Establishing a Correlative Model for Improving NC Machining process

Chang-Tzuoh Wu

***Abstract*—**Based on the machining conditions of the uses of flat end cutter (FC), ball-end cutter (BC) and disc cutter with a concave end (DCC) in the numerically controlled machining (NC) process, current studies aimed to establish related geometry model for efficiency evaluation and demonstrate qualitative and quantitative analysis by utilizing computer modeling. The compound surfaces are divided into three kinds of regions by the theory of differential geometry. The correlative mathematical models for defining the cutter parameters, the step-forward length, and the path intervals in the NC machining of the surfaces are described based on the feature of the regions. The analyzed parameters such as efficiency, accuracy and economical utility for different machined surfaces will be discussed to provide reliable selection principles of FC and DCC in order to improve NC machining.

***Keywords*—**Freeform Surface, NC Machining, Disc Cutter with Concave End, Geometry Model.

I. Introduction

WITH the sculptured surface being widely applied in the design of airplane, ship-hulls, car bodies and dies, the surfaces of the objects for NC machining are usually variable. Generally, these compound surfaces do not include the plane, cylinder surface, cone surface, sphere surface, and the spiral surface, since all these surfaces can be expressed by quadric analytical equations and can be machined by non-NC machining. Neither do the complex surfaces, which can be machined by copy machining technology. In current paper, NC machining would be applied on these compound surfaces to obtain optimal results.

Bedi (1997) developed a method with flat end cutter to improve material removing rate while the direction of machining process was parallel to that of principle curvature. In addition to, scallop height (SH) was suggested to be smaller while using toroidal cutter compared to flat end cutter. Vickers (1989) discussed the differences of curved surface machining between ball-mills and end-mills. Li (2004, 2006) discussed the problems of cutter selection and stated that the cutter selection is closely related to the tool's orientation, tool path topology and tool path parameters. Most of the current methods avoid modeling of this problem and assume that the tool has been selected before tool path generation. Chang et. al. (1998) proposed that the revolving surface which forms the strips of envelope would degenerate into a circle- the trace circle of the tool noses of a disc cutter. Therefore, the uses of DCC may enhance the degree of closeness between the strip envelopes formed by the cutting edges of a tool in every pass and the required surfaces. Since these studies were aim on the concave surfaces, current studies were aimed to further explore mathematically the utilization of two cutters on the other types of surface.

For the purpose of improving the machining efficiency and accuracy, the selection of NC machine tool related to the surface feature should be considered. The region division of the surfaces to be machined is first described. The calculating process of principal curvature of the surfaces is then briefly discussed. Based on the surface feature, the parameters of the cutters are determined. The models used to calculate step-forward length and path interval are introduced according to the minimal radius of normal curvature and the maximal admissible machining error. The transmission of the maximal error caused by the step-forward linearization to the scallop height between the two adjacent toolpaths is also considered. The NC machining of compound surfaces with different types of regions are also investigated.

The geometrical parameters of this surface are easily to be controlled by changing the tilting angle of the tool axes for both FC and DCC. Furthermore, the contact-order for such given contact point of two surfaces would be secondary. In the other words, the surfaces would be in the same curve curvature or their derivatives under the circumstances of concave to the same direction. The step-forward length and path interval are the direct indication for the machining efficiency and determined by contact point and SH. Therefore, the mathematical analysis of those parameters required for providing correct evaluation will be reported in this paper.

II. Analysis of Compound Surface

According to the theory of differential geometry [2-4,7], it is known that the points on the surface may be classified by Gauss curvature into three types: positive, minus or zero. For the smooth continuous surface, if the Gauss curvature $K > 0$ exists for a certain point, then there must be a region around the point that $K < 0$ exists for a certain point, then there must be a region around the point that $K < 0$ exist for all the points in the region. Such region is called hyperbolic region. If the Gauss curvature $K = 0$ for all the points in a certain region, such region is called parabolic region. A smooth continuous surface consists of three types of regions at most.

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A. The main curvature analysis of the surface

The Gauss curvature may be obtained by the product of k_1 and k_2, which is maximal main curvature and minimal main curvature respectively. Therefore, the coefficients of the first and secondary fundamental form of the surface should be calculated first, then the main curvatures may be obtained by solving the equation of main curvature.

Let the equation of the surface be:

$$r(u,v) = \{x(u,v), y(u,v), z(u,v)\} \tag{1}$$

The coefficients of the first and second fundamental form of the surface may be determined by the following equation

$$\begin{cases} E = \mathbf{r}_u^{\,2} = x_u^{\,2} + y_u^{\,2} + z_u^{\,2} \\ F = \mathbf{r}_u \mathbf{r}_v = x_u x_v + y_u y_v + z_u z_v \\ G = \mathbf{r}_v^{\,2} = x_v^{\,2} + y_v^{\,2} + z_v^{\,2} \end{cases}$$

$$D = \sqrt{EG - F^2} \tag{2}$$

$$\begin{cases} L = (\mathbf{r}_u, \mathbf{r}_v, \mathbf{r}_{uu}) / D \\ M = (\mathbf{r}_u, \mathbf{r}_v, \mathbf{r}_{uv}) / D \\ N = (\mathbf{r}_u, \mathbf{r}_v, \mathbf{r}_{vv}) / D \end{cases}$$

Substituting equation (2) into the equation of main curvature

$$\begin{vmatrix} k_n E - L & k_n F - M \\ k_n F - M & k_n G - N \end{vmatrix} = 0 \tag{3}$$

The two main curvature k_1 and k_2 may be obtained easily form equation (3).

B. Region division of surface

For the points in the elliptical region, it is known from K = $k_1 \cdot k_2 > 0$ that both the two main curvatures are positive or minus, that is to say the surface bends to the same side of the tangent plane at the point. As for the points in the hyperbolic region, since the two main curvatures have different sign, according to the continuity, it is known that there must be two asymptotic directions for $k_n = 0$. For the whole hyperbolic region, there must be two groups of asymptotic curves, and the surface is divided into two groups of angle regions. The sub-surfaces in the adjacent angle regions bend toward the different side of the tangent plane at the point, but the sub-surfaces in the opposite angle regions bend toward the same side. In the parabolic region, since Gauss curvature $K = 0$, there must be a main curvature equal to zero. Assume that $k_1 = 0$, then the main curvature direction corresponding to $k_1 = 0$ is an asymptotic direction. The surface will be developable from the feature $K = 0$. Besides, from the Euler eqation:

$$k_n = k_1 \cos^2 \phi + k_2 \sin^2 \phi \tag{4}$$

It is known that all the points in the parabolic region have the same sign, and the surface bends towards the same side of the tangent plane at the point. From the features of different regions, it is obvious that the features must be considered in the NC machining of compound surface so that a good result may be obtained.

