Vibrations of elastic stretched strips with cracks

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Abstract— Vibrations of elastic plate strips supported at two opposite edges and free at the other edges are studied. It is assumed that the thickness of the strip is piece wise constant whereas stable part-through cracks are located at the re-entrant corners of steps. Making use of the basic concepts of the fracture mechanics a method for determination of eigenfrequencies of stepped plates with cracks is developed. The influence of a crack on the behavior of the strip is modeled as a change of the local flexibility or as a distributed line spring. Numerical results are presented for strips with cracks and without any crack subjected to the tension applied at an edge of the strip.
Keywords— Crack, Eigenfrequency, Plate, Strip, Vibration

I. Introduction

Due to the growing interest for the use of non-destructive testing technique and vibration monitoring of structures and machines there exists a growing need for the vibration analysis of structural elements with flaws and cracks.

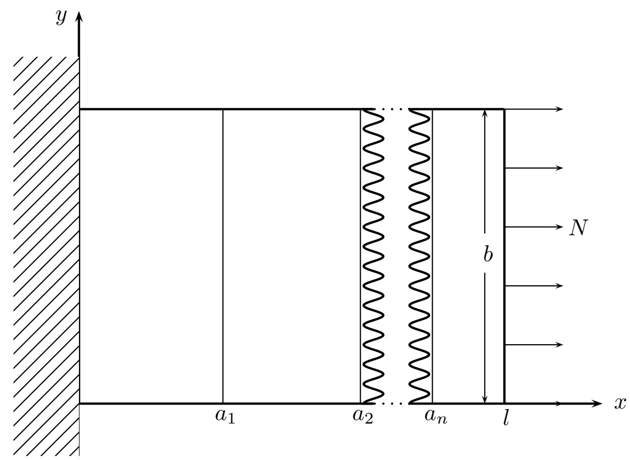
The matter that the presence of surface flaws or intrinsic cracks in a machine element is a source of local flexibility which in turn influences the dynamic behavior of the whole system was recognized long ago. The idea of an equivalent elastic spring, a local compliance, was used first to quantify the relation between the applied load and the strain concentration in the vicinity of the crack tip by Irwin [1].

Later Rice and Levy [2] computed the local flexibility in the case of a combined loading consisting of the bending and tension.

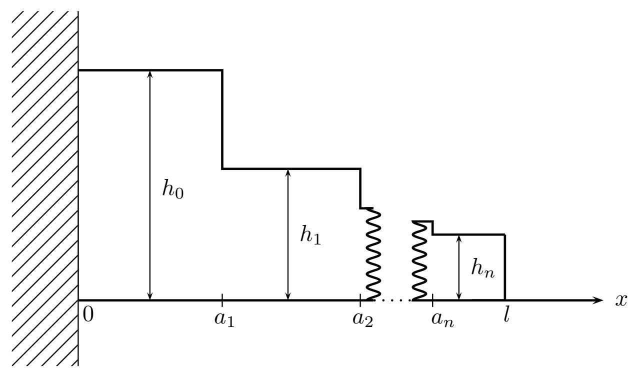
Dimarogonas and Paipetis [3], Dimarogonas [4], Chondros *et al.* [5], [6] combined this spring model in the case of a vibrating beam with the methods of the elastic fracture mechanics. As a result the frequency spectral method was developed. This idea was exploited by Rizos *et al.* [7], Kukla [8], Chondros *et al.* [6] for the analysis of cracked beams. It was extended by Lellep *et al.* [9], [10], [11] for axisymmetrical vibrations of cylindrical shells.

The effect of cracks on the free vibration of uniform beams with arbitrary number of cracks was studied by Lin, Chang, Wu [12] making use of the transfer matrix method. Liang, Choy, Hu [13], [14] developed a method of detection of cracks in beams making use of measurements of natural frequencies. This idea was exploited also by Nandwana and Maiti [15].

De Rosa [16] investigated the influence of cracks on the free vibrations of stepped beams with flexible ends. It was assumed that following forces were applied at the steps. Prestressed beams with fixed ends were studied by Masoud *et al.* [17]. The authors of [18] presented theoretical and experimental results concerning an axially loaded beam weakened with cracks.



(a) Dimensions of the strip



(b) Piece wise constant thickness

Fig. 1. Plate strip

In the present paper free vibrations of stepped plate strips are studied in the case of presence of cracks.

II. Formulation of the problem

Let us consider natural vibrations of a plate strip subjected to the in-plane tension N (Fig. 1). Let the dimensions of the strip in x and y direction be l and b, respectively. The plate under consideration is clamped at the edge $x = 0$, the other edges are free.

The thickness h of the plate us assumed to be piece wise constant. Thus

$$h(x, y) = h_j \tag{1}$$

for $x \in (a_j, a_{j+1})$, where $j = 0, \ldots, n$. The quantities a_j and h_j $(j = 0, \ldots, n)$ are given constants whereas $a_0 = 0$, $a_{n+1} = l$.

It is assumed that at cross sections $x = a_j$ $(j = 1, \ldots, n)$ where the thickness has jumps cracks of constant depth c_j are located. These flaws or cracks are treated as stable surface cracks.

In the present study like in Rizos *et al.* [7], Chrondos *et al.* [6], Dimarogonas [4] the problem of crack propagation in the body during vibrations is disregarded.

The material of plates is considered as a linear elastic material. Both, homogeneous elastic plates and these made of non-homogeneous composite materials are studied.

The aim of the paper is to elucidate the sensitivity of natural frequencies on the crack parameters and geometrical parameters of the plate.

