Controller choice for car active suspension

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Abstract — Automotive suspension system design is very important part of the passengers comfort and safety. In this article automotive active suspension with electric linear motor as an actuator is designed. Due to many reasons H-infinity control is used. This paper is focused on comparison of different controller designed for quart, half or full-car controller (and always used for "full" car). Each controller configuration is simulated and then verified on the hydraulic quarter car test bed.

Keywords — robust control, active vehicle suspension, linear motors, energy control.

I. Introduction

RECENTLY there is an increased demand on automotive suspension system design. The basic function of the vehicle suspension is to provide comfort to passengers, maximize the friction between the tires and the road surface and provide steering stability with good handling.

In the time of growing interest of renewable energy resources the minimization of energy consumption can be small contribution to better utilization of energy resources. Especially for the car application the energy consumption play important role of the design process. In this paper the linear electric motor is used as an actuator and then there is possibility to recuperate energy during specific movement of suspension, accumulate it and use it later when necessary.

All suspension systems are designed to meet specific requirements. In suspension systems, usually two most important features are expected to be improved - disturbance absorbing (i.e. passenger comfort) and attenuation of the disturbance transfer to the road (i.e. car handling). The first requirement could be presented as an attenuation of the damped mass acceleration or as a peak minimization of the damped mass vertical displacement. The second one is characterized as an attenuation of the force acting on the road or - in simple car model - as an attenuation of the unsprung mass acceleration. It is obvious that there is a contradiction between these two requirements. Effort devoted to passive suspension design is ineffective, because there is a contradiction between both requirements. The best result (in sense of requirements improvement) can be achieved by active suspension, which means that some additional force can act on system.

With respect to these contradictory requirements the best results can be achieved using active suspension systems generating variable mechanical force acting in the system using a linear electrical motor as the actuator. Compared to traditional drives using rotational electro-motors and lead screw or toothed belts, the direct drive linear motor exhibits the property of contact-less transfer of electrical power according to the laws of magnetic induction. The electromagnetic force is applied directly without the intervention of a mechanical transmission. Low friction and no backlash resulting in high accuracy, high acceleration and velocity, high force, high reliability and long lifetime enable not only effective usage of modern control systems but also represent the important attributes needed to control vibration suspension efficiently.

Fundamental information about an active suspension and the important practical aspects are in [1] and [2]. The articles summarize elementary facts beginning with the road profiles and ending with different configurations of an active suspension systems and their control. In the papers [3] and [4] an H1 approach to the active suspension with hydraulic actuator has been proposed. Articles linearize the nonlinear actuator and apply the linear H1 theory with satisfactory results. Of course there exist other approaches to the active suspension control than H1. Optimal preview control with integral constraints is in [5]. In [6] can be found L1 state feedback. Special approach to the half car model is in [7], where authors decompose the model and then put it together in another way to simplify definitions of the performance requirements and cost functions. Car model can be divided into front and rear part and rear part is delayed during driving. Moreover this delay is variable. Interesting study on time-varying delay control is in [8].

II. Model configuration

Most important part of controller design is to develop and use appropriate mathematic model, which is later validated during experiments. This paper is prepared to compare different system configurations – with quarter, half and full-car model controller.

Generally, the suspension system consists of a spring and a damper connected to the sprung and unsprung masses

Manuscript received August 16th, 2010.

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(Fig. 1). Bottom spring represents a character of the car tire. In semi-active suspension a controllable damper is used and therefore the controller has possibility to change the damping quotient. Therefore the damping characteristics of the system can be changed dynamically during driving.

Interesting study on symbolic analysis of mechanical models is in [9].

A. *Quarter car model*

So let's start with the simplest quarter car model (see Fig. 1). It contains two springs (one in suspension and second representing car tires), one dumper and source of power as actuator.

In fact a linear electric motor is used as an actuator but presumption to use ideal source of power can be achieved, because the linear motor in its working range has nearly linear characteristic and very small time constants.