III. Cutting Contact Analysis

At present, ball-end milling cutter is most often used tool in the surface numerical control processing. It is because that the normal vector of any point on the ball-nose surface passes through the center, i.e., tool center is on the isometric surface of machined surface so as to calculate the cutter position easily. Besides, gouge will be avoided while the radius of ball-nose is small than the minimal radius of normal curvature. Numerical control programming is being simple, interference inspection easy, and only needs the three-axis NC machine are the advantages of using ball-end cutter so as to be widely used in NC machining. However, ball-end cutter still had many shortcomings, such as,

1. The contact condition of cutter and machined surface is point contact. In addition, the cutting velocity changes according to the different cutter contact position. It should be noted that the cutting velocity at tip of ball-nose is approaching zero. The bad cutting condition will induce accuracy.
2. The tool wear is non-uniform, moreover after wearing, repairs with difficulty.
3. The cutting path interval should be small to conform to the required scallop height. Therefore, the cutting efficiency will be reduced.

As compared to ball-end cutter, flat-end cutter has the advantages of convenient adjustments, the long service life, high cutting efficiency and good processing quality. Especially for 4-axis and 5-axis NC machine, the effective cutting radius to close to the machined surface can be obtained by tilting or yawing an angle of cutting tool. In recent years what displaced was toroidal cutter which not only has the advantages of using flat-end cutter but also reducing the wear because of the torus end to make it be the more attractive selection.

It should be noted that optimal selection of cutter should be made by considering not only the accuracy of machined surface but also the NC machine types and the region types of surfaces. The basic rules of tool selection are suitable size of cutter for gouge-avoidance, reducing frequency of changing tool and the evaluation of accuracy.

A. Contact performances between cutter and surface

The current tendency of research is to explore the methods of increasing the efficiency of machining the compound surface. The feasible direction has specifically,

1. Increase feeding rate.
2. Optimize tool-path planning.
3. Let the envelope of cutting tool approaches more closely to the design surface.

Contact order of the cutter and machined surface can be used to discuss the relationships between the envelope of cutting tool and the design surface by utilizing differential geometry. Based on the region division of surface, the evaluation of selection of cutter and corresponding parameters can be determined by quantitative analysis of scallop height caused by linear interpolation.

In order to discuss the contact degree between two surfaces, the definition of contact order developed by Zhang[16] is introduced as follows, Definition: For the contact point of two surfaces, there are series of common normal planes through this point. The normal intersecting curves obtained by common normal plane intersecting with any surfaces have corresponding contact orders. The contact order is defined as the lowest contact order.

Based on this definition, the contact order of two intersecting curves, obtained by any inclined plane intersecting with a pair of contact surfaces through the contact point, will not lower than the contact order of surface at this point. Therefore, we can inference that the intersecting curve L_1 obtained by a plane which normal to the feeding direction intersecting with cutter trace envelop and the intersecting curve L_2 obtained by a plane which normal to the feeding direction intersecting with design surface have the same k order derivative.

Ball-end cutter, the frequently used cutter, obviously has the first contact order with machined surface so that the envelope of cutting trace cannot more closely to the machined surface by inclined the cutter. However, for flat-end cutter and toroidal cutter used in 4-axis and 5-axis NC machine, the envelop of cutter trace will be more closed to machined surface by tilting the axis of cutter toward the feeding direction to reduce the scallop height as shown in Fig.1. As discussed by Zhang(1998), by using a disk cutter with a concave end, and by adjusting the cutter to move relatively to machine bed under some special rules, an entirely new concept of machining sculptured surfaces has be proposed. The intersecting lines on the plane normal to the feeding direction, which are formed by both the required surface and the strip envelope, are ensured to have the same curve curvatures and their derivatives. Thus, the envelope formed by the trace of the tool-nose in each pass and the required surface have the same derivatives up to the second order in the plane normal to the feeding direction.

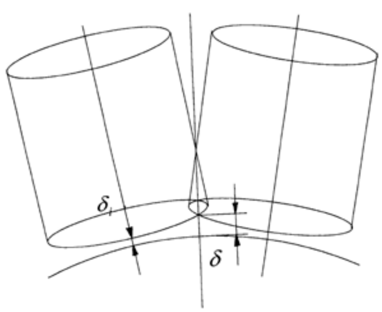


Fig.1 The features of NC machining using FC with tilting angle

B. Error Transmission and Scallop Height

Since a NC machining for compound surface consists of several linearly interpolated movements, care must be taken in setting step-forward distance. The error caused by the step-forward linearization movement needs to be controlled. There are two types of tolerance: inner and outer tolerance. The selection of tolerance type depends upon the surface demanding and error distribution. Since current studies focus on the outer tolerance, the chord (tangent) method would be used for approaching origin curve in concave (convex) region, respectively. Those errors would be transmitted into scallop heights introduced by two adjacent tool paths as shown in Fig.2. Therefore, the error transmission should be taken into account for step-forward distance and path interval determination.

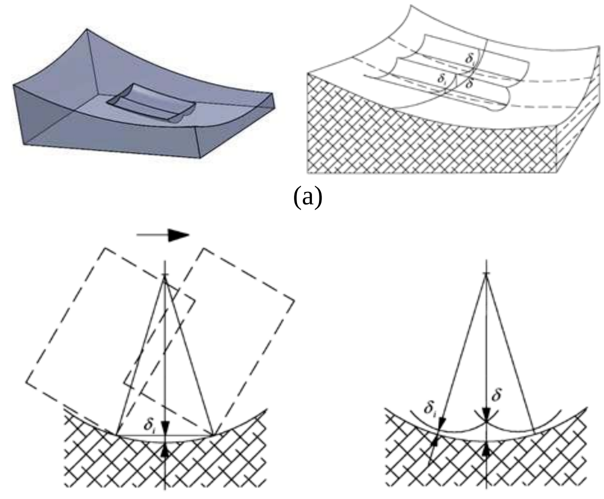


Fig.2 (a) Step-forward linearization and adjacent toolpath, (b)Error caused by the step-forward linearization, (c) Error δ_1 transmitted into scallop height

IV. Machining Properties of Cutters

Since different cutters providing exclusively characteristics, NC machining scheme and models for each cutter would be different. Therefore, it is necessary to discuss each cutter individually. According to above analysis of the surface, the evaluation model of cutters involved in three different surface regions will be discussed as follows.

A. Elliptical region

For the points in the elliptical region, it is known from Gauss curvature $K > 0$ that the surface bends to the same side of the tangent plane at the point. Therefore, there are only two types of surface exist. One is concave surface and another one is convex. The features of NC machining discussed in Reference 1, 2 were involved in the points on the elliptical region on the concave surface. The mathematical analysis will be discussed later.

Flat-end cutter

In 5-axis milling, the flat-end cutter can be tipped at an angle so that the machined surface approaches closely to the design surface. The calculation procedures for such effective radius were described by Vickers[14] and Li[10]. When the flat-end cutter tilts an angle β along the feeding direction of cutter, the projection of the cutter bottom face to the plane perpendicular to the feeding direction will be an ellipse (as shown in Fig.3). It is obvious that the main radius of the ellipse is the radius of flat-end cutter R, and the minor radius is $R\sin\beta$. If an arc, which passes through the cutter contact point and the two end points of the main radius, is used to replace the lower half ellipse, the radius of the arc is called the effective radius R^*, and satisfies

$$R^* = \frac{1+\sin^2\beta}{2\sin\beta}R \tag{5}$$

In this way, the machining effect of a flat-end cutter with a tilting angle is similar to that of a ball-end cutter. Therefore, when calculating the path interval of flat-end cutter, the cutter may be treated as a ball-end cutter with an effective radius of R^*.