III. Basic equations

In the present case of the plate it is reasonable to assume that the stress-strain state of the plate depends on the time t and coordinate x only, provided the stresses mean generalized stresses (bending moments and membrane forces). However, due to the tension N applied at the edge of the plate in-plane forces have to be taken into account. If, moreover, the inertia of the rotation is not neglected as well the equilibrium equations of a plate element can be presented as (Reddy [19])

$$\begin{aligned} \frac{\partial M_x}{\partial x} &= Q_x \\ \frac{\partial Q_x}{\partial x} &= -N\frac{\partial^2 W}{\partial x^2} + \rho h_j \frac{\partial^2 W}{\partial t^2} - I_j \frac{\partial^4 W}{\partial x^2 \partial t^2} \end{aligned} \tag{2}$$

for $x \in (a_j, a_{j+1})$, where $j = 0, \ldots, n$.

In (2) $W = W(x, t)$ stands for the transverse deflection corresponding to the point with coordinate x at the middle plane of the plate whereas M_x is the bending moment and Q_x - the shear force. Herein

$$I_j = \frac{\rho h_j^3}{12} \tag{3}$$

where ρ stands for the density of the material. According to this theory the membrane force in the direction of the axis x $N_x = N$ in the present case.

Eliminating the shear force Q_x from (2) one easily obtains

$$\frac{\partial^2 M_x}{\partial x^2} + N\frac{\partial^2 W}{\partial x^2} = \rho h_j \frac{\partial^2 W}{\partial t^2} - I_j \frac{\partial^4 W}{\partial x^2 \partial t^2} \tag{4}$$

for $x \in (a_j, a_{j+1})$ where $j = 0, \ldots, n$.

It is well known that (Reddy [19], Soedel [20])

$$M_x = -D_j \frac{\partial^2 W}{\partial x^2} \tag{5}$$

where $D_j = Eh_j^3/12(1 - \nu^2)$; $j = 0, \ldots, n$.

Substituting (5) in (4) yields

$$D_j \frac{\partial^4 W}{\partial x^4} - N\frac{\partial^2 W}{\partial x^2} = \rho h_j \frac{\partial^2 W}{\partial t^2} - I_j \frac{\partial^4 W}{\partial x^2 \partial t^2} \tag{6}$$

for $x \in (a_j, a_{j+1})$ where $j = 0, \ldots, n$. The latter will be considered as the equation of motion for the segment (a_j, a_{j+1}); $j = 0, \ldots, n$. It can be solved accounting for appropriate boundary conditions. In the case of a free edge of a strip the boundary conditions are

$$\frac{\partial^2 W}{\partial x^2} = 0, \ \frac{\partial^3 W}{\partial x^3} = 0. \tag{7}$$

However, in the case of the clamped edge one has

$$\frac{\partial W}{\partial x} = 0, \ W = 0. \tag{8}$$

Let at the initial moment

$$\frac{\partial W}{\partial t} = 0, \ W = \varphi(x) \tag{9}$$

where φ is a given function.

IV. Solution of the equation of motion

It is reasonable to look for the general solution of (6) in the form

$$W(x, t) = w_j(x) T(t) \tag{10}$$

for $x \in (a_j, a_{j+1})$ where $j = 0, \ldots, n$.

Differentiating (10) with respect to variables x, t and substituting in (6) one easily obtains

$$D_j w_j^{IV} T - N w_j'' T = \rho h_j w_j \ddot{T} - I_j w_j'' \ddot{T} \tag{11}$$

for $x \in (a_j, a_{j+1})$ where $j = 0, \ldots, n$. Here prims denote the differentiation with respect to the coordinate x and dots – with respect to time t.

Separating variables in (10) yields

$$D_j w_j^{IV} + (I_j \omega^2 - N) w_j'' - \rho h_j \omega^2 w_j = 0 \tag{12}$$

for $j = 0, \ldots, n$ and

$$\ddot{T} + \omega^2 T = 0 \tag{13}$$

where ω stands for the frequency of natural vibrations. Evidently, the solution of (13) which satisfied according to (9) initial conditions $T(0) = d$, $\dot{T}(0) = 0$ has the form

$$T = d \cos \omega t \tag{14}$$

where d is a constant.

The equation (12) is a linear fourth order ordinary equation with respect to the variable w_j. The characteristic equation corresponding to (12) is

$$D_j r_j^4 + (I_j \omega^2 - N) r_j^2 - \rho h_j \omega^2 = 0. \tag{15}$$

Form (15) one easily obtains the roots

$$r_j = \pm \sqrt{\frac{-I_j \omega^2 + N}{2D_j} \pm \sqrt{\frac{(I_j \omega^2 - N)^2}{4D_j^2} + \frac{\rho h_j}{D_j}}}. \tag{16}$$

Introducing the notation

$$\begin{aligned} (r_j^2)_1 &= -\lambda_j^2 \\ (r_j^2)_2 &= \mu_j^2 \end{aligned} \tag{17}$$

one can present the general solution of (12) as

$$\begin{aligned} w_j(x) &= A_{1j}\sin\lambda_j x + A_{2j}\cos\lambda_j x + \\ &\quad + A_{3j}\sinh\mu_j x + A_{4j}\cosh\mu_j x \end{aligned} \tag{18}$$

which holds good for $x \in (a_j, a_{j+1})$, $j = 0, \ldots, n$. Here $A_{1j}, \ldots, A_{4j}$ stand for unknown constants of integration. These will be determined from boundary conditions and requirements on the continuity of displacements and generalized stresses.

However, it appears that the quantity W' can not be continuous at $x = a_j$ according to the model of distributed line springs developed by Rice and Levy [2]; Dimarogonas [4], Chondros *et al.* [5].

V. Local compliance of the plate strip

Let us consider the influence of the crack located at the cross section $x = a$ on the stress-strain state of the sheet in the vicinity of the crack. For the conciseness sake we shall study the case when $n = 1$ and thus in the adjacent segments to the crack the thickness equals to h_0 and h_1, respectively. Let $h = min(h_0, h_1)$.

