Actual behaviour of composite externally CFRP-reinforced timber beams – stress analysis

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***Abstract*—**The paper is focused on the problems of composite CFRP-reinforced timber beams. CFRP reinforcement is represented by the lamella externally bonded on the tensile part of the beam for the strengthening the member to increase in the resistance from the viewpoint of both the ultimate and serviceability limit states. Usually for the resistance determination the concept of composite structural members based on the substitute (ideal, respectively) cross-section is applied, but the usage of this principle is depending on the efficiency of the shear connection (here bonded) between CFRP reinforcement and basic material – timber. It is mainly influenced by the glue type and also by the bonded connection quality. The practical experiences obtained from the tests oriented to the experimental verification of the actual behaviour show that the efficiency is very different in dependence on the particular structural member. The paper presents selected results of the measurement of the stresses in CFRP reinforcement and in timber base, which can give the information on the level of the mutual interaction between both materials and on the rightness of the substitute cross-section approach mentioned above in the case of timber beams. The paper deals with the verification of the actual normal stresses, mainly in the cross-section part of the contact between timber and CFRP reinforcement (here bottom tensile edge). Actual normal stresses obtained from the tests have been used for the evaluation of actual characteristics of the substitute cross-section, especially the second moment of the area. Applying those derived cross-section characteristics the effects of the longitudinal shear, that means shear forces and stresses, have been calculated aimed to the verification of the bonded shear connection.

***Keywords*—**Timber beam, composite, strengthening, CFRP reinforcement, bonded lamella, actual behaviour, elasticity, normal stress, shear connection, longitudinal shear stress, experimental verification, theoretical analysis, interaction.

I. INTRODUCTION

THE important attention to the research activities directed towards the usage of progressive high-strength materials combined with traditional materials in one structural member is paid on the author's workplace in the recent period. Within the framework of the experimental and theoretical research the capacity and serviceability of load-carrying CFRP-timber and CFRP-steel members are investigated and solved from the viewpoint of the increase in bending resistance due to strengthening by external CFRP lamella reinforcement bonded to timber or steel surface of the tension beam edge.

Except the usually measured standard parameters (loading, deflections) during the loading tests also the stresses in the cross-section have been monitored and verified by way of the strain measurement. For this reason the strain gauges bonded on the surface both on CFRP lamella and on base material have been applied. The results obtained from the measurement have been taken as the base for the verification, accuracy improvement and, correction of the theoretical tenseness analysis.

This experimental research has been realized for the test specimens with the several timber beam cross-sections: 100/220, 100/200, 100/180, 100/160, 100/140, 100/120. The cross-section dimension of used CFRP lamellas was 50/1.2 mm. The basic description and results of this research can be obtained from primary information, which has been already presented, for example, in [7], [8], [9], [11], [14].

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II. PRINCIPLES OF THEORETICAL ANALYSIS OF STRESSES IN CROSS-SECTION OF COMPOSITE EXTERNALLY CFRP-REINFORCED TIMBER BEAM

A. Normal Stress

Strengthening bended structural members by reinforcement based on fibre-reinforced polymers is given by advanced material properties of used fibres. CFRP composites usually use carbon fibres with very high tensile strength and high modulus of elasticity (unlike glass fibres). Generally it is assumed, strengthening by CFRP can be efficient for timber because of its higher Young's modulus than timber one.

Assuming the rigid shear connection between timber and CFRP lamella the relative deformations of timber and CFRP reinforcement in the contact are the same, i.e. $\varepsilon_{timber} = \varepsilon_{CFRP}$. Then, for the calculation of normal stresses or, respectively, for the determination of predicted elastic bending moment resistance, the general concept of composite substitute cross-section (see, for example, [1], [8], [9], [10], [12], [13], [15], [33]) based on the parameter n given as the ratio of

$$n = \frac{E_{timber}}{E_{CFRP}} \ , \tag{1}$$

where E_{timber}, E_{CFRP} are Young's modulus of elasticity of timber and CFRP reinforcement, can be applied.