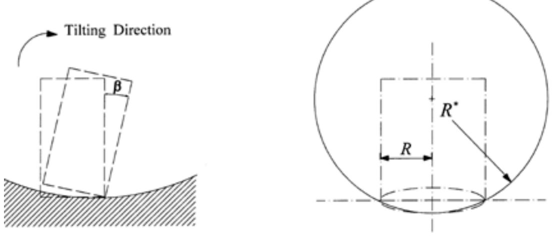
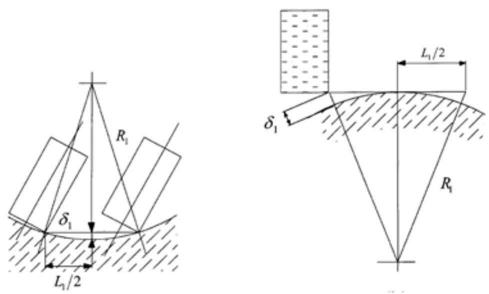


Fig.3 Tilting angle and effective tool radius



(a) Concave elliptical region (b) Convex elliptical region

Fig.4 The step-forward of flat-end cutter

For the points in the elliptical region, bending to the same side of the tangent plane is their common characteristic. In order to construct the model, first let

$$\begin{cases} R_1 = 1/\max\{|k_1|\} \\ R_2 = 1/\max\{|k_2|\} \end{cases} \tag{6}$$

where R_1 and R_2 is the minimal principal curvature radius for any point on the surface along two principal curvature direction respectively. This is to ensure that the models along the direction of step-forward and the directions of path intervals are reliable.

Here are two cases, one is concave elliptical region as shown in Fig.4(a), and the other is convex elliptical region as shown in Fig.4(b). As for the case in Fig.4, while a flat-end cutter is used to machine the concave elliptical region, the outer radius of flat-end cutter should satisfy the following relation

$$R < \min\left\{\sqrt{R_i^2 - (R_i - \delta_1)^2}\right\} \qquad (i = 1, 2) \tag{7}$$

The angle between the bottom face of the cutter and the tangent line of the normal section curve along the step-forward direction must satisfy

$$\alpha > \max\left\{\sin^{-1}\frac{R}{R_i}\right\} \qquad (i = 1, 2) \tag{8}$$

so that the interference may be avoided. In order to assure the machining quality, the cutter should tilt a certain angle

$$\beta = \max\left\{\sin^{-1}\left(\frac{R}{R_i}\right)\right\} + \gamma_0 \qquad (i = 1, 2) \tag{9}$$

where γ_0 should be chosen about $2 \sim 3$ degree according to the experience.

As for the case in Fig.4(b), the radius of flat-end cutter need not be constrained by the above condition. But, for both cases, the determination of radius of cutter should consider the standard size of the cutter and the scallop height at the edges of machining region. That is to say, the actual radius of the cutter cannot be too large.

If the maximal admissible machining error in Fig.2(c) is δ, notice that the error δ_1 caused by the linearization of step-forward will be transmitted to the scallop height between the two adjacent tool paths, so the error of linearization of the step-forward should be

$$\delta_1 < \delta \tag{10}$$

From Fig.4(a), the step-forward length should be

$$L_1 < 2\sqrt{R_1^2 - (R_1 - \delta_1)^2} \tag{11}$$

In defining the path interval (see Fig.5(a)), let the normal curvature radius of the surface along the direction of path intervals be R_2, considering the transmission of error in the step-forward, the actual path interval L_2 should be less than the path interval $L_2^{'}$ determined by the following equation.

$$R_2 - \left(\frac{R_2 - R^* - \delta_1}{2R_2}\right)\sqrt{4R_2^{\,2} - L_2^{'\,2}} - \sqrt{R^{*2} - \frac{(R_2 - R^* - \delta_1)^2}{4R_2^{\,2}}L_2^{'\,2}} - \delta = 0 \tag{12}$$

As for the convex elliptical region in Fig. 4(b), the cutter radius need not be constrained by the curvature of the surface, and let $\alpha = 0$, the step-forward must satisfy the following relation:

$$L_1 < 2\sqrt{(R_1 + \delta_1)^2 - R_1^{\,2}} \tag{13}$$

Referring to Fig.5(b), the actual path interval L_2 should be determined by the following condition,

$$L_2^{'} = 2\left(\frac{R_2}{R_2 + \delta}\right)\sqrt{(R_2 + \delta)^2 - (R_2 + \delta_1)^2} \tag{14}$$

The determination of the tilting angle of flat-end cutter according to R_1 and R_2 particularly for a concave elliptical region surface is critical. Therefore, a 5-axis NC machining tool is required when a flat-end cutter is used to machine an elliptical surface.

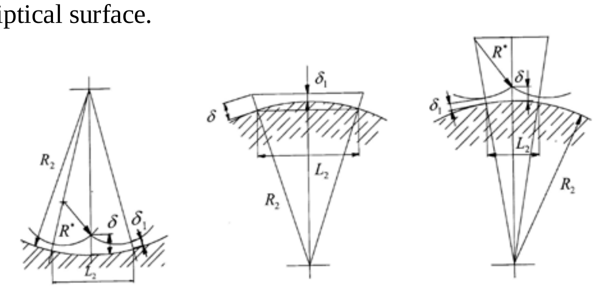
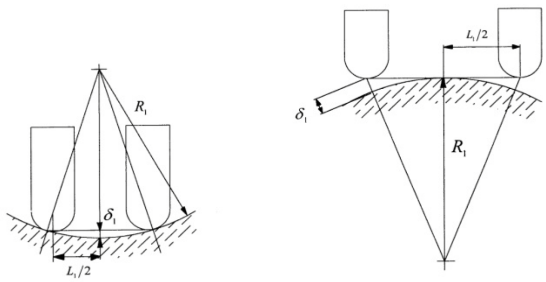


Fig.5 The path interval of flat-end cutter, (a) and (c) the cutter tilts an angle, (b) the axis of cutter aligned with surface normal

Ball-end cutter



(a) Concave (b) Convex

Fig.6 The step-forward of ball-end cutter

As for the case in Fig. 6(a), when a ball-end cutter is used to machine concave elliptical region, the radius of ball-end cutter should satisfy the following condition:

$$R < \min\{R_1, R_2\} \tag{15}$$

otherwise, cutting interference will happen and the machining result cannot be ideal. R_1 and R_2 is the minimal principal curvature radius for any point on the surface along two principal curvature directions respectively.

Let the curvature radius of the surface along the direction of step-forward be R_1, then the step length should be:

$$L_1 < 2\sqrt{(R_1 - R)^2 - (R_1 - R - \delta_1)^2} \tag{16}$$

Referring to Fig.6(a), the actual path interval L_2 should be less than the path interval $L_2^{'}$ determined by Eq.(12) with R^* being replaced by R.