According to the distributed line spring method the slope of the deflection has a jump

$$\Theta = w'(a+0) - w'(a-0) \tag{19}$$

at the cross section $x = a$. The angle Θ can be treated as a generalized displacement corresponding to the generalized stress M_x. Thus

$$\Theta = CM_x(a) \tag{20}$$

or

$$C = \frac{\partial\Theta}{\partial M_x(a)} \tag{21}$$

where C is the local compliance due to the crack. It is known in the linear elastic fracture mechanics that (see Anderson [21], Broberg [22])

$$\Theta = \frac{\partial U_T}{\partial M_x(a)} \tag{22}$$

where U_T is the extra strain energy caused by the crack. Combining (20)–(22) one obtains

$$C = \frac{\partial^2 U_T}{\partial M_x^2(a)}. \tag{23}$$

According to the concept of the distributed line spring $1/C = K$, where K stands for the stress intensity coefficient. It is known in the fracture mechanics that (see Anderson [21])

$$K_M = \sigma_M\sqrt{\pi c}F_M\left(\frac{c}{h}\right). \tag{24}$$

In (24) c is the crack depth and

$$\sigma_M = \frac{6M_x(a)}{bh^2}, \tag{25}$$

provided the element involving the cross section $x = a$ is loaded by the bending moment M_x only. Here the function F_M is to be approximated on the basis of experimental data [23].

If the element is loaded by the axial tension N then the stress intensity coefficient

$$K_N = \sigma_N\sqrt{\pi c}F_N\left(\frac{c}{h}\right). \tag{26}$$

where

$$\sigma_N = \frac{N}{bh}. \tag{27}$$

In the case of a combined loading the stress intensity coefficient

$$K_T = K_M + K_N. \tag{28}$$

Note that (28) holds good under the condition that (24)–(27) refer to the common mode of fracture (see Anderson [21] and Broek [24]).

In the present case this requirement is fulfilled, K_M and K_N regard to the first mode of the fracture. It was shown in the previous studies (Lellep, Roots [11]; Lellep, Puman, Tungel, Roots [9]; Lellep, Sakkov [25]) that in the case of loading by the moment

$$K_M = \frac{E'h^2b}{72\pi f(s)} \tag{29}$$

where $E' = E$ for plane stress state and $E' = E/(1-\nu^2)$ in the case plane deformation state.

Here $s = c/h$ and the compliance

$$C = \frac{72\pi}{E'h^2b}\int_0^s sF_M^2(s)ds \tag{30}$$

whereas

$$f(s) = \int_0^s sF_M^2(s)ds. \tag{31}$$

The function F_M was taken in the studies by Dimarogonas [4]; Rizos, Aspragathos, Dimarogonas [7] as

$$F_M = 1.93 - 3.07s + 14.53s^2 - 25.11s^3 + 25.8s^4. \tag{32}$$

According to the handbook by Tada, Paris, Irwin [23] the function F_N can be approximated as

$$F_N = 1.122 - 0.23s + 10.55s^2 - 21.71s^3 + 30.38s^4. \tag{33}$$

VI. Determination of natural frequencies

In the case when the plate has a unique step the deflected shape of the plate can be presented according to (18) as

$$w(x) = A_1\sin\lambda_0 x + A_2\cos\lambda_0 x + A_3\sinh\mu_0 x + A_4\cosh\mu_0 x \tag{34}$$

for $x \in [0, a]$ and

$$w(x) = B_1\sin\lambda_1 x + B_2\cos\lambda_1 x + B_3\sinh\mu_1 x + B_4\cosh\mu_1 x \tag{35}$$

for $x \in [a, l]$.

Arbitrary constants A_i, B_i $(i = 1, \ldots, 4)$ have to meet boundary requirements and intermediate conditions at $x = a$. The latter can be presented as (Lellep, Roots [11])

$$\begin{aligned} w(a-0) &= w(a+0) \\ w'(a-0) &= w'(a+0) - pw''(a+0) \\ h_0^3 w''(a-0) &= h_1^3 w''(a+0) \\ h_0^3 w'''(a-0) &= h_1^3 w'''(a+0) \end{aligned} \tag{36}$$

where according to (28), (29)

$$p = \frac{Eh^3}{12(1-\nu^2)K_T}. \tag{37}$$

It is worthwhile to mention that the third and the fourth equality in (36) express the continuity of the bending moment and the shear force, respectively, when passing the step at $x = a$. It is known from the solid mechanics that these quantities must be continuous (Soedel [20]).

Boundary conditions (8) at $x = 0$ admit to eliminate from (34) the unknown constants

$$\begin{aligned} A_4 &= -A_2, \\ A_3 &= -A_1 \frac{\lambda_0}{\mu_0}. \end{aligned} \tag{38}$$

The intermediate conditions (36) with boundary requirements (8) at $x = l$ lead to the system of six equations which will be presented in the matrix form. The continuity of the deflection leads to the equation

$$\begin{bmatrix} A_1 \\ A_2 \\ B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix}^\top \times \begin{bmatrix} \sin\lambda_0 a - \frac{\lambda_0}{\mu_0}\sinh\mu_0 a \\ \cos\lambda_0 a - \cosh\mu_0 a \\ -\sin\lambda_1 a \\ -\cos\lambda_1 a \\ -\sinh\mu_1 a \\ -\cosh\mu_1 a \end{bmatrix} = 0. \tag{39}$$

According to the second relation in (36) one has

$$\begin{bmatrix} A_1 \\ A_2 \\ B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix}^\top \times \begin{bmatrix} \lambda_0(\cos\lambda_0 a - \cosh\mu_0 a) \\ -\lambda_0\sin\lambda_0 a - \mu_0\sinh\mu_0 a \\ \lambda_1(p\lambda_1\sin\lambda_1\alpha - \cos\lambda_1\alpha) \\ \lambda_1(\sin\lambda_1\alpha + p\lambda_1\cos\lambda_1\alpha) \\ -\mu_1(\cosh\mu_1\alpha + p\mu_1\sinh\mu_1\alpha) \\ -\mu_1(\sinh\mu_1\alpha + p\mu_1\cosh\mu_1\alpha) \end{bmatrix} = 0. \tag{40}$$