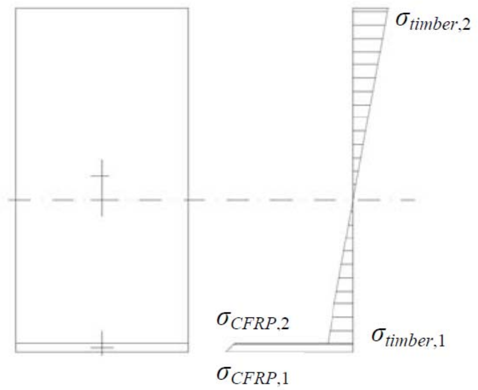

Fig. 1 elastic stress distribution in CFRP-timber beam cross-section

The assumed normal stress distribution in CFRP-timber cross-section based on the elastic approach is shown in Fig. 1, where σ_{timber} is timber stress on the tensile edge of the beam and σ_{CFRP} is the stress in CFRP reinforcement. The values of normal stresses anywhere in the cross-section can be generally calculated using the following formulas:

$$\sigma_{timber} = \frac{M}{I_i} \cdot z_{timber} \ , \tag{2}$$

$$\sigma_{CFRP} = \frac{M}{n \cdot I_i} \cdot z_{CFRP} \ , \tag{3}$$

where M is the bending moment, I_i is the second moment of area of the substitute (ideal) cross-section and z_{timber} and z_{CFRP} are the corresponding distances from the gravity centre of the substitute cross-section. Basic characteristics of the substitute (ideal, respectively) cross-section of the composite member, that means cross-section area A_i and the second moment of area I_i, are generally given as follows:

$$A_i = A_{timber} + \frac{A_{CFRP}}{n} \ , \tag{4}$$

$$I_i = I_{timber,i} + \frac{I_{CFRP,i}}{n} \ . \tag{5}$$

Of course, the formulas for the cross-section characteristics of CFRP parts and for normal stresses in CFRP parts include parameter n, which arises from the principle described above and it is also evident from equations (3), (4), (5).

B. Longitudinal Shear Stress

In addition to the normal stresses and their effects, also shear connection between cross-section parts is very important for the design of composite beams in general. Based on the substitute (ideal) cross-section conception, the longitudinal shear force $V_{long,1}$ (per unit of the length) at the interface between CFRP reinforcement and timber can be calculated as

$$V_{long,1} = \frac{V \cdot S_i}{I_i} \qquad [\mathrm{N.mm^{-1}}], \tag{6}$$

where V is the (transverse) shear force, S_i is the first (static) moment of the cut off part of the area, I_i is the second moment of area. Then, the longitudinal shear stress in the interface between timber and CFRP can be given as

$$\tau_{long,1} = \frac{V \cdot S_i}{b \cdot I_i} \ , \tag{7}$$

where b is the width in the corresponding place of the cross-section, that means in this case it is the width of the glued area, i.e. the width of CFRP lamella, because it is bonded over its all the width.

III. Experimental Verification of Stresses in Cross-Section of Composite Externally CFRP-Reinforced Timber Beam

For the verification of the objective resistance and actual stresses in the member, CFRP-timber beams have been tested. During the loading tests, stresses have been monitored and subsequently compared with the predicted calculated values.

Material of the beams was structural timber of the class C22 with the characteristic bending-tensile strength of 22 MPa and mean value of Young's modulus of 10 GPa. Tensile strength of CFRP reinforcement was, according to the information of the producer, 3 000 MPa and Young's modulus was 155 GPa.

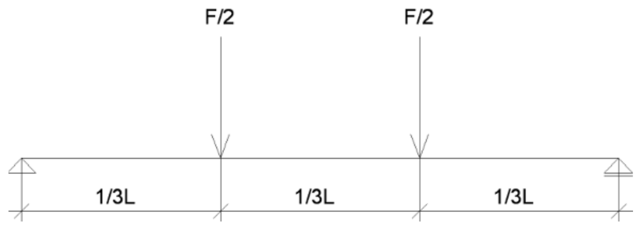

Fig. 2 tested specimen scheme: four-point bending

The test specimens – beams of the span of 3 or 4 meters – have been simply supported and loaded by the forces introduced in the beam thirds (four-points bending) – see scheme in Fig. 2 and illustration of the test arrangement and performance of loading test in Fig. 3.

The testing using four-points bending have been chosen with respect to the testing equipment, but as, if possible, to the best simulate the real loading, which is in usual cases approximately uniform.

During loading process, except of forces F and deflections w also stresses σ_{timber}, σ_{CFRP} in timber and in CFRP on the tensile edge have been measured through strain gauges – for the illustration of strain gauges see Figs. 4, 5.