The model is described by the following motion equations:

$$\begin{aligned} m_b\ddot{z}_b &= F - k_b(z_b - z_w) - b_b(\dot{z}_b - \dot{z}_w) \\ m_w\ddot{z}_w &= -F + k_b(z_b - z_w) - k_w(z_w - z_r) + b_b(\dot{z}_b - \dot{z}_w) \end{aligned} \quad (1)$$

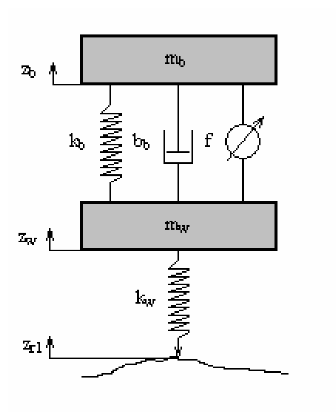

Fig. 1. Quarter car model schema.

B. *Half car model*

Natural expansion of quarter car model is half car model for the left and right part of the car. Model is shown in the Fig. 2. Then the following motion equations are added for pitching, centre of gravity movement and body speed, respectively:

$$\begin{aligned} m_b\ddot{z}_{b1}L_1 - m_b\ddot{z}_{b2}L_2 - J_P\dot{\omega} &= 0 \\ m_b\ddot{z}_{b1} + m_b\ddot{z}_{b2} - m_b\dot{v}_T &= 0 \\ \dot{z}_{b1} &= v_T + \omega L_1 \\ \dot{z}_{b2} &= v_T - \omega L_2 \end{aligned} \quad (2)$$

Of course, there is possibility to prepare model for the front and rear part also. In fact we can use the same equations and rotate the model for 90°.

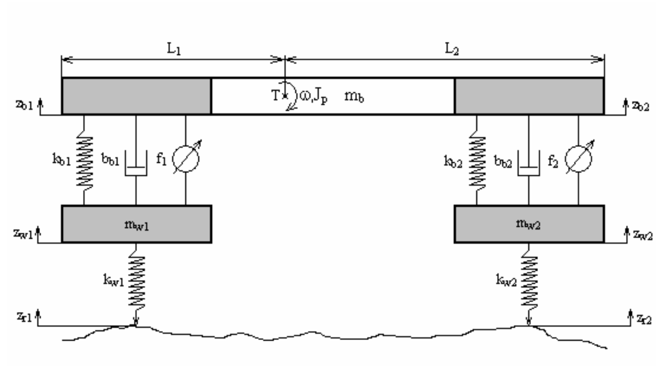

Fig. 2. Half car model schema.

C. *Full car model*

Final model in this paper is full car model. It means two half car model are linked together. Then the following motion equations are added for pitching, rolling, centre of gravity movement and body speed, respectively:

$$\begin{aligned} F_1L_1 - F_2L_2 + F_3L_1 - F_4L_2 - J_P\dot{\omega} &= 0 \\ F_1D_1 + F_2D_2 - F_3D_1 - F_4D_2 - J_{PR}\dot{\Omega} &= 0 \\ F_1 + F_2 + F_3 + F_4 - m_b\dot{v}_T &= 0 \\ \dot{z}_{b1} &= v_T + \omega L_1 + \Omega D_1 \\ \dot{z}_{b2} &= v_T - \omega L_2 + \Omega D_2 \\ \dot{z}_{b3} &= v_T + \omega L_1 - \Omega D_3 \\ \dot{z}_{b4} &= v_T - \omega L_2 - \Omega D_4 \end{aligned} \quad (3)$$

More about car behavior and modeling can be found in [10].

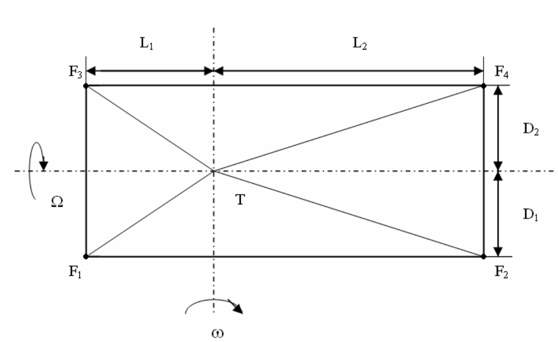

Fig. 3. Full car model schema.