The continuity requirements imposed on the bending moment and the shear force, respectively, lead to the equations

$$\begin{bmatrix} A_1 \\ A_2 \\ B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix}^\top \times \begin{bmatrix} -h_0^3\lambda_0(\lambda_0\sin\lambda_0 a + \mu_0\sinh\mu_0 a) \\ -h_0^3(\lambda_0^2\cos\lambda_0 a + \mu_0^2\cosh\mu_0 a) \\ h_1^3\lambda_1^2\sin\lambda_1 a \\ h_1^3\lambda_1^2\cos\lambda_1 a \\ -h_1^3\mu_1^2\sinh\mu_1 a \\ -h_1^3\mu_1^2\cosh\mu_1 a \end{bmatrix} = 0 \tag{41}$$

and

$$\begin{bmatrix} A_1 \\ A_2 \\ B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix}^\top \times \begin{bmatrix} -h_0^3\lambda_0(\lambda_0^2\cos\lambda_0 a + \mu_0^2\cosh\mu_0 a) \\ h_0^3(\lambda_0^3\sin\lambda_0 a - \mu_0^3\sinh\mu_0 a) \\ h_1^3\lambda_1^3\cos\lambda_1 a \\ -h_1^3\lambda_1^3\sin\lambda_1 a \\ -h_1^3\mu_1^3\cosh\mu_1 a \\ -h_1^3\mu_1^3\sinh\mu_1 a \end{bmatrix} = 0. \tag{42}$$

The boundary conditions (7) can be expressed as

$$\begin{bmatrix} A_1 \\ A_2 \\ B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix}^\top \times \begin{bmatrix} 0 \\ 0 \\ -\lambda_1^2\sin\lambda_1 l \\ -\lambda_1^2\cos\lambda_1 l \\ \mu^2\sinh\mu_1 l \\ \mu^2\cosh\mu_1 l \end{bmatrix} = 0 \tag{43}$$

and

$$\begin{bmatrix} A_1 \\ A_2 \\ B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix}^\top \times \begin{bmatrix} 0 \\ 0 \\ -\lambda_1^3\sin\lambda_1 l \\ \lambda_1^3\cos\lambda_1 l \\ \mu^3\sinh\mu_1 l \\ \mu^3\cosh\mu_1 l \end{bmatrix} = 0. \tag{44}$$

The system (39)–(44) is a linear homogeneous system of algebraic equations. It has a non-trivial solution only in the case, if its determinant Δ equals to zero. The equation $\Delta = 0$ is solved up to the end numerically.

VII. Numerical results

Numerical results are obtained for the strip which is symmetrical with respect to the central cross section ($h_0 = h_2$, $l - a_2 = a_1$). The length of the strip is taken $2l$ and the origin of coordinates is shifted at the center of the plate strips. Thus l means the semi length of the strips.

The results of calculations are presented in Fig. 2–8. In calculations the plate with dimensions $l = 0.5$ m, $h_0 = 0.02$ m was considered. The material parameters are $\rho = 7860$ kg/m^3, $E = 2.1 \cdot 10^{11}$ N/m^2, $\nu = 0.3$.

In following the notation

$$\alpha = \frac{a}{l},\ \gamma = \frac{h_1}{h_0}$$

is used.

In Fig. 2 the frequency of natural vibrations is shown versus a where a is the location of the crack. Different curves in Fig. 2 correspond to different values of the crack depth.

The curves depicted in Fig. 2 are associated with the case when no tension is applied to the plate, e.g. $N = 0$. It can be seen from Fig. 2 that when the crack depth increases then the natural frequency decreases, as might be expected.

The natural frequency versus α and γ in the case of a plate of piece wise constant thickness is presented in Fig. 3, 4. Here $N = 0$ whereas Fig. 3 and 4 are associated with $\alpha = 0.5$ and $\gamma = 0.5$, respectively.

Similarly to the case of a plate of constant thickness it reveals from Fig. 3 and 4 that the natural frequency is maximal for the intact plate in comparison to that for a cracked sheet. It is interesting to note that when increasing the ratio of thickness in the case of fixed position of the step then the natural frequency ω increases until a certain value and in the course of subsequent increase of γ the quantity ω slowly decreases (Fig. 3). For instance, if $c = 0$, the point of maximum is achieved for $\gamma = 0.5$ and if $c = 0.6h_1$ then $\gamma = 0.7$.