Fig. 3 arrangement and realization of loading tests

Fig. 4 strain gauges on the bottom (tensile) edge

Fig. 5 view to the bottom edge with strain gauges on CFRP lamella and timber surface

A. *Normal Stresses*

The normal stresses in timber and CFRP reinforcement measured on the tensile beam edge are presented in Table I. For the comparison and verification of the theoretical approach the stresses calculated applying the elastic method are presented in Table I, too. This table shows stress values for the bending moments equal to $0.25\,M_u$, $0.5\,M_u$ and $0.75\,M_u$, where M_u is the maximum (ultimate) bending moment reached within the tests, when the failure occurred, both experimental values and calculated values.

Table I Stresses $\sigma_{timber,1}$, $\sigma_{CFRP,1}$ [MPa] in timber and CFRP on the tensile edge: theory vs. tests (obtained from strain gauges)

100/220-C $M_u = 36.44$ kNm		$0.25\,M_u$	$0.5\,M_u$	$0.75\,M_u$
theory	$\sigma_{timber,1}$	10.7	21.4	32.1
	$\sigma_{CFRP,1}$	167.7	335.4	503.1
test	$\sigma_{timber,1}$	8.7	17.8	27.6
	$\sigma_{CFRP,1}$	142.9	310.0	475.9
100/200-C $M_u = 30.71$ kNm		$0.25\,M_u$	$0.5\,M_u$	$0.75\,M_u$
theory	$\sigma_{timber,1}$	10.8	21.7	32.5
	$\sigma_{CFRP,1}$	170.2	340.5	510.7
test	$\sigma_{timber,1}$	7.3	14.6	22.2
	$\sigma_{CFRP,1}$	100.3	203.1	306.9
100/180-C $M_u = 24.69$ kNm		$0.25\,M_u$	$0.5\,M_u$	$0.75\,M_u$
theory	$\sigma_{timber,1}$	10.7	21.4	32.1
	$\sigma_{CFRP,1}$	167.9	335.9	503.8
test	$\sigma_{timber,1}$	8.5	17.2	26.7
	$\sigma_{CFRP,1}$	136.7	279.0	435.6
100/160-C $M_u = 20.13$ kNm		$0.25\,M_u$	$0.5\,M_u$	$0.75\,M_u$
theory	$\sigma_{timber,1}$	10.9	21.9	32.8
	$\sigma_{CFRP,1}$	172.1	344.2	516.2
test	$\sigma_{timber,1}$	8.4	17.7	29.3
	$\sigma_{CFRP,1}$	112.8	216.4	300.5
100/140-C $M_u = 16.37$ kNm		$0.25\,M_u$	$0.5\,M_u$	$0.75\,M_u$
theory	$\sigma_{timber,1}$	11.5	22.9	34.4
	$\sigma_{CFRP,1}$	181.1	362.2	543.3
test	$\sigma_{timber,1}$	9.6	19.5	30.4
	$\sigma_{CFRP,1}$	153.8	315.9	493.4
100/120-C $M_u = 14.02$ kNm		$0.25\,M_u$	$0.5\,M_u$	$0.75\,M_u$
theory	$\sigma_{timber,1}$	13.2	26.3	39.5
	$\sigma_{CFRP,1}$	208.4	416.7	625.1
test	$\sigma_{timber,1}$	9.3	18.9	29.5
	$\sigma_{CFRP,1}$	153.7	304.6	471.2

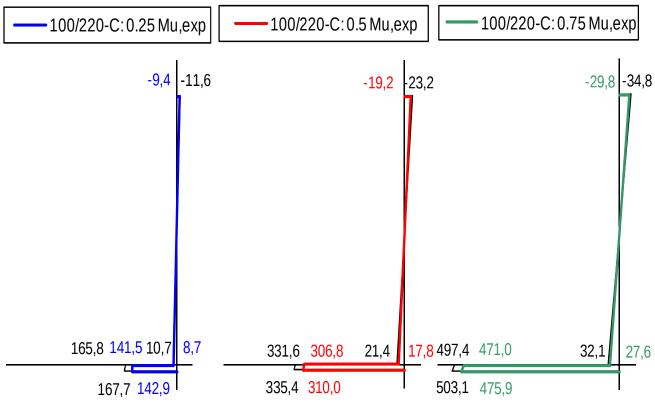

Fig. 6 theoretical (calculated) and actual (derived from the tests) stresses in composite CFRP-timber beam subjected to bending moment: cross-section 100/220-C