D. *Simulation model*

The full-car active suspension model has been developed for complex analysis of suspension behavior. This model is considered as a vehicle model with four wheels. Moreover influence of braking and cornering is included. Finally passenger model is integrated to complete this mathematical car model.

In generally, this model can be considered as arbitrary kind of suspension, because in fact, braking and cornering simulate influences of the wind acting on the system and passenger is additional mass in the system only.

Described model has been implemented into Matlab Simulink as additional library (Fig. 4) and consequently has been used for analysis of the problem. Interested reader can find detailed description in [11].

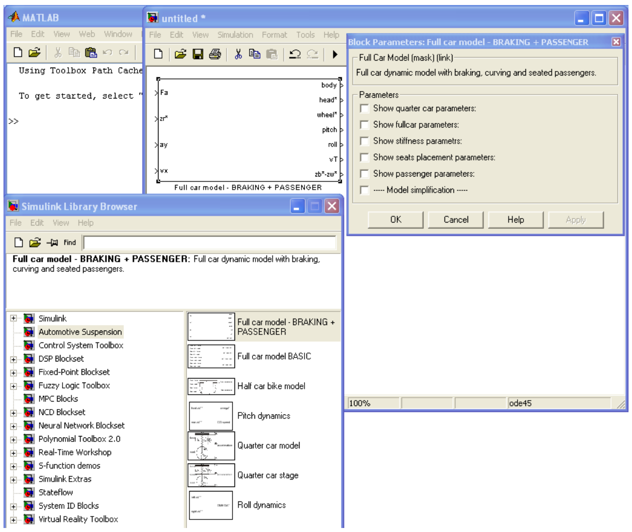

Fig. 4. Developed Matlab Simulink library.

E. Experiment configuration

All experiments with the active damper have been performed using the linear motor TBX3810 Copley Controls Cooperation equipped with two different control units. First of them uses two microcontrollers AT90S4434 Atmel (control unit, communication) and the second one DSP 1104 dSPACE. PWM amplifier 7426ACH Copley Controls Cooperation proved to be a satisfactory power amplifier. As an energy dissipater the model No. 145 Copley Controls Cooperation has been used. All the experiments have been measured to verify validity of the results obtained using the Simulink model.

The experiments have been performed using the experiment stand representing a one-quarter-car and road disturbances. Mechanical configuration is obvious from the Figure 5. The test bed consists of hydraulic source of power (as an input road disturbance), one quarter of the car and linear electric motor (as actuator).

Fig. 5. Experimental test bed.

III. LINEAR ELECTRIC MOTOR

Linear electric motor can transform electrical energy to straight motion of the rotor. In fact, linear motor works on the same basis as conventional electric motor, the only difference is that linear motor has unfolded windings to straight direction. It can be imagined as common rotary motor with diameter equal to infinity. Basic principle is shown in Fig. 6 (figure has been adapted from manufacturer spreadsheets). The same principle in larger scale is used for example in magnetic levitation trains

In this paper another property of motor has been utilized. The motor is able to recuperate energy, thus motor can transform energy from straight motion to electrical energy. That means linear motor use vertical motions to produce electrical energy.

The objective of controller design is to find the balance between energy demands and supply. In other words to find the trade-off between performance and energy consumption.

All experiments have been done using tubulus linear electric motor TBX3810 (ThrustTube). The important properties are:

- peak force 2027N
- peak current 21.8A
- continuous stall force 293.2N
- electrical time constant 1.26ms
- continuous working voltage 320V ac
- maximum phase temperature 100 ◦C

The beauty of linear motors is that they directly translate electrical energy into usable linear mechanical force and motion, and vice versa. The motors are produced in synchronous and asynchronous versions. Compared to conventional rotational electro motors, the stator and the shaft (translator) of direct-drive linear motors are linear-shaped.