As for the case in Fig.6(b), when a ball-end cutter is used to machine convex elliptical region, the radius of ball-end cutter need not be constrained by the above condition. The step length should be satisfied the following relation:

$$L_1 < 2\sqrt{(R_1 + \delta_1)^2 - R_1^{\,2}} \tag{17}$$

Referring to Fig.5(c), the actual path interval L_2 should be less than the path interval $L_2^{'}$ determined by Eq.(18) with R^* being replaced by R.

$$\left(\frac{R_2 + R^* + \delta_1}{2R_2}\right)\sqrt{4R_2^{\,2} - L_2^{'\,2}} - \sqrt{R^{*2} - \left(\frac{R_2 + R^* + \delta_1}{2R_2}\right)^2 L_2^{'\,2}} - R_2 - \delta = 0 \tag{18}$$

Disc Cutter with Concave End

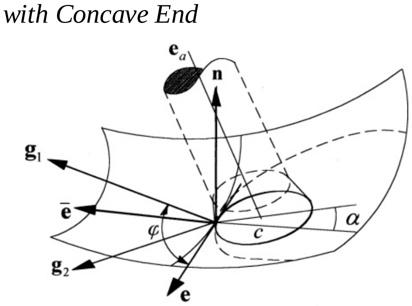
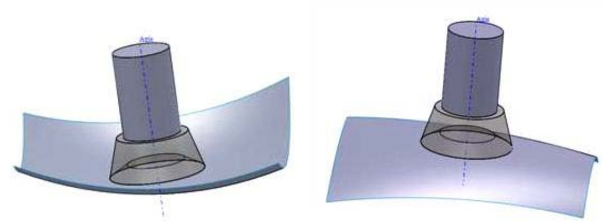


Fig.7 Curvature catering of DCC



(a) (b)

Fig.8 The features of NC machining on the convex and concave elliptical region using DCC

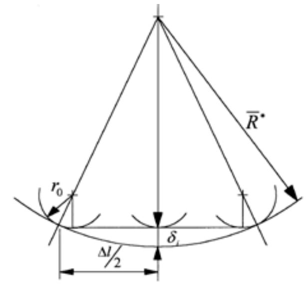


Fig.9 Error caused by linear interpolation for DTC

The features of machining a concave surface and a convex surface by using DCC are illustrated as Fig.7 and Fig.8. Thus, the scallop heights of using DTC or FC were still higher than that resulting from DCC machining even if the tilting angle was adjusted. If taking maximal admissible error into account, the step-forward length and path interval would be smaller while using DTC or FC. So forth, the machining efficiencies were lower than using DCC. Obviously, it is more appropriate to utilize DCC, compared to FC and TDC, for the elliptical surface machining.

The features of NC machining on the convex elliptical region, application of FC results in smooth surface. However, there was higher SH remained on the surface while using disc-flat end cutter (DFC) and it is totally improper to use DCC. The SHs produced by DTC or DCC were still higher than that resulting from TC machining even if the tilting angle was adjusted. If taking maximal admissible error into account, the step-forward length and path interval would be smaller while using DTC or DCC. So forth, the machining efficiencies were lower than using FC. Obviously, it is more appropriate to utilize FC, compared to DCC and TDC, for the convex elliptical surface machining.

Concave Elliptical Region

As for the case in Fig. 8(a), when a DCC is used to machine concave elliptical region, the radius of DCC should satisfy the following condition for gouge-avoidance.

$$\bar{R} < \frac{1}{|k_n|} \tag{19}$$

where k_n is the normal curvature.

The radius of DCC should be under such limitation to prevent gouge. For the purpose of having the 2-order curvature catering, cutter should inclined an angle α,

$$\alpha = \sin^{-1}(|k_n|\bar{R}) \tag{20}$$

Referring to Fig. 9, the error δ_1 caused by the linearization of step-forward will be transmitted to the scallop height between the two adjacent tool paths, so the error of linearization of the step-forward should be

$$\delta_i = \bar{R}^* - \frac{(\bar{R}-r_0)\sqrt{4\bar{R}^{*2}-\Delta l^2}}{2\bar{R}^*} - r_0 \tag{21}$$

While considering the milling width problem, i.e., path interval. Within the coordinate shown in Fig. 7., DCC tilts an angle α around the $\mathbf{e}$ axis and interpolates a distance λ toward $\bar{\mathbf{e}}$ direction. Thus, the surface groups will be

$$\bar{\mathbf{r}} = \begin{Bmatrix} (\bar{R}+r_0\cos\theta_1)\cos\varphi_1\cos\alpha - r_0(1+\sin\theta_1)\sin\alpha + \lambda \\ (\bar{R}+r_0\cos\theta_1)\sin\varphi_1 \\ (\bar{R}+r_0\cos\theta_1)\cos\varphi_1\sin\alpha + r_0(1+\sin\theta_1)\cos\alpha \end{Bmatrix} = \{\bar{x},\bar{y},\bar{z}\} \tag{22}$$

Considering of the enveloping condition $(\bar{\mathbf{r}}_{\theta_1},\bar{\mathbf{r}}_{\varphi_1},\bar{\mathbf{r}}_{\lambda})=0$, we can obtain that

$$\varphi_1 = \cos^{-1}(\tan\theta_1\tan\alpha) \tag{23}$$

For the path interval direction, we substitute Eq.(23) into Eq.(22) and let component in $\bar{\mathbf{e}}$ direction be 0. Thus, intersecting curve $\tilde{C}$ can be obtained. Based on the transmission of δ_i as shown in Fig.10, the equation of circle with radius $R-\delta$ at center Q can be expressed as,

$$r_{Q_1} = \{0\,,\ (R-\delta)\cos\zeta_1\,,\ R-(\bar{R}\sin\alpha+r_0)-\delta_i+(R-\delta)\sin\zeta_1\} \tag{24}$$

By solving Eq.(24) simultaneous with Eq.(22) and Eq.(23) the coordinate of point P can be obtained as,

$$\mathbf{r}_P = \{0\,,\pm\bar{y}_0\,,\bar{z}_0\} \tag{25}$$

Thus, the path interval l_1 should be satisfied

$$l_1 \le \frac{2\bar{y}_0 R}{R-\delta} \tag{26}$$

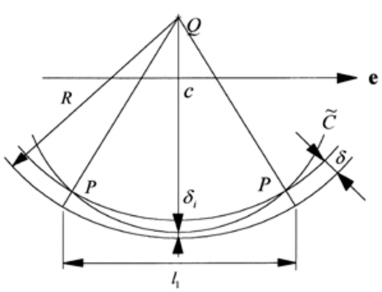
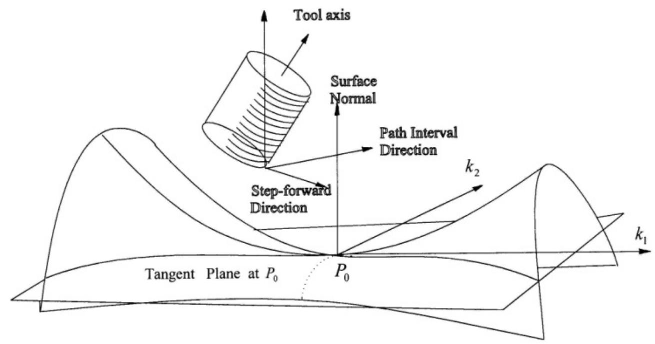


Fig.10 Path interval analysis using envelope

Convex Elliptical Region

For the convex elliptical region, the analysis is relatively simple. It is needless to use DCC, flat-end cutter can be used instead. The analytical results of using DCC can be referred to that of flat-end cutter discussed above.



