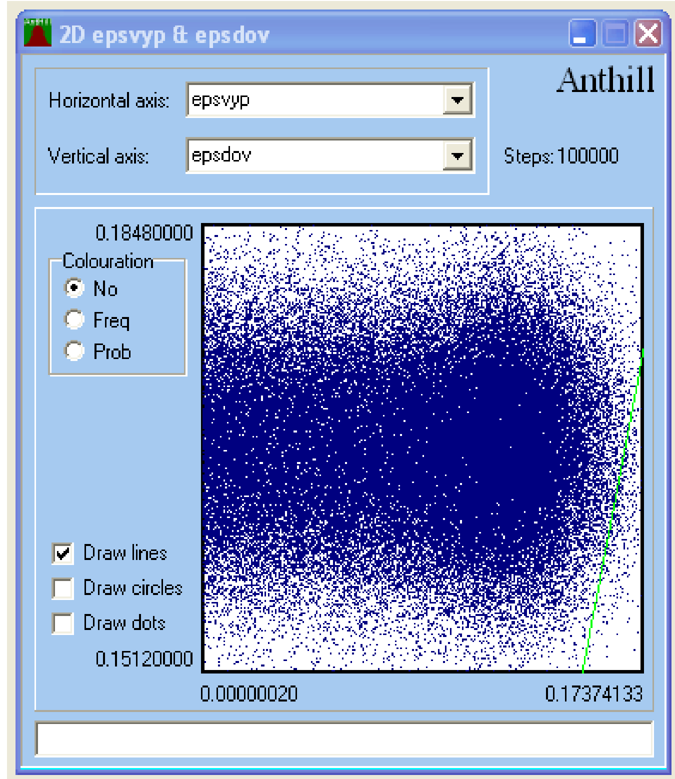

Fig. 4.7 Distribution admissible values ε_{dov} and critical values ε_{vyp}

V. NOTE ABOUT ANOTHER APPLICATIONS OF SBRA METHOD

Another application of SBRA Method are presented in books [2] [4], [5] and [14] to [19], for example see Fig. 5.1, 5.2 (i.e. application in designing in mechanical engineering) and Fig. 5.3 and 5.4 (i.e. application in designing in rock mechanics).

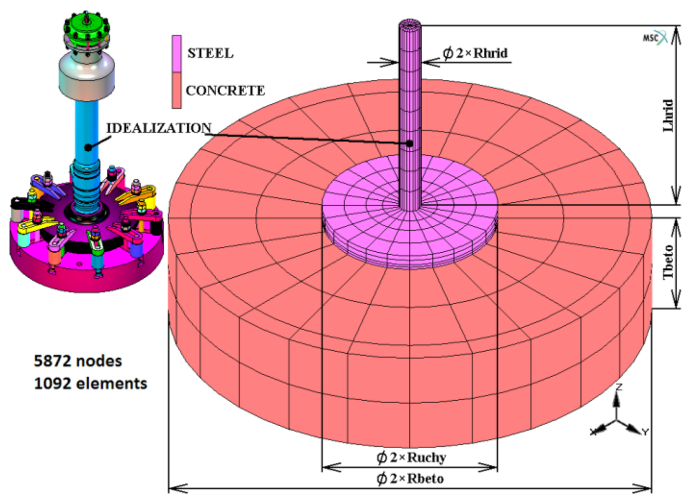

Fig. 5.1 Fatigue testing of railway axles (numerical model)

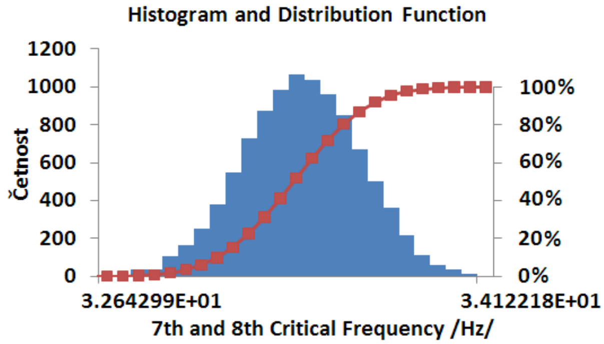

Fig. 5.2 Fatigue testing of railway axles (histogram and distribution function for 8th critical frequency)