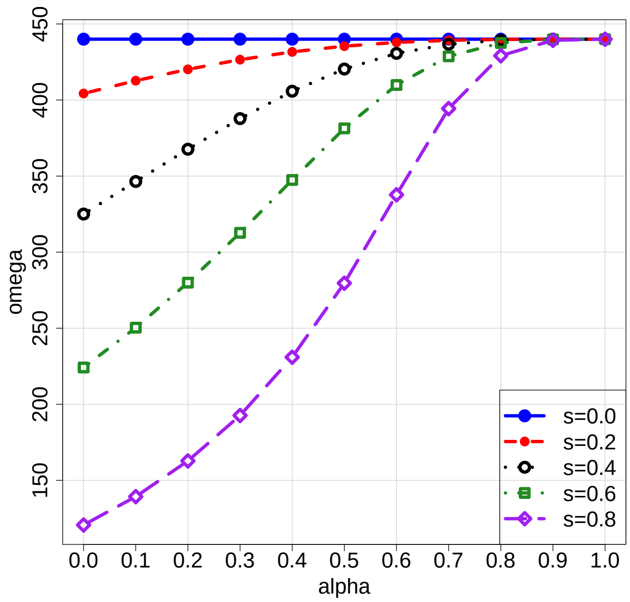


Fig. 2. Natural frequency vrs a/l, $N = 0$, $\gamma = 1$

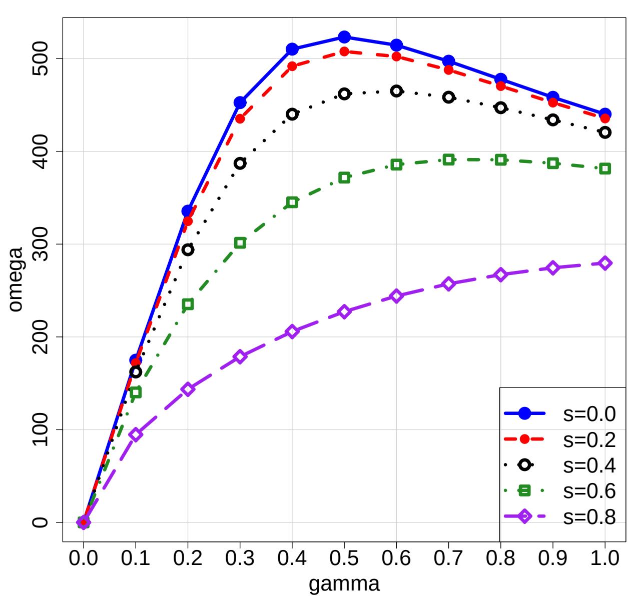


Fig. 3. Natural frequency vrs h_1/h_0, $N = 0$, $\alpha = 0.5$

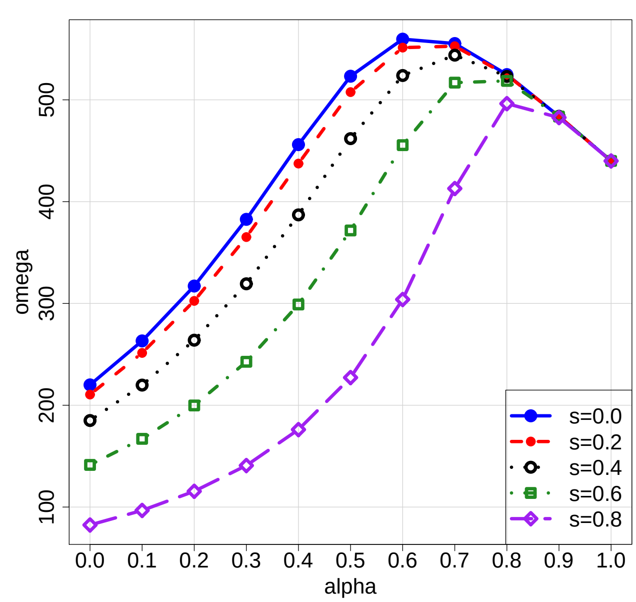


Fig. 4. Eigenfrequency vrs a/l, $N = 0$, $\gamma = 0.5$

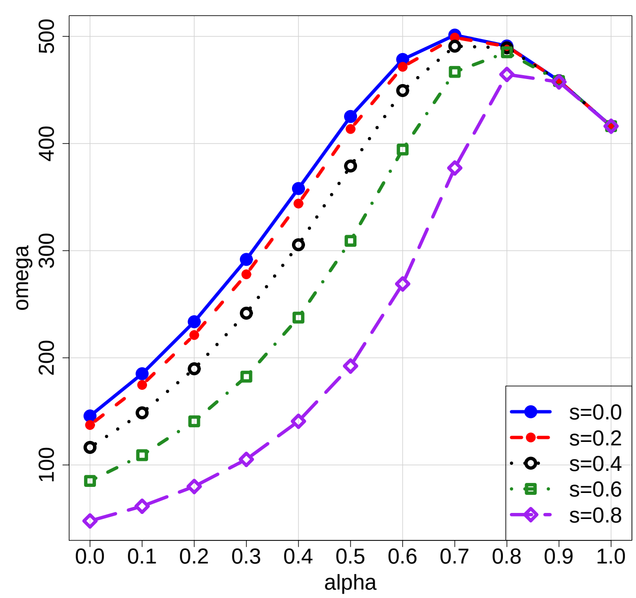


Fig. 5. Stretched strip, $N = 10^6$, $\gamma = 0.5$

If the ratio of thickness γ is fixed then there exists also a maximum of ω with respect to the step location α. If, for instance, $N = 0$, $\gamma = 0.5$ (Fig. 4) then the maximum points are $\alpha = 0.65$ (for uncracked plate, $s = 0$), $\alpha = 0.7$ (for $s = 0.4$) and $\alpha = 0.85$ (for $s = 0.9$).

Calculations carried out showed that the natural frequency depends quite weakly on the edge tension N if $N < N_*$ where N_* is a critical value of the edge tension. The dependence of the frequency ω on the ratio of thickness and on the crack length is depicted in Fig. 5. Here $N = 10^6$ and $h_1 = 0.5h_0$.

Comparing the results presented in Fig. 3 and Fig. 5 one can see that the corresponding curves are relatively close each other.

Similar results are presented in Fig. 6 and 7 for $N = 5 \cdot 10^6$. The curves depicted in Fig. 6 correspond to the case $a = 0.5l$ and these shown in Fig. 7 are associated with the fixed value of the ratio of thicknesses $h_1 = 0.5h_0$.