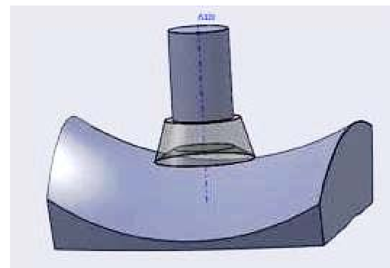


Fig.11 NC machining in hyperbolic region (Saddle like)

B. Hyperbolic region

As for the points in the hyperbolic region (Fig.11), there are two asymptotic directions for $k_n = 0$ and the surface is divided into two groups of angle regions. The sub-surfaces in the adjacent angle regions bend toward the different side of tangent plane at the point. The intersecting line of the points on hyperbolic region is either concave or convex which depends upon the directions. The tension angle of concave or convex field is not necessarily the same.

If the tension angle of convex and concave field on the hyperbolic region were equivalent while NC machining, there would be two different features: a. Directions of step-forwarding linearization is within convex field and path interval is within concave field. b. Direction of step-forwarding linearization is within concave field and path interval is within convex field. Under these two situations, mathematical analysis is required for cutter selection. While the tension angle is unequivalent between convex and concave field, there would be the other two situations existing. Direction of step-forwarding linearization and path interval is within convex field. However, that of the other angle region is within concave field. Therefore, it is necessary to tilt along path interval to avoid interference with concave field instead of using FC for machining. The reason is due to the overlapping of axis by FC on machined intersecting line. The selection of cutter would depend upon the mathematical analysis.

Flat-end cutter

For the points in the hyperbolic region, two principal curvatures have different sign. Assume that $k_1 > 0$, $k_2 < 0$ and let

$$\begin{cases} R_1 = 1/\max\{k_1\} \\ R_2 = -1/\min\{k_2\} \end{cases} \tag{27}$$

Two situations will be discussed respectively in the followings.

Concave-Convexity Case

When the direction of step-forward linearization is toward concave field, theoretically a tilting angle α of flat-end cutter axis is used to avoid interference between tool and machined surface. Tool radius and angle α are restricted as Eq.(7) and Eq.(8). In practical machining, a tilting angle is chosen by Eq.(9) to ensure proper cutting.

If the step-forward moves along the direction of R_1, the length of step should satisfy Eq.(11). When the path interval is being determined, the transmission of error caused by the step-forward linearization to the scallop height should be taken into account. Referring to Fig.5(c), the actual path interval L_2 should be less than the path interval L'_2 determined by the following equation.

$$\left(\frac{R_2+R^*+\delta_1}{2R_2}\right)\sqrt{4R_2^{\,2}-L_2^{'\,2}} - \sqrt{R^{*2}-\left(\frac{R_2+R^*+\delta_1}{2R_2}\right)^2 L_2^{'\,2}} - R_2 - \delta = 0 \tag{28}$$

Convex-Concavity Case

When linear interpolation is applied on movement in convex region, it seems that a flat-end cutter, with its axis aligned with the normal to the surface, can be directly used for machining. In fact, since the direction of path interval is toward concave field, a tilting angle is important to decrease interference with machined surface in concave region. Tilting angle must satisfy Eq.(9) with $R_i = R_1$

If the step-forward moves along the direction of R_2, the length of step should satisfy

$$L_1 < 2\sqrt{(R_2+\delta_1)^2-(R_2)^2} \tag{29}$$

Referring to Fig.5(a), the actual path interval L_2 should be less than the path interval L'_2 determined by the following equation.

$$R_1-\left(\frac{R_1-R^*-\delta_1}{2R_1}\right)\sqrt{4R_1^{\,2}-L_2^{'\,2}} - \sqrt{R^{*2}-\frac{(R_1-R^*-\delta_1)^2}{4R_1^{\,2}}L_2^{'\,2}} - \delta = 0 \tag{30}$$

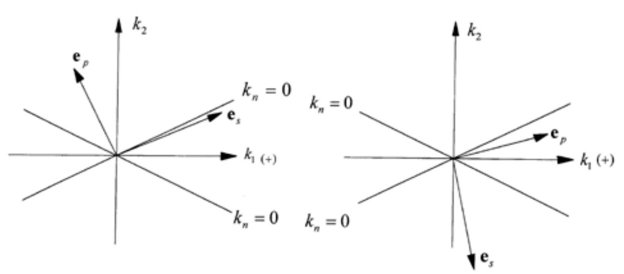


Fig.12 (a) Step-forward direction is toward concave field and path interval direction is toward convex field
(b) Step-forward direction is toward convex field and path interval direction is toward concave field

Ball-end cutter

The definition of R_1 and R_2 are the same as Eq.(27). The radius of ball-end cutter should be smaller than the minimal radius of curvature, i.e.

$$R < R_1 \tag{31}$$

Concave-Convexity Case

If the step-forward moves along the direction of R_1, the length of step should satisfy Eq.(16). Referring to Fig.5(c), the actual path interval L_2 should be less than the path interval L'_2 determined by Eq.(28) with R^* being replaced by R.

Convex-Concavity Case

If the step-forward moves along the direction of R_2, the length of step should satisfy relation(29). Referring to Fig.5(a), the actual path interval L_2 should be less than the path interval L'_2 determined by Eq.(30) with R^* being replaced by R.

Disc Cutter with Concave End

Relies on the preceding way, we discuss the relative problems about machining error and path interval caused by linear interpolation.

Concave-Convexity Case

When linear interpolation is applied on movement in convex region, it seems that a DCC, with its axis aligned with the normal to the surface, can be directly used for machining. In fact, since the direction of path interval is toward concave field, a tilting angle α is important to decrease interference with machined surface in concave region. It should be noted that the tilting angle α is not used for 2-order curvature catering but for gouge-avoidance. As shown in Fig.13, α must be larger than the solution of following equation.

$$\frac{\overline{R}\overline{R}^*}{\overline{R}^*-r_0} - \frac{\Delta l}{2\cos\alpha} - \sqrt{\overline{R}^{*2}-\left(\frac{\overline{R}\overline{R}^*}{\overline{R}^*-r_0}\right)^2}\tan\alpha = 0 \tag{32}$$

Referring to Fig. 9, the error δ_1 caused by the linearization of step-forward will be transmitted to the scallop height between the two adjacent tool paths, so the error of linearization of the step-forward should be the same as Eq.21.