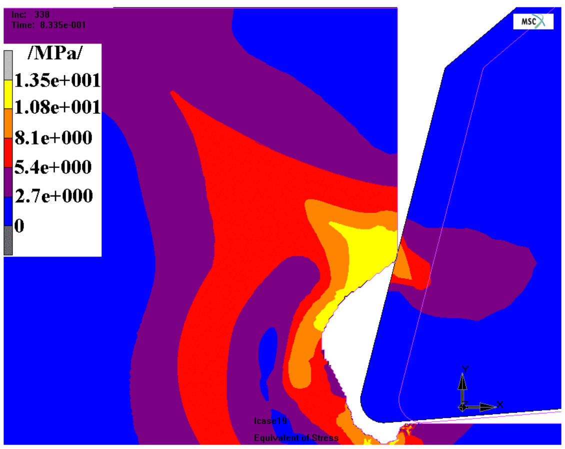

Fig. 5.3 Disintegration of the ore and movement of the bit (equivalent von Mises stresses distributions, result of one Monte Carlo simulation)

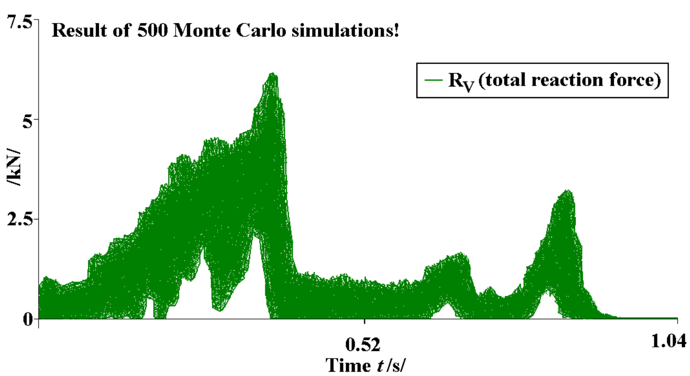

Fig. 5.4 Disintegration of the ore (reaction forces in the bit, probabilistic outputs – result of 500 Monte Carlo simulations)

VI. Conclusion

The aim of this paper was to test the possibility of using probabilistic calculations for modelling the behaviour of the lithosphere of the Earth (the primary methodology for setting the limit values, which leads to the irreversible movement of the lithospheric plates - creep). We can say that the method can be used for the modelling of the massif, when its parameters do not enter as mean values, but by using Monte Carlo method (i.e. SBRA applications). The method could be used directly inside the geomechanical model.

Acknowledgment

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Assoc. Prof. M.Sc. Karel FRYDRÝŠEK, Ph.D., ING-PAED IGIP – corresponding author (Department of Mechanics of Materials, Faculty of Mechanical Engineering, VŠB – Technical University of Ostrava, Czech Republic) - born in June 27^{th} 1972, married, one daughter.

Study: M.Sc. - 1995, Ph.D. - 1999, Assoc. Prof. - 2009 in the branch of "Applied Mechanics" all at the Faculty of Mechanical Engineering VŠB – Technical University of Ostrava. He also studied pedagogy in the branch of "Academic Pedagogy for Teachers-Engineers According to the European Standards IGIP" at the Centre for Study of Academic Teaching in the Prague - 2003.

Scientific-research activities and cooperation with industry: Theory and practice of FEM and other numerical methods, strength and elasticity, plasticity, material tests, fatigue, thermal stresses, creep, comparing of experiments and calculations, stress-strain analyses in bodies, proposition of testing machines and its parts, rock mechanics, geomechanics, mechanics of composites and structures on elastic foundation. He has a rich cooperation with industry (automotive industry, railway industry, civil engineering, mining, metallurgy, forming, casting, heat technology, steel structures, pipe systems, biomechanics etc.). In the last years, he is focused on probabilistic reliability assessment (SBRA Method applications) and biomechanics (problems of design of external ans internal fixators for treatment of open and unstable fractures in traumatology and orthopaedics).