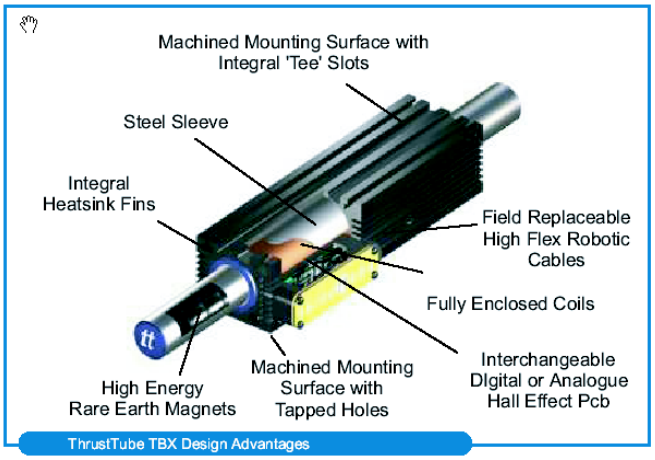

Fig. 6. Linear electric motor TBX3810.

Linear motor translator movements take place with high velocities (up to approximately 200m/min), large accelerations (up to g multiples), and forces (up to kN). As mentioned above, the electromagnetic force can be applied directly to the payload without the intervention of a mechanical transmission, what results in high rigidity of the whole system, its higher reliability and longer lifetime. In practice, the most often used type is the synchronous three-phase linear motor. In this research the ThrustTube TBX3810 motor is used (see [12]).

The force/velocity profile is in Fig. 7. This profile assumes the continuous working voltage is available across the motor (there are no amplifier limitations).

It is necessary to answer one important question – if it is more advantageous to include the model of the linear electric motor in the model for active suspension synthesis or if it should be used only for simulations.

Comparing advantages and disadvantages of the model inclusion, it can be said that the closed-loop provides more information so that better control results can be achieved. Unfortunately, there are also some significant disadvantages in such a solution (complexity, nonlinearity etc.).

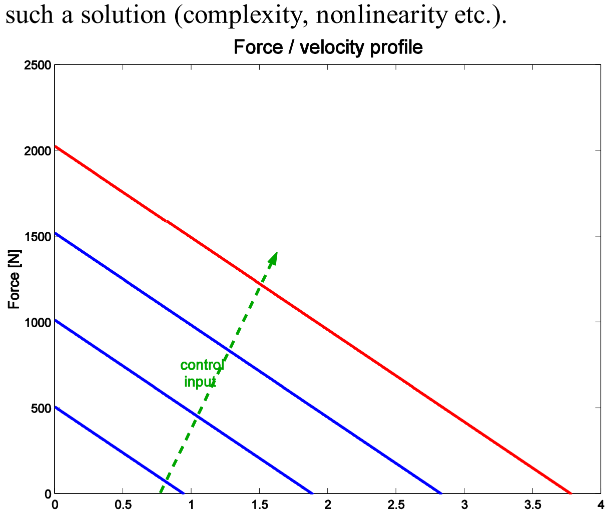

Fig. 7. Force / velocity profile

There is another important question whether the linear motor model could be omitted and a linear character of the desired force could be supposed. The answer is “yes”. Both the mechanical and the electrical constants are very small – just about 1ms.

The detailed non-linear motor model used for simulation (not for synthesis) is described in [13] or [14].

IV. CONTROLLERS

There is a basic question which model should be used for controller design? According to mathematical models described in the previous section, three different controllers have been designed. Starting with easiest quarter car and ending with full car model controller.

What can be expected? If the design model is not full then obviously the results cannot be optimized for real full car. But is it really an issue? Are the results for quarter car model so bad in comparison with full model? Next paragraphs describe such design and then the controllers are compared.

A. H-infinity controller

An H-infinity controller is computed as $\|T_{y1u1}\|_\infty$ norm minimization (see Fig. 9). So it is possible to shape a closed loop characteristic in open branch to improve performance of the whole system. The basic schematic diagram of a plant augmentation is shown in the Fig. 8.

System has two inputs – road disturbance and acting signal for linear motor. The first output is used to modify performance and robustness behavior of resulted closed loop. Second output is the feedback signal for controller.

The first output can be weighted by either by constant or by frequency function to achieve the desired closed loop characteristic. How the function or constant was developed is described the following sections (they are different for each controller configuration).