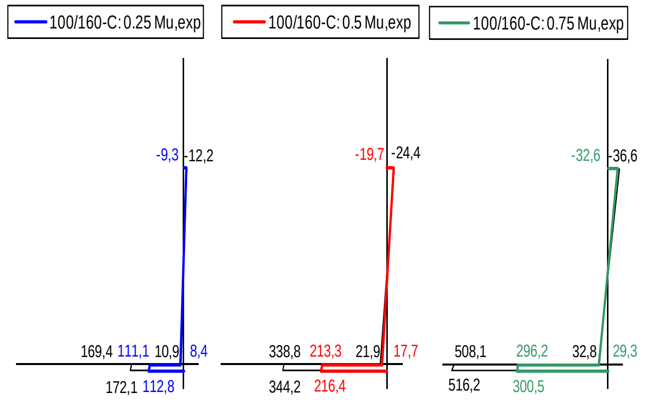

Fig. 9 theoretical (calculated) and actual (derived from the tests) stresses in composite CFRP-timber beam subjected to bending moment: cross-section 100/160-C

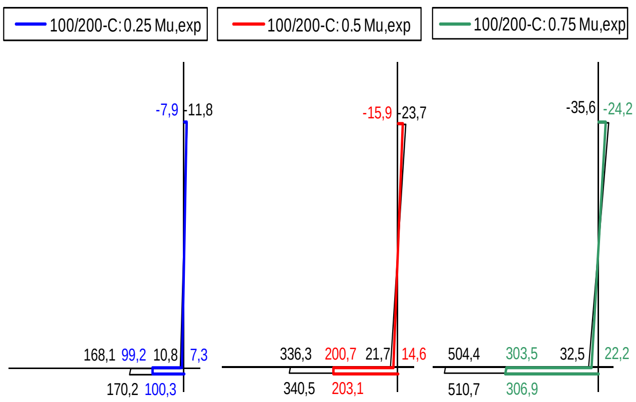

Fig. 7 theoretical (calculated) and actual (derived from the tests) stresses in composite CFRP-timber beam subjected to bending moment: cross-section 100/200-C

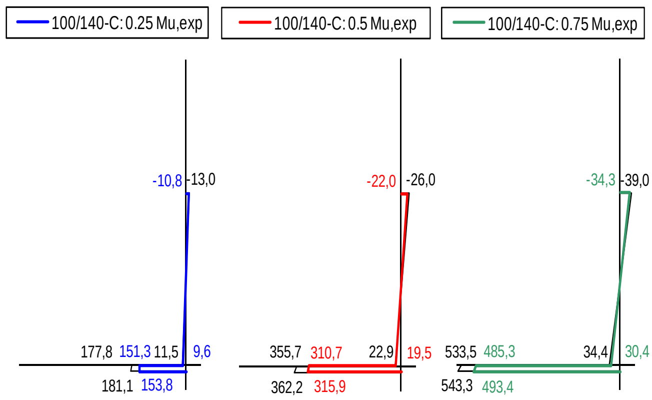

Fig. 10 theoretical (calculated) and actual (derived from the tests) stresses in composite CFRP-timber beam subjected to bending moment: cross-section 100/140-C

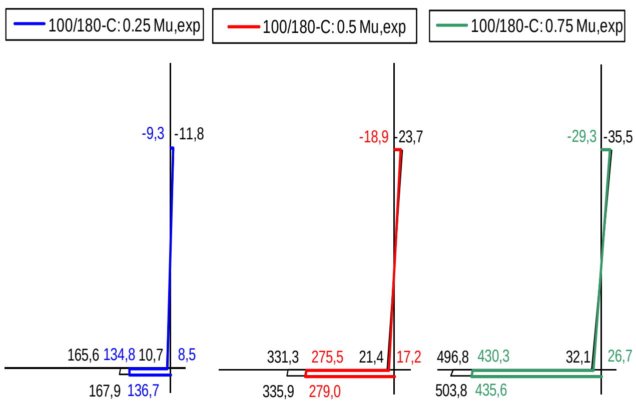

Fig. 8 theoretical (calculated) and actual (derived from the tests) stresses in composite CFRP-timber beam subjected to bending moment: cross-section 100/180-C