Relies on the preceding way, the surface groups and corresponding enveloping condition should be the same as Eq.22 and Eq.23. Based on the transmission of δ_i as shown in Fig.13, the equations of circle with radius R and $R+\delta$ at center Q can be expressed as following equations, respectively.

$$r_Q = \{0,\ R\cos\zeta_2,\ -R-(\bar{R}\sin\alpha+r_0)-\delta_i+R\sin\zeta_2\} \tag{33}$$

$$r_{Q_1} = \{0,(R+\delta)\cos\zeta_3,-R-(\bar{R}\sin\alpha+r_0)-\delta_i+(R+\delta)\sin\zeta_3\} \tag{34}$$

By solving Eq.(34) simultaneous with Eq.(22) and Eq.(23) the coordinate of point P can be obtained as,

$$\mathbf{r}_P = \{0,\pm\bar{y}_{01},\bar{z}_{01}\} \tag{35}$$

Thus, the path interval l_1 should be satisfied

$$l_1 \le \frac{2\bar{y}_{01}R}{R+\delta} \tag{36}$$

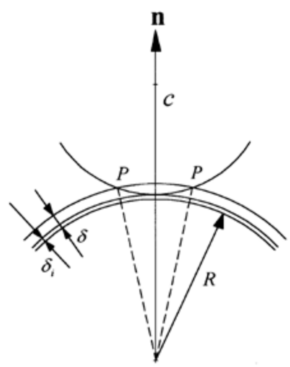


Fig.13 Path interval analysis using envelope

Convex-Concavity Case

While machining in such surface, the cutting width can be enlarged by the application of curvature catering. Therefore, DCC is thought to be the optimal selection. The error of linearization of the step-forward should be

$$\delta_i = \frac{2\bar{R}^{*2}}{\sqrt{4\bar{R}^{*2}-\Delta l^2}} - \bar{R}^* \tag{37}$$

The corresponding parameters of $\bar{R}$, α, $\mathbf{e}_a$, $\mathbf{r}_c$ and l_1 can also be obtained according to the situation of DCC working in concave elliptical region.

C. Hyperbolic region

Since the parabolic region is an expendable surface and it has a group of straight generating line, the surface bends to the same side of the tangent plane except in the direction of straight generating line. Thus the parabolic region may be classified into concave parabolic region and convex parabolic region. If a 2D region of S is parabolic, that area is said to be expendable. A flat-end tool may have access to an expendable region depending upon the sign of the second principal curvature[7]. Let us investigate the shape of a surface in the neighborhood of a parabolic point P_0. If $K = k_1 \cdot k_2 = 0$, it is obvious that one of the principal curvatures vanishes at least. If $k_1 = k_2 = 0$, P_0 is a planar umbilical point, and every normal section has a contact of order ≥ 2 with its tangent at P_0. If $k_1 \ne 0$, $k_2 = 0$, let $R_1 = 1/\max\{|k_1|\}$, the direction with respect to $k_2 = 0$ is asymptotic direction and (sign k_n) = (sign k_1) for all direction except that of asymptotic direction. One should not claim that the normal section along the asymptotic direction must be a straight line. In fact, the normal section might be a curve or straight line. It can be realized easily according to the following statement. We consider a point Q^* : $\mathbf{r}(u+\Delta u, v+\Delta v)$ of S. According to Taylor's formula we have

$$\begin{aligned}
&\mathbf{r}(u+\Delta u, v+\Delta v)\\
&=\mathbf{r}(u,v)+\left(\frac{\partial \mathbf{r}}{\partial u}\Delta u+\frac{\partial \mathbf{r}}{\partial v}\Delta v\right)\\
&\quad+\frac{1}{2!}\left(\frac{\partial^2 \mathbf{r}}{\partial u^2}\Delta u^2+2\frac{\partial^2 \mathbf{r}}{\partial u\partial v}\Delta u\Delta v+\frac{\partial^2 \mathbf{r}}{\partial v^2}\Delta v^2\right)\\
&\quad+o\left[\left(|\Delta u|^2+|\Delta v|^2\right)^2\right]
\end{aligned} \tag{38}$$

The distance of Q^* from the tangent plane $\mathbf{E}(P_0)$ to S is therefore

$$\begin{aligned}
\delta(Q^*) &= [\mathbf{r}(u+\Delta u, v+\Delta v)-\mathbf{r}(u,v)]\cdot\mathbf{n}\\
&=\frac{1}{2}\left(\mathbf{r}_{uu}\Delta u^2+2\mathbf{r}_{uv}\Delta u\Delta v+\mathbf{r}_{vv}\Delta v^2\right)\cdot\mathbf{n}\\
&\quad+o\left[\left(|\Delta u|^2+|\Delta v|^2\right)^2\right]
\end{aligned} \tag{39}$$

setting $\Delta u = du$, $\Delta v = dv$ in consequence of Eq.(39), we obtain

$$\begin{aligned}
\delta(Q^*) &= \frac{1}{2}\left(\mathbf{r}_{uu}du^2+2\mathbf{r}_{uv}dudv+\mathbf{r}_{vv}dv^2\right)\cdot\mathbf{n}\\
&\quad+o\left[\left(|du|^2+|dv|^2\right)^2\right]
\end{aligned} \tag{40}$$

That second fundamental form vanishes does not imply that $\delta(Q^*) = 0$. It is clear that the normal section might be a curve with large curvature radius or straight line.

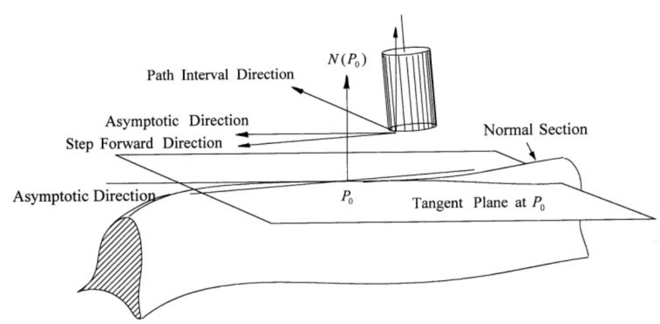


Fig.14 NC machining in parabolic region

Flat-end cutter and DCC

For a parabolic region, if any normal section bands the same direction with the one along the asymptotic direction and obvious a convex type. Thus, flat-end cutter is better than the DCC because of the DCC should incline an angle and result in scallop height. If the machined surface is concave for all direction except that of asymptotic direction, flat-end cutter and DCC are suitable.

Convex Parabolic Region

If the step-forward posses the direction, which k_1 lays, for a convex parabolic region, referring to Fig.4(b), the step length should be constrained by relation(13).Since the normal section curve might be a curve or straight line. The author suggests that the path intervals should be determined by the method presented as following. The direction of path interval should move toward the asymptotic direction but turn an angle (about $1^0 \sim 2^0$) to ensure the radius of normal section curve, which the path interval being deduced (see Fig.12). The unit vector with respect to the new direction du_1/dv_1 can be determined by

$$\mathbf{g_1} = (\mathbf{r}_u du_1 + \mathbf{r}_v dv_1)\Big/\sqrt{Edu_1^2 + 2Fdu_1dv_1 + Gdv_1^2} \tag{41}$$

The plane containing $\mathbf{g_1}$ and normal vector $\mathbf{N}(P_0)$ is

$$(\mathbf{g}_1 \times \mathbf{N}) \cdot (\boldsymbol{\rho} - \mathbf{r}_0) = 0 \tag{42}$$

where $\mathbf{r_0}$ is the vector of machining point P_0 and the vector of any point on the plan is $\boldsymbol{\rho}$.

The radius of normal section curve $\overline{c}$ with respect to the path interval direction can be obtained by Eq.(43)

$$\hat{R} = \max\left\{\frac{|\mathbf{r}'|^3}{|\mathbf{r}' \times \mathbf{r}''|}\right\} \tag{43}$$

If the direction of path interval is toward the convex region along curve $\overline{c}$, the actual path interval L_2 should be less than the path interval $L_2^{'}$ determined by the following equation.

$$L_2^{'} = 2\left(\frac{\hat{R}}{\hat{R}+\delta}\right)\sqrt{\left(\hat{R}+\delta\right)^2 - \left(\hat{R}+\delta_1\right)^2} \tag{44}$$

If the direction of path interval is toward the concave region along curve $\overline{c}$, in order to avoid interference and assure the machining quality, the cutter should tilt a certain angle as described in Eq.(9) with R_i being replaced by $\hat{R}$.