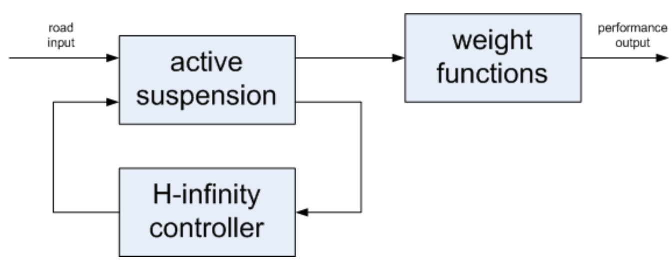

Fig. 8. Standard H-infinity configuration

Detailed schema of H-infinity controller design is in Fig. 9. The first input signal v represents the road disturbance and is scaled by factor Sv. The first group (port) includes the input f, which is disturbance signal acting on measured feedback. The second input is desired value for actuator. And as for outputs, the first output y1 consists of two parts: (1) nominal system outputs (in this case states weighted by MIMO function Wperf and scaled by constant matrix Sy and (2) actuating signal weighted by Wrob. The second output is formed as sum of a nominal output y2 and disturbance input f.

Suspension speed has been chosen as a measured output (y2), because for a linear motor control the speed has to be measured as well. Moreover signals, which affect the important characteristics, have to be weighted in controlled output (y1). Influence of weighting functions and constants is mainly evident; moreover it has been checked by simulations. So the first output is following: suspension displacement, wheel-road displacement, vertical body speed (Wperf1), vertical wheel speed (Wperf2) and Wrob. Robustness function Wrob is supposed to be total weighting by reason of linear motor nonlinearities, feedback branch disturbances and other uncertainties.

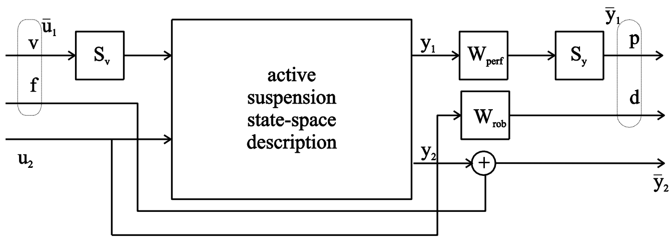

Fig. 9. Detailed H-infinity schema configuration

The controllers have been developed using H-infinity control theory (for more details see [15]). Interesting case study on H-inifinity controllers is in [16] and [17].

B. *Quarter car controller*

The first developed controller is for quarter car model. Then the same controller is used four times for each suspension element (front – rear, left – right). Configuration is obvious from the Figure 10. Of course the disadvantage is the controller has not the information about other elements. It means there is only small (and indirect) possibility to influence rolling and pitching.

Following weights have been used:

- road to wheel deviation – help to improve steady state deviation, influence stability of the car
- body acceleration – improve comfort of passenger, weighted by frequency function
- acting signal – provide possibility to limit desired actuator force, weighted by function
- wheel acceleration

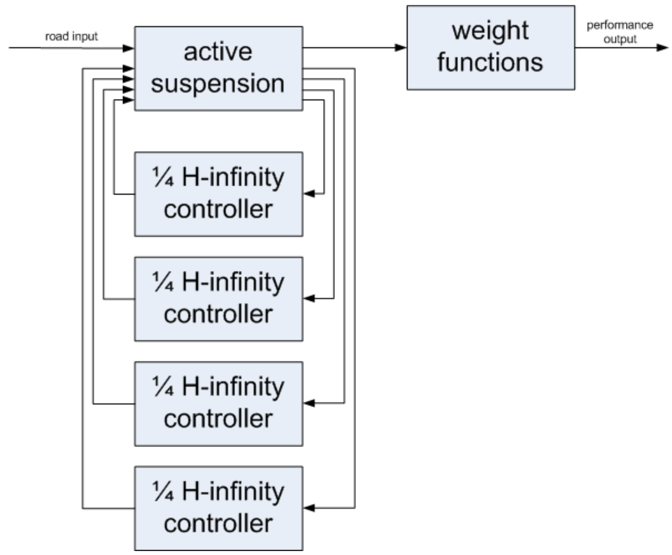

Fig. 10. Quarter car controller configuration.