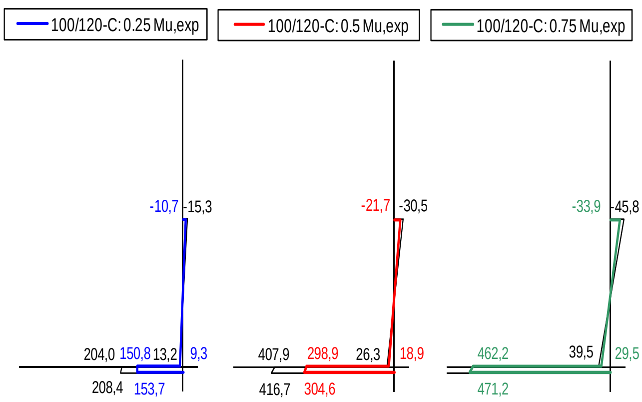

Fig. 11 theoretical (calculated) and actual (derived from the tests) stresses in composite CFRP-timber beam subjected to bending moment: cross-section 100/120-C

Table II Actual values of parameter n obtained from the tests using actual normal stresses: $n = E_{timber} / E_{CFRP} = \sigma_{timber,1} / \sigma_{CFRP,2}$

100/220-C	0.25 M_u	0.5 M_u	0.75 M_u	$n_{mean,220}$
$\sigma_{timber,1}$	8.7	17.8	27.6	0.0595
$\sigma_{CFRP,2}$	141.5	306.8	471.0	
n	0.0616	0.0581	0.0587	
100/200-C	0.25 M_u	0.5 M_u	0.75 M_u	$n_{mean,200}$
$\sigma_{timber,1}$	7.3	14.6	22.2	0.0732
$\sigma_{CFRP,2}$	99.2	200.7	303.5	
n	0.0737	0.0728	0.0732	
100/180-C	0.25 M_u	0.5 M_u	0.75 M_u	$n_{mean,180}$
$\sigma_{timber,1}$	8.5	17.2	26.7	0.0623
$\sigma_{CFRP,2}$	134.8	275.5	430.3	
n	0.0631	0.0616	0.0622	
100/160-C	0.25 M_u	0.5 M_u	0.75 M_u	$n_{mean,160}$
$\sigma_{timber,1}$	8.4	17.7	29.3	0.0860
$\sigma_{CFRP,2}$	111.1	213.3	296.2	
n	0.0757	0.0831	0.0991	
100/140-C	0.25 M_u	0.5 M_u	0.75 M_u	$n_{mean,140}$
$\sigma_{timber,1}$	9.6	19.5	30.4	0.0631
$\sigma_{CFRP,2}$	151.3	310.7	485.3	
n	0.0636	0.0629	0.0627	
100/120-C	0.25 M_u	0.5 M_u	0.75 M_u	$n_{mean,120}$
$\sigma_{timber,1}$	9.3	18.9	29.5	0.0631
$\sigma_{CFRP,2}$	150.8	298.9	462.2	
n	0.0618	0.0634	0.0640	
Mean value of parameter n: $\boldsymbol{n_{mean}}$ **= 0.068**				

The actual values of normal stresses verified using test results have been determined through the relative deflections ε, which have been obtained from the strain gauges. Measured values of strains (per unit of the length) in [N.mm^{-1}] have been recalculated to the stresses according to the relationship between stress and strain given by Hooke's law. For the calculation Young's modulus have been used as described above (see paragraph III).

In Table I the maximum stresses $\sigma_{timber,1}$ and $\sigma_{CFRP,1}$ in timber and CFRP on the tensile edge (see Fig. 1) are listed. The complete overview of normal stresses in the cross-section is depicted in Figs. 6 to 11, where normal stress distributions for both theoretical and actual values (derived from the tests) are drawn, for the comparison (theoretical stress distribution rendered by black lines, actual stress distribution rendered by colour lines).

The particular values of normal stress show the following: the actual normal stresses obtained from the tests are less than the theoretical ones calculated using the concept of substitute cross-section, both for timber and CFRP reinforcement, while the differences in timber normal stresses usually are in the range from 10 up to 30 % and differences in CFRP normal stresses are in the range from 5 to 40 %, too.