The actual path interval L_2 should be less than the path interval $L_2^{'}$ determined by the following equation.

$$\hat{R} - \left(\frac{\hat{R} - R^* - \delta_1}{2\hat{R}}\right)\sqrt{4\hat{R}^2 - L_2^{'2}} - \sqrt{R^{*2} - \frac{(\hat{R} - R^* - \delta_1)^2}{4\hat{R}^2} L_2^{'2}} - \delta = 0 \tag{45}$$

Concave Parabolic Region

If the step-forward posses the direction, which k_1 lays, for a concave parabolic, referring to Fig.4, the outer radius of flat-end cutter should satisfy the following relation

$$R < \sqrt{R_1^2 - \left(R_1 - \delta_1\right)^2} \tag{46}$$

The tilting angle for cutter axis should satisfy Eq.(9) with $R_i = R_1$. Therefore, the step-forward length should satisfy relation (11).The path interval in concave parabolic region can also be determined with the same approach used in convex parabolic region. The actual path interval L_2 should be less than the path interval $L_2^{'}$ determined by the following Eq.(47), (48).

As the direction of path interval is toward the convex region along curve $\overline{c}$, Eq.:

$$\left(\frac{\hat{R} + R^* + \delta_1}{2\hat{R}}\right)\sqrt{4\hat{R}^2 - L_2^{'2}} - \sqrt{R^{*2} - \left(\frac{\hat{R} + R^* + \delta_1}{2\hat{R}}\right)^2 L_2^{'2}} - \hat{R} - \delta = 0 \tag{47}$$

As the direction of path interval is toward the concave region along curve $\overline{c}$, Eq.:

$$\hat{R} - \left(\frac{\hat{R} - R^* - \delta_1}{2\hat{R}}\right)\sqrt{4\hat{R}^2 - L_2^{'2}} - \sqrt{R^{*2} - \frac{(\hat{R} - R^* - \delta_1)^2}{4\hat{R}^2} L_2^{'2}} - \delta = 0 \tag{48}$$

If the step-forward posses the direction, which k_2 lays, and the cross section along the direction is a straight line, the length of step need not be constrained. The path interval may be determined by Eq.(12) and (14). If the cross section along the direction is a curve, the discussion procedures are similar. It is noticed that the cross section with respect to k_2 may be a curve with large curvature radius or a straight line, i.e., the actual step-forward length can be increased and the machining efficiency can be improved.

Ball-end cutter

The discussion for ball-end cutter is similar to that for flat-end cutter. If $k_1 \neq 0$, $k_2 = 0$, let $R_1 = 1/\max\left\{|k_1|\right\}$

Convex Parabolic Region

If the step-forward takes the direction, which k_1 lays, for a convex parabolic region, referring to Fig.6(b), the step length should be constrained by relation(17). Imitating the procedure of determining the path interval for flat-end cutter, the path interval in convex parabolic region can be determined. The actual path interval L_2 should be less than the path interval $L_2^{'}$ determined by Eq.(47) and (48) with R^* being replaced by R.

Concave Parabolic Region

If the step-forward takes the direction, which k_1 lays, for a concave parabolic region, the radius of ball-end cutter should be constrained by $R < R_1$ and the step length should be constrained by relation(16). The path interval may be determined by Eq.(47) and (48) with R^* being replaced by R.

V. Accuracy Evaluation and Geometry Model

In order to clarify the comparison between FC and DCC, a small tilting angle is applied to an initial tool orientation in the tool-center plane to provide equal curvature radius for the normal intersecting line for tool curvature surface and cutter contact point. As Fig.15 shown below, the cutter tools are given the coordinates [**c;g,n**] and the projection of the FC surface on the plane (**g-n**) vertically to the tool feeding direction is elliptical circle (EC). Therefore, let the large radius of the EC as *R*, the flat-end surface radius, and the small radius of that as *Rsinα*, the equation of this EC is

$$\mathbf{r}_1 = \left\{\bar{R}\cos\bar{\theta}\,,\ \bar{R}\sin\alpha\sin\bar{\theta}\right\} \tag{41}$$

where the origin c of the coordinates is the center of the flat-end circle, **n** is normal vector of the machined surface at the cutter-contact point, **g** is the unit vector of the step-forward distance at the machined point M.

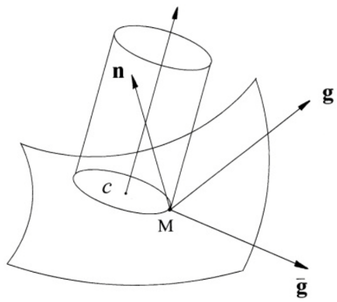


Fig.15 coordinates [**c;g,n**]

For the DCC in the process of rotation, the analysis is explained while the center of the normal intersecting line at cutter-contact point M is expressed as *c*. Different from the characteristics of FC, that intersecting line for working DCC is the cross line of step-forwarding linerization-formed strip envelope and **g-n** plane. Therefore, within the coordinates $\sigma = [\mathbf{c};\mathbf{g},\bar{\mathbf{g}},\mathbf{n}]$, the equation of the generating circle in parametric form is then given by:

$$\mathbf{r}_2 = \begin{Bmatrix} (R + r_0\cos\theta_1)\cos\varphi_1 \\ (R + r_0\cos\theta_1)\sin\varphi_1\cos\alpha + r_0\sin\theta_1\sin\alpha + \lambda \\ (R + r_0\cos\theta_1)\sin\varphi_1\sin\alpha - r_0\sin\theta\cos\alpha \end{Bmatrix} \tag{42}$$

Assuming the circumstances of strip envelope $\left(\mathbf{r}_{2\theta_1}, \mathbf{r}_{2\varphi_1}, \mathbf{r}_{2\lambda}\right) = 0$, we obtained

$$\varphi_1 = -\sin^{-1}(\tan\theta_1\tan\alpha) \tag{43}$$

While equation (43) is substituted with equation (42), the equation of the intersecting line at coordinates $\bar{\sigma} = [\mathbf{c};\mathbf{g},\mathbf{n}]$ is given as

$$\begin{cases} \bar{\mathbf{r}}_2 = \left\{(R + r_0\cos\theta_1)\cos\varphi_1,\ (R + r_0\cos\theta_1)\sin\varphi_1\sin\alpha - r_0\sin\theta\cos\alpha\right\} \\ \varphi_1 = -\sin^{-1}(\tan\theta_1\tan\alpha) \end{cases} \tag{44}$$

where r_0 is the radius of cutter, *R* is the distance between the circle center and the axis of propeller, φ_1 and θ_1 are the parameters of the surface.