As mentioned above, two outputs are weighted by function. First the body acceleration which is weighted according to different passenger sensitivity in different frequency ranges. Weighting function is plotted in Fig. 11.

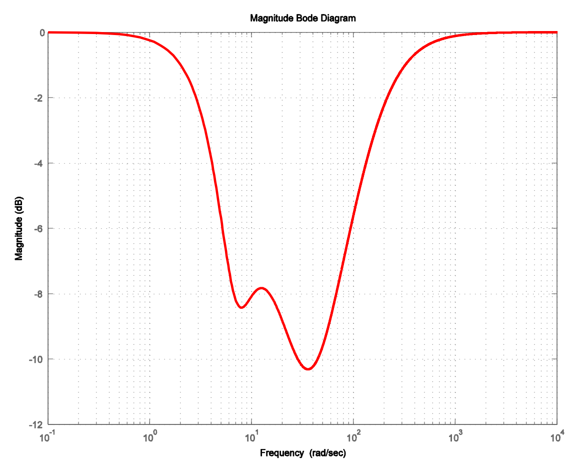

Fig. 11. Human sensitivity function

Second the acting signal where the higher frequencies are limited by this function. In fact high pass filter (because of inverse) is used for weighting. Robustness function is plotted in Fig. 12.

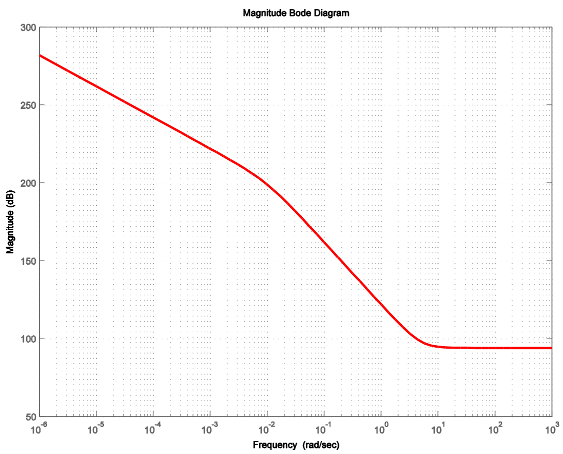

Fig. 12. Robustness weighting function.

C. *Half car controller*

Half car controller has one advantage against quarter car – it can directly influence either the rolling or pitching of the car. It implies we have two possibilities during the half car controller design. Configuration is drawn in Figure 13.

The first possibility is to divide the car to the left and right half. Then the pitching can be influenced and resulted controller has been used two times for each element (left – right). For weighting the same outputs have been used as for quarter car (of course the values for weighting are different! because of influence between front and rear part of the car). Additional weighting constant is added – angle of pitching.

The second possibility is to divide the car to the front and rear half. Then the rolling can be influenced and resulted controller has been used two times for each element (front – rear). Additional weighting constant is added – angle of rolling.

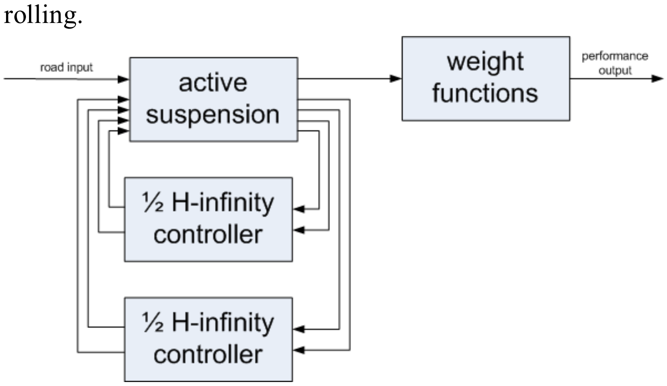

Fig. 13. Half car controller configuration.

D. *Full car controller*

Of course last possibility how to control full suspension configuration is to develop controller for exactly this model. Then there is possibility to control all – each suspension element properties, rolling and pitching. Obviously such a controller is most difficult to design. Configuration is in Figure 14. The weights are the same as quarter car together with roll and pitch angle.