Table III Values of the second moments of area I_i: theory vs. tests

100/220-C	I_i [mm^4]	$I_{i,test} / I_{i,theory}$
theory	$99.651 \cdot 10^6$	1.119
test	$100.528 \cdot 10^6$	
100/200-C	I_i [mm^4]	$I_{i,test} / I_{i,theory}$
theory	$75.663 \cdot 10^6$	0.986
test	$74.637 \cdot 10^6$	
100/180-C	I_i [mm^4]	$I_{i,test} / I_{i,theory}$
theory	$55.861 \cdot 10^6$	1.004
test	$56.104 \cdot 10^6$	
100/160-C	I_i [mm^4]	$I_{i,test} / I_{i,theory}$
theory	$39.845 \cdot 10^6$	0.966
test	$38.476 \cdot 10^6$	
100/140-C	I_i [mm^4]	$I_{i,test} / I_{i,theory}$
theory	$27.215 \cdot 10^6$	1.054
test	$28.677 \cdot 10^6$	
100/120-C	I_i [mm^4]	$I_{i,test} / I_{i,theory}$
theory	$17.571 \cdot 10^6$	1.004
test	$17.636 \cdot 10^6$	
Mean value of the ratio: $\boldsymbol{I_{i,test} / I_{i,theory}}$ **= 1.022**		

From the values of actual stresses in the place of the contact of CFRP reinforcement and timber the actual values of the parameter $n = E_{timber} / E_{CFRP}$ can be determined. These values may be calculated using the ratio $n = \sigma_{timber,1} / \sigma_{CFRP,2}$, as Table II shows. The resulting mean value determined from the results of tested specimens is $n = 0.068$, which is by about 5 % only different (more) with the theoretical value.

Using the actual values of parameter n obtained from the tests the second moments of the area corresponding to the calculated n values can be calculated as their actual values for particular test specimens. Overview of the second moments of area $I_{i,theory}$ calculated for theoretical value of n parameter (taken from modulus of elasticity of both materials given by producer information) and $I_{i,test}$ calculated for n parameter derived from the tests is in Table III. From the values for particular test specimens and their cross-sections the ratio of test to theoretical values is 1.022, as shown in Table III.

B. Longitudinal Shear Stresses

Load-carrying capacities of all test specimens were higher than predicted values calculated according to elastic approach (see e.g. [7], [8], [9], [11], [13]) and actual stresses obtained from the tests and parameters derived from them indicate, that the actual behaviour of investigated members very good correspond with the elastic principles assumed as the suitable approach for the stress and resistance analysis. The results also show that the interaction between timber and CFRP reinforcement is very satisfactory for using the assumption of the rigid shear connection and the concept of the substitute cross-section following from this fact.

However, in the case of some test specimens the rupture of CFRP lamella (see Fig. 12) and, mainly, the separation of the lamella from timber occurred (see Figs. 13 and 14). Therefore, the attention has been paid to the shear connection, too, from the viewpoint of the actual values of longitudinal shear force and from that arising longitudinal shear stress, respectively, between CFRP reinforcement and timber base. In this context, currently research is also oriented to the analysis of glue properties aimed to select the most suitable glue for CFRP lamella to timber bonded connection.

The longitudinal shear forces $V_{long,1}$ (per unit of the length) and corresponding longitudinal shear stresses τ_{long} at the interface between CFRP and timber calculated using both the actual cross-section characteristics derived from the tests (see e.g. the second moments of area in Table III) and theoretical ones, are presented in Table IV. Shear stresses have been calculated for the lamella width, which is corresponding width of resisting cross-section part in the level of longitudinal shear actions. Table IV shows stress values for the shear forces equal to 0.25 V_u, 0.5 V_u and 0.75 V_u, where V_u is the maximum (ultimate) shear force reached within the tests, when the test specimen collapse occurred.

Table IV shows that the actual affects of longitudinal shear between CFRP lamella and timber (longitudinal shear forces and shear stresses) vary in the range of ±10 %, in average, with theoretical values.