Comparing Figures 6 and 7 with Fig. 3 and 4, respectively,

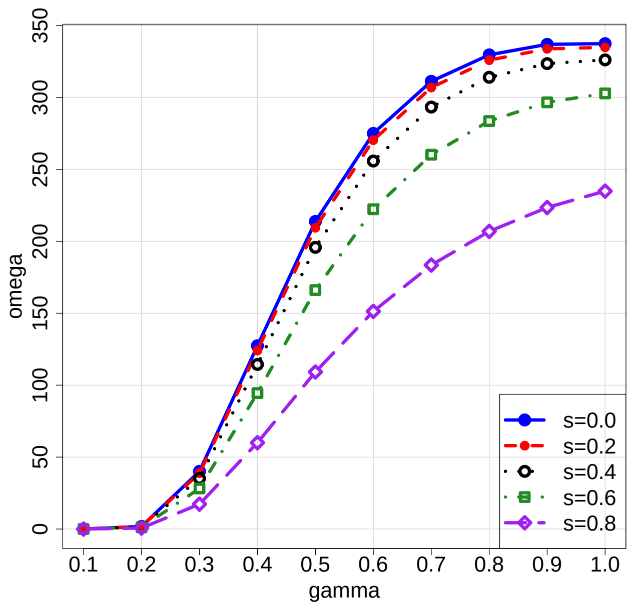


Fig. 6. Eigenfrequency vrs h_1/h_0, $N = 5 \cdot 10^6$, $\alpha = 0.5$

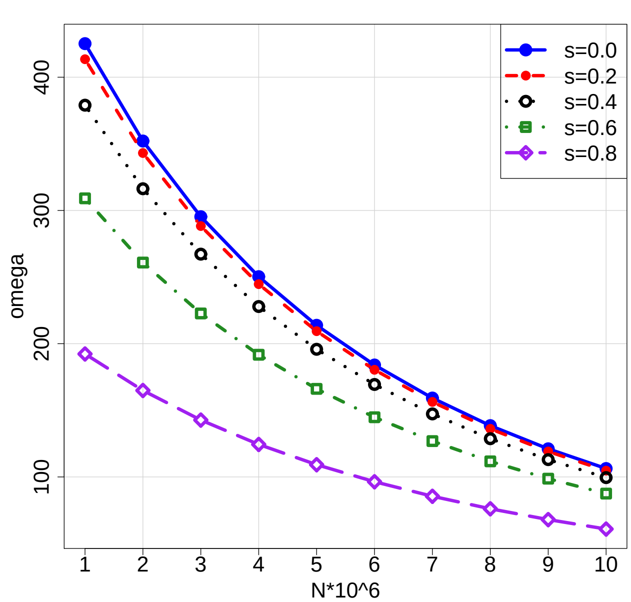


Fig. 8. Eigenfrequency vrs edge tension, $\alpha = 0.5$, $\gamma = 0.5$

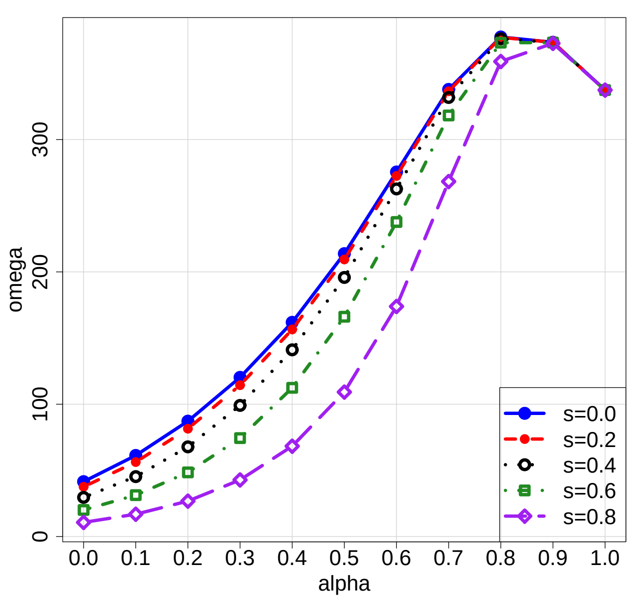


Fig. 7. Eigenfrequency vrs a/l, $N = 5 \cdot 10^6$, $\gamma = 0.5$

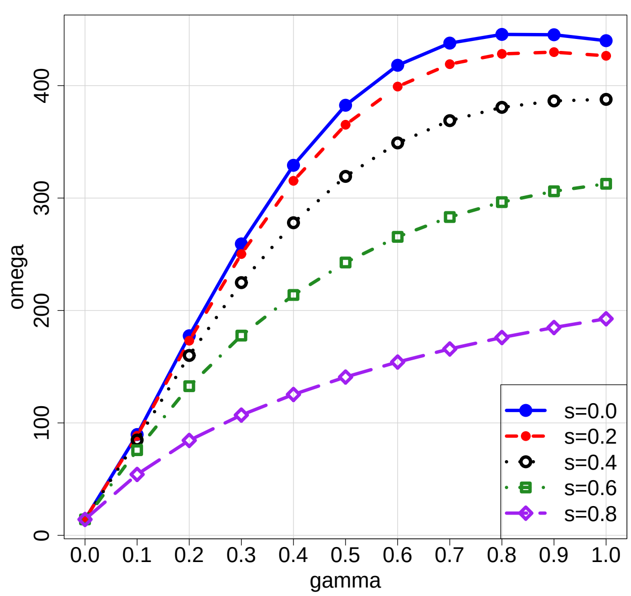


Fig. 9. Natural frequency vrs h_1/h_0, $\alpha = 0.3$, $N = 0$

one can see that in the case of larger edge tension values of the natural frequency are smaller than those corresponding to the plate without edge tension. Naturally, it is assumed herin that the strips with the same geometrical parameters and with the same crack parameters are compared. It can be seen from Fig. 6, 7 as well, that the increase of the crack length entails reduced values of the natural frequency.

The influence of the edge loading N on the natural frequency ω is presented in Fig. 8. Here $\alpha = 0.5$ and $\gamma = 0.5$ whereas the units in the horizontal axis are taken in millions.

It can be seen from Fig. 8 that in this scale the natural frequency ω does depend on the crack length and on the edge tension. The larger is the edge load the smaller is the natural frequency of the plate.

In Fig. 9 and Fig. 11 the natural frequency is shown as a function of the ratio of thickness h_1 and h_0.

In Fig. 9 $a/l = 0.3$ whereas in Fig. 11 $a/l = 0.7$. In both cases the tension $N = 0$.