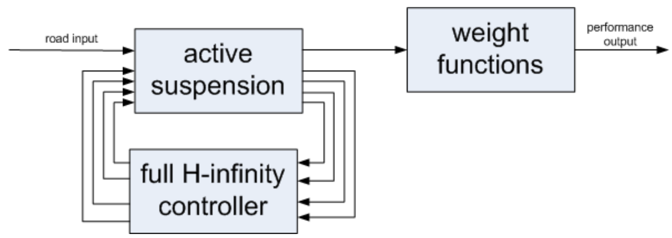

Fig. 14. Full car controller configuration.

V. INPUT SIGNAL

There are many opportunities how to simulate road disturbance and there exists many possible models. In this paper two of them have been used.

A. *Slow-down retarder*

One specific situation important for car comfort and road friendliness has been chosen – the slow down retarder. Size of this jump is 0.5 x 0.45 x 0.05m.

As a mathematical model the half of sinus function sin(75,36 t) is used and of course, the longitude velocity has to be taken into consideration.

B. *Common road (deterministic)*

One possibility how to model common road is to use deterministic “random” signal. It can be described by the equation (4).

Resulted input signal is plotted in Figure 15.

$$z_r = \sum_{i=1}^{n} \sqrt{\frac{\dot{\omega}}{\pi \cdot v_x}} \left\{ \mathrm{Re}\left(\frac{b_o}{-\omega^2 + a_1 j\omega + a_o}\right) \cdot \cos(\omega t + \alpha_i) + \right.$$

$$\left. + \mathrm{Im}\left(\frac{b_o}{-\omega^2 + a_1 j\omega + a_o}\right) \cdot \sin(\omega t + \alpha_i) \right\}$$

$$b_o = 0.121 \cdot v_x \qquad (4)$$
$$a_o = 2.249 \cdot v_x$$
$$a_1 = 30.36 \cdot v_x$$

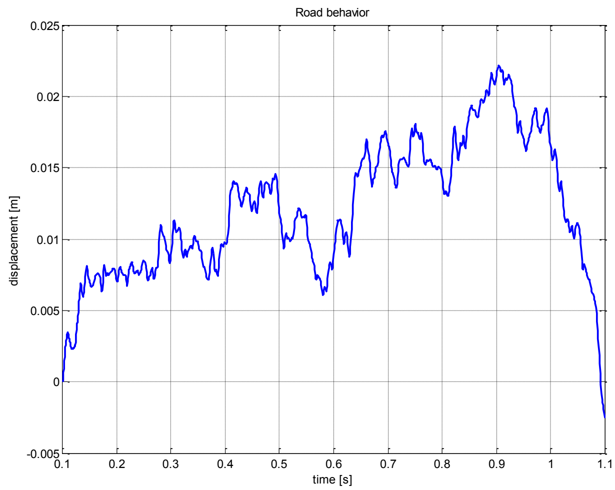

Fig. 15. Deterministic road disturbance signal.

VI. QUANTIFICATION

Again as for road model there are many possibilities how to quantify simulation or experimental results. Of course the results are only one so the quantification always must give the same comparison. In this paper the RMS value for body acceleration (comfort) and wheel-road deviation (stability).

The RMS for stability is defined as RMS of road to wheel deviation:

$$J_{stab} = \sqrt{\int_0^T \left(z_w(t) - z_r(t)\right)^2 dt} \quad (5)$$

The RMS for comfort is defined as:

$$J_{comfISO} = \sqrt{\int \left(G_w(t-\tau) * \ddot{z}_b(t)\right)^2 dt} \quad (6)$$

Where G_w in (6) is transfer function defined by the ISO norm 2631 as a sensitivity of human being to different vibration frequency and * means the convolution.

VII. RESULTS

The stability has not been affected by the different controllers, because the design was driven to achieve stability level and improve comfort. So only comfort is evaluated bellow.

The main objective of the paper was to compare different H-infinity controller. If we are thinking about each condition in controllers then the results are not surprising, but some aspects are at least important from the practical aspects. Let's take a look at it.