Fig. 12 CFRP lamella rupture together with timber failure

Fig. 13 CFRP lamella separation

Fig. 14 delaminated CFRP lamella

Table IV Longitudinal shear in glued interface between timber and CFRP lamella: shear forces $V_{long,1}$ [N.mm^{-1}], stresses τ_{long} [MPa]

100/220-C V_u = 27.33 kN		0.25 V_u	0.5 V_u	0.75 V_u
theory	$V_{long,1}$	6.77	13.53	20.31
	τ_{long}	0.135	0.271	0.406
derived from tests	$V_{long,1}$	7.25	14.50	20.74
	τ_{long}	0.145	0.290	0.435
100/200-C V_u = 23.03 kN		0.25 V_u	0.5 V_u	0.75 V_u
theory	$V_{long,1}$	6.80	13.60	20.41
	τ_{long}	0.136	0.272	0.408
derived from tests	$V_{long,1}$	6.11	12.22	18.33
	τ_{long}	0.122	0.244	0.367
100/180-C V_u = 18.52 kN		0.25 V_u	0.5 V_u	0.75 V_u
theory	$V_{long,1}$	6.64	13.28	20.51
	τ_{long}	0.133	0.266	0.398
derived from tests	$V_{long,1}$	6.84	13.67	20.51
	τ_{long}	0.137	0.273	0.410
100/160-C V_u = 20.13 kN		0.25 V_u	0.5 V_u	0.75 V_u
theory	$V_{long,1}$	8.95	17.91	26.86
	τ_{long}	0.179	0.358	0.537
derived from tests	$V_{long,1}$	7.05	14.09	21.14
	τ_{long}	0.141	0.282	0.423
100/140-C V_u = 16.37 kN		0.25 V_u	0.5 V_u	0.75 V_u
theory	$V_{long,1}$	9.26	18.52	27.78
	τ_{long}	0.185	0.370	0.556
derived from tests	$V_{long,1}$	10.06	20.13	30.19
	τ_{long}	0.201	0.403	0.604
100/120-C V_u = 14.02 kN		0.25 V_u	0.5 V_u	0.75 V_u
theory	$V_{long,1}$	10.43	20.86	31.29
	τ_{long}	0.209	0.417	0.626
derived from tests	$V_{long,1}$	10.60	21.19	31.79
	τ_{long}	0.212	0.424	0.636

IV. Conclusions

From the test verification, which results – actual normal and shear stresses and from them calculated cross-section characteristics – are listed in Tables I to IV and illustrated in Figs. 6 to 11, the following conclusions can be deducted:

A. *Normal Stresses*

- The actual normal stresses in timber are by about from 10 to 30 % less than the stresses determined by elastic calculation; the usual differences with theoretical values are about 20 % in average; these facts indicate that the elastic approach can be taken as the right and suitable for the safe calculation.
- The actual normal stresses in CFRP reinforcement are by about from 5 (sporadically) up to 40 % less than the calculated elastic stresses; the usual differences vary around the values from 20 to 30 %, that means also in the case the calculation using elastic approach can be applied, even as safer than in the case in timber.
- The values of the parameter n derived from the tests using the measured stresses are in the range from 0.0595 to 0.0860 with the mean value of 0.068, in comparison with the theoretical value of 0.0645, that the difference is 5 % only; this shows relatively very good interaction between CFRP reinforcement and timber and confirms the rightness of the elastic calculation based on the concept of the substitute (ideal) cross-section arising from the parameter n as a ratio of Young's modulus of both section materials.
- The values of the substitute cross-section characteristics, namely the second moment of area, calculated using verified n parameter (see above) are different from +12 to -4% with the theoretical values, while the mean value of the difference 2.2 % is very low, so that the second moment of area and other section characteristics can be calculated according to the formulas mentioned above – see, for example, (4), (5).

B. *Longitudinal Shear Stresses*

- The actual values of longitudinal shear forces at the interface between CFRP lamella and timber and longitudinal shear stresses arising from them, which have been derived using the actual parameters from the tests (see above), are different in the range ±10 %, in average, with theoretical values; this shows very good match of the theoretical calculation of shear connection with the reality and the rightness of this principle for investigated structural members composed of timber and externally reinforced by CFRP lamellas; but in this case of bonded connections "CFRP – timber" the attention must be more paid to the investigation of suitable glue and verification of its properties from the viewpoint of the coherence of both materials, because there is one of the important problems which can significantly influence load-carrying capacity and namely interaction between both parts.

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Within the elaboration of the paper the realized loading tests have been evaluated applying the procedures using the experimental results including methods of the design assisted by testing based on the statistical approach and using the experiences obtained by the similar evaluation of the large sets of test data for various structural members and parts composed of variety of materials, which are described in [16] - [19], [23] - [32], [35]. In partial, the methods based on the probabilistic approaches have been taken into account, too – see e.g. [20], [21], [22], [27], [28]. Author thanks the staff of the test room at the authors' workplace for the works within the preparation and realization of large number of tests mentioned above.

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