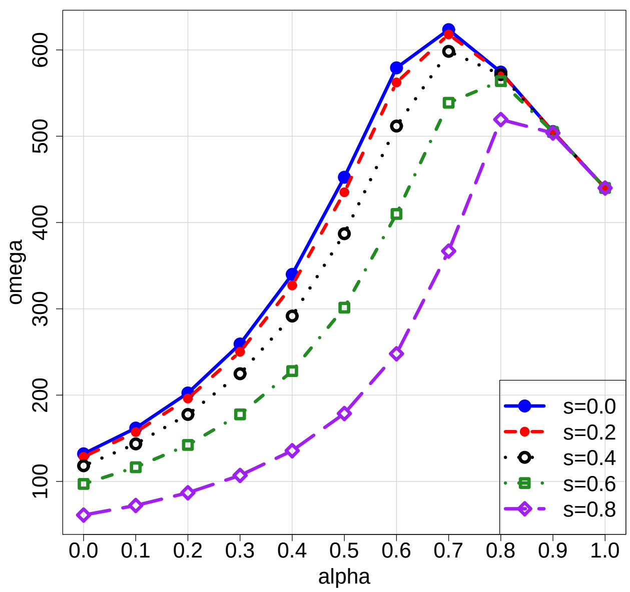


Fig. 10. Natural frequency vrs a/l, $\gamma = 0.3$, $N = 0$

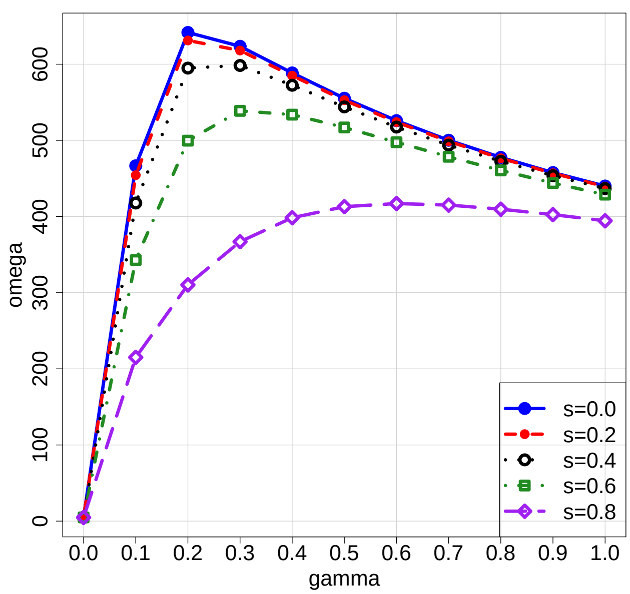


Fig. 11. Natural frequency vrs h_1/h_0, $\alpha = 0.7$, $N = 0$

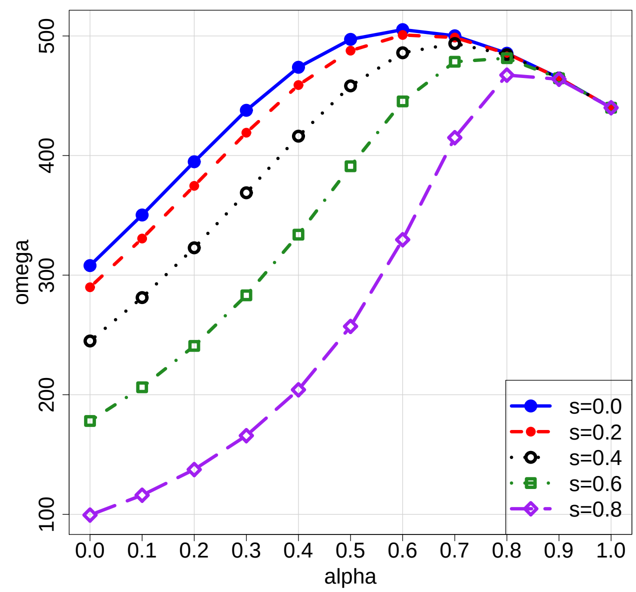


Fig. 12. Natural frequency vrs a/l, $\gamma = 0.7$, $N = 0$

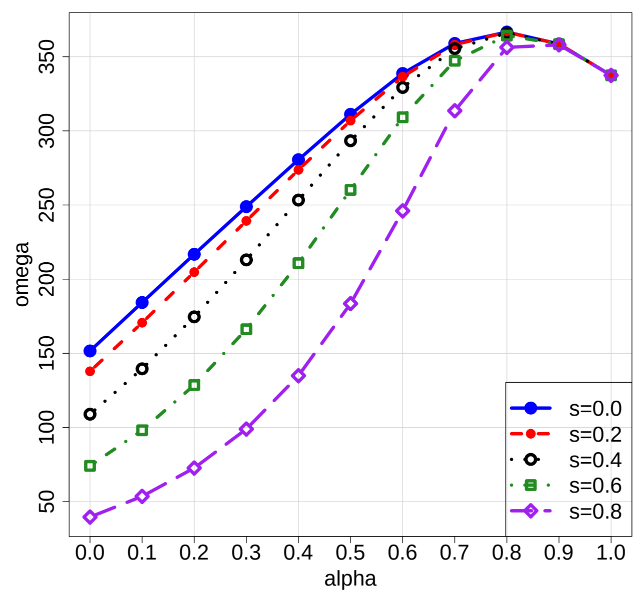


Fig. 13. Natural frequency vrs a/l, $\gamma = 0.7$, $N = 5 \cdot 10^6$

The natural frequency versus $\alpha = a/l$ is depicted in Fig. 10 and Fig. 12 in the cases $h_1/h_0 = 0.3$ and $h_1/h_0 = 0.7$, respectively. It can be seen from Fig. 9 – 12 that the highest curves correspond to plate strips without defects.