For closed-loop illustration there is body acceleration (in the Fig. 16) response on a driving through slow-down retarder described in previous section for quarter car model configuration (full controller = green, half controller=red, passive=blue).

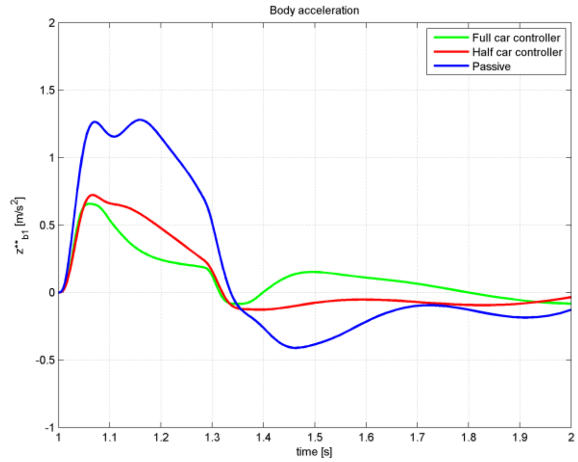

Fig. 16. Body acceleration on a slow-down retarder.

Next figure (Fig. 17) illustrates the angular velocity of pitching of passive suspension (blue), half car controller (red) and full car controller (green) as a response during driving through slow-down retarder. There is an obvious improvement of half and full car controller against passive suspension and there is minimum difference between half and full controller (as is proven in Tab. 2).

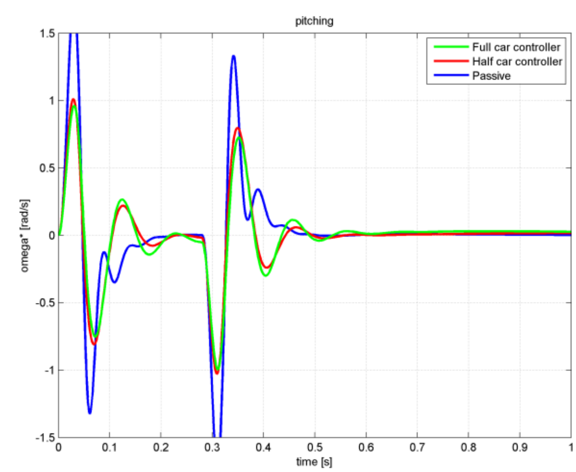

Fig. 17. Angular velocity on a slow-down retarder.

Finally is the time to compare each method of control used for automotive active suspension. Both described signal was tested during simulations (and for quarter car during experiment). The results are summarized in the Table 1. The RMS for comfort (mentioned above) is compared for each controller, whereas "passive" controller means without control.

	Slow-down retarder	Common road
Passive	0.2738	0.3860
¼ controller	0.2201	0.2931
½ controller	0.2674	0.3438
Full controller	0.2698	0.3622

Tab. 1. Comfort (RMS) comparison.

Second table compares the rolling of the car for each controller. This gives us the picture how seems the results in case there are not (or there are) information about each element. Results are in Table 2.

Both tables acknowledge the results illustrated on slow down retarder in Figures 18 and 19. Similar results have been obtained for simulations on “common road” described in section 5.2.

	Slow-down retarder	Common road
Passive	0.0207	0.0362
¼ controller	0.0190	0.0254
½ controller	0.0133	0.0194
Full controller	0.0138	0.0196

Tab. 2. Pitch angle (RMS) comparison.

VIII. CONCLUSION

In this paper several H-infinity controllers (with different complexity) for active suspension with linear electric motor have been designed and then compared together. The experiment signal for real road simulation has been developed and then it has been used for simulations experiments.

It has been proved that it is not necessary to design the complex controller to achieve the best result. It can be observed that quarter car controller in full car simulations shows the better results for passenger comfort at a cost of the worse pitching (or rolling) of the car.

This is true for H-infinity control due to its robustness, but probably it will not be valid for some type of predictive control, where the signal from the front wheel can be used in the rear of the car. This should be inspiration for future research.

ACKNOWLEDGEMENT

This research has been supported by the MSMT project No. 1M6840770002 ”Josef Bozek’s Research Center of Combustion Engines and Automobiles II”

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