Comparing the results presented in Fig. 3, Fig. 9 and Fig. 11 one can see that when the step tends to the edge of the strips then the peaks of curves tend to the center of the strips. However, the peaks of corresponding curves in Fig. 4, Fig. 10, Fig. 12 are less sensitive with respect to the parameter $\gamma = h_1/h_0$. The relationship between the natural frequency and location of the step a/l is shown in Fig. 13 for the case $h_1/h_0 = 0.7$. Here the tension $N = 5 \cdot 10^6$.

VIII. Conluding Remarks

Free vibrations of elastic plate strips of piece wise constant thickness have been studied accounting for flaws and cracks. The cracks are considered as stationary surface cracks which are located at the corners of steps and which have not penetrated the plate thickness.

Combing the methods of the theory of elastic plates and of the linear elastic fracture mechanics an approximate technique for determination of natural frequencies of plate strips is developed. This technique admits to account for the tension applied at the edge of the plate weakened by part-through surface cracks occurring at the re-entrant corners of steps.

Calculations carried out showed that the crack location and the crack depth have an essential influence in the frequency of free vibrations. It was established numerically that when the crack extends then the natural frequency decreases, provided the other geometrical parameters remain unchanged. It is interesting to remark that the reduce of the frequency takes place independently of the ratio of thicknesses, of the location of the step and of other geometrical parameters.

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REFERENCES

[1] G. Irwin, “Fracture mechanics,” in *Structural Mechanics*. Oxford: Pergamon Press, 1960.

[2] J. Rice and N. Levy, “The part-through surface crack in an elastic plate,” *Journal of Applied Mechanics*, vol. 39, pp. 185–194, 1972.

[3] A. Dimarogonas and S. Paipetis, *Analytical methods in rotor dynamics*. London: Elsevier, 1983.

[4] A. D. Dimarogonas, “Vibration of cracked structures: A state of the art review,” *Engineering Fracture Mechanics*, vol. 55, no. 5, pp. 831–857, 1996.

[5] T. Chondros, A. Dimarogonas, and J. Yao, “A continuous cracked beam vibration theory,” *Journal of Sound and Vibration*, vol. 215, pp. 17–34, 1998.

[6] ——, “Vibration of a beam with a breathing crack,” *Journal of Sound and Vibration*, vol. 239, pp. 57–67, 2001.

[7] P. Rizos, N. Aspragathos, and A. Dimarogonas, “Identification of crack location and magnitude in a cantilever beam from the vibration modes,” *Journal of Sound and Vibration*, vol. 138, no. 3, pp. 381–388, 1990.

[8] S. Kukla, “Free vibrations and stability of stepped columns with cracks,” *Journal of Sound and Vibration*, vol. 319, no. 3-5, pp. 1301–1311, 2009.

[9] J. Lellep, E.Puman, L. Roots, and E.Tungel, “Optimization of rotationally symmetric shells,” in *Proc. of the 14th WSEAS Int. Conf. Applied Mathematics*. WSEAS Press, 2009, pp. 233–238.

[10] J. Lellep and L. Roots, “Vibration of stepped cylindrical shells with cracks,” in *3rd WSEAS Int. Conf. Eng. Mech., Structures, Eng. Geology (WORLDGEO’10)*. WSEAS Press, 2010, pp. 116–121.

[11] ——, “Vibrations of cylindrical shells with circumferential cracks,” *WSEAS Trans. Math.*, vol. 9, no. 9, pp. 689–699, 2010.

[12] H. Lin, S. Chang, and J. Wu, “Beam vibrations with an arbitrary number of cracks,” *Journal of Sound and Vibration*, vol. 258, no. 5, pp. 987–999, 2002.

[13] R. Y. Liang, F. K. Choy, and J. Hu, “Detection of cracks in beam structures using measurements of natural frequencies,” *Journal of the Franklin Institute*, vol. 328, no. 4, pp. 505–518, 1991.

[14] R. Y. Liang, J. Hu, and F. K. Choy, “Theoretical study of crack-induced eigenfrequency changes on beam structures,” *Journal of Engineering Mechanics*, vol. 118, no. 2, pp. 384–396, 1992.

[15] B. P. Nandwana and S. K. Maiti, “Detection of the location and size of a crack in stepped cantilever beams based on measurements of natural frequencies,” *Journal of Sound and Vibration*, vol. 203, no. 3, pp. 435–446, 1997.

[16] M. A. DeRosa, “Free vibration of stepped beams with flexible ends, in the presence of follower forces at the step,” *Journal of Sound and Vibration*, vol. 194, no. 4, pp. 631–635, 1996.

[17] S. Masoud, M. A. Jarrah, and M. Al-Maamory, “Effect of crack depth on the natural frequency of a prestressed fixedfixed beam,” *Journal of Sound and Vibration*, vol. 214, no. 2, pp. 201–212, 1998.

[18] J. Lellep and E. Kägo, “Vibrations of stepped plate strips with cracks,” in *Proc. of the Int. Conf. on Mathematical Models for Engineering Science*. WSEAS Press, 2010, pp. 244–249.

[19] J. N. Reddy, *Theory and Analysis of Elastic Plates and Shells*. Boca Raton: CRC Press, 2007.

[20] W. Soedel, *Vibrations of Shells and Plates*. New York: Marcel Dekker, 2004.

[21] T. Anderson, *Fracture Mechanics*. Boca Raton: CRC Press, 2005.

[22] K. Broberg, *Cracks and Fracture*. New York: Academic Press, 1999.

[23] H. Tada, P. C. Paris, and G. R. Irwin, *The Stress Analysis of Cracks Handbook*. New York: ASME Press, 2000.

[24] D. Broek, *The Practical Use of Fracture Mechanics*. Dordrecht: Kluwer, 1990.

[25] J. Lellep and E. Sakkov, “Buckling of stepped composite columns,” *Mechanics of Composite Materials*, vol. 42, no. 1, pp. 63–72, 2006.