Therefore, we may utilize above results to access the comparison of two cutters. At the convenience of analysis, we assume that the radius of outer shape of FC and DCC are equal. That is $\bar{R} = R + r_0$, equation (44) is expressed as

$$\begin{cases} \bar{\mathbf{r}}_2 = \{(\bar{R} + r_0\cos\theta_1 - r_0)\cos\varphi_1, \\ \qquad (\bar{R} + r_0\cos\theta_1 - r_0)\sin\varphi_1\sin\alpha - r_0\sin\theta_1\cos\alpha\} \\ \varphi_1 = -\sin^{-1}(\tan\theta_1\tan\alpha) \end{cases} \tag{45}$$

Cutter-contact points for the FC and DCC are overlapped. That is equation (41) is modified as

$$\bar{\mathbf{r}}_1^* = \left\{\bar{R}\cos\bar{\theta}\,,\ \bar{R}\sin\alpha\sin\bar{\theta} - r_0(1 - \sin\alpha)\right\} \tag{46}$$

Additionally, the differences in SH are expressed by the ones on the **n** plane while at the same coordinate **g**. $\bar{\theta}$ in the equation (45) may be determined by the followings:

$$\bar{R}\cos\bar{\theta} = (\bar{R} + r_0\cos\theta_1 - r_0)\cos\varphi_1 \tag{47}$$

$$\bar{\theta} = \cos^{-1}\left(\frac{(\bar{R} + r_0\cos\theta_1 - r_0)\cos\varphi_1}{\bar{R}}\right) \tag{48}$$

Finally, the differences in SH may also provide information for the machined accuracy evaluation and are expressed as

$$\begin{aligned} \Delta = {} & \bar{R}\sin\alpha\sin\bar{\theta} - r_0(1 - \sin\alpha) - \\ & (\bar{R} + r_0\cos\theta_1 - r_0)\sin\varphi_1\sin\alpha + r_0\sin\theta_1\cos\alpha \end{aligned} \tag{49}$$

If the value $\Delta > 0$, the smaller the SH is while using the DCC, the machined accuracy is higher; if the value $\Delta < 0$, FC would be the better choice.

Based on the machined efficiency, the value of step lengths and path intervals may be used as determinants while the maximal admissible error and the transmission of error caused by step length to SH are taken into account. As figure shown below, within the same step length, the path interval of FC is expressed as

$$L_2 = 2\bar{R}\cos\bar{\theta} \tag{50}$$

while

$$\bar{R}\sin\alpha\sin\bar{\theta} - r_0(1 - \sin\alpha) < \delta_{all} - \delta_1 \tag{51}$$

where δ_1 is the maximal error at the step length, δ_{all} is the admissible error. The path interval of DCC is expressed as

$$L_2 = 2(\bar{R} + r_0\cos\theta_1 - r_0)\cos\varphi_1 \tag{52}$$

while

$$(\bar{R} + r_0\cos\theta_1 - r_0)\sin\varphi_1\sin\alpha - r_0\sin\theta_1\cos\alpha < \delta_{all} - \delta_1 \tag{53}$$

VI. Example

Propeller blade is taken as an example to verify the above models of machining parameters. For the propeller blade (Fig.16) with a constant pitch, the driving face consists of three areas, area 1 is freeform surface near leading edge, area 2 is spiral surface with a constant pitch, and area 3 is freeform surface near trailing edge. The analytical equation of area 2 will be deduced by the method of analytical geometry. The equation of area 1 and 3 can be obtained by surface fitting method in the theory of computational geometry. So the blade surface may be determined by the following equation:

$$\mathbf{S}(r,\theta) = \{r\cos\theta, r\sin\theta, b\theta - r\tan\varphi\} \tag{53}$$

where r is the distance between any point on the driving face of blade and axis of propeller,

θ is the angle parameter,

β is the rake angle of propeller blade,

b is spiral parameter, if the pitch is T, then $b = T/2\pi$

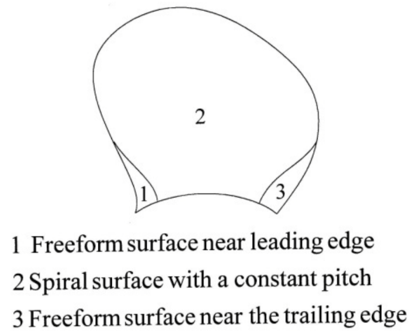


Fig.16 Different region of driving face

As for the freeform surface area, its analytic equation cannot be provided directly. Since the surface is expressed by the discrete points, so the numeric equation may be obtained by the method of surface fitting using the theory of computational geometry. The absolute coordinate of each point in the workpiece coordinate system should be calculated by the following equations:

$$\begin{cases} x_{[i,j]} = r_{[i]} \cos\theta_{[i,j]} \\ y_{[i,j]} = r_{[i]} \sin\theta_{[i,j]} \\ z_{[i,j]} = (b\theta_{[i,j]} - r_{[i]} \tan\varphi) - C_{td_{[i,j]}} t_{m[j]} \end{cases} \tag{54}$$

The surface defined by the above discrete points can be expressed by surface fitting method such as 3 order Bezier method[8]. So the equation of freeform surface may be as following:

$$\mathbf{S}^* = \mathbf{S}^*(r,\theta) = \mathbf{UMBM}^T\mathbf{V}^T \tag{55}$$

Since equation (55) is the function of discrete points $P_{ij}(x_{[i,j]}, y_{[i,j]}, z_{[i,j]})$, and the above discrete points are the function of a certain position (r,θ) on the blade surface, so equation (55) is also the function of (r,θ).

For region 2,

The fundamental coefficients can be calculated as following

$$\begin{cases} E = \sec^2\beta \\ F = -b\tan\beta \\ G = r^2 + b^2 \\ D = \sqrt{r^2\sec^2\beta + b^2} \end{cases} \tag{56}$$

$$\begin{cases} L = 0 \\ M = -b/D \\ N = -r^2\tan\beta/D \end{cases} \tag{57}$$

The main curvature equation can be expressed as following:

$$(r^2\sec^2\beta + b^2)K_n^2 + \frac{(r^2\sec^2\beta + 2b^2)\tan\beta}{D}K_n - \frac{b^2}{D^2} = 0 \tag{58}$$

From the Gauss curvature

$$K = K_1K_2 = \frac{LN - M^2}{EG - F^2} = \frac{-b^2}{r^2\sec^2\beta + b^2} < 0 \tag{59}$$

it is known that all the points in region 2 are hyperbolic points. It is the same case for region 3. But for region 1, it is known that Gauss curvature $K > 0$, all the points in the region 1 are elliptical points. We should determine the maximal radius of cutter and minimum tilting angle of the flat-end cutter, so that the interference could be avoid.

The given parameters of a propeller blade with a constant pitch are

$$\begin{cases} r_{in} = 400\ mm & \text{(inner radius of blade)} \\ \beta = 15^0 & \text{(rank angle)} \\ b = T/2\pi = 6000/2\pi = 954.93\ mm & \text{(spiral parameter)} \end{cases}$$

Thus, $k_1 = -0.00115$ and $k_2 = 0.006757$ at $r = 400mm$. Considering the accuracy problem, the radii of FC and DCC are $\overline{R} = 60mm$ and $R = 50mm$, $r_0 = 10mm$, respectively. The feeding step-forward direction is toward the convex field and the path interval is toward the concave field. The tilting angle of the cutter is 5° to avoid gouging. The comparison of surface intersecting curve (scallop height) between FC and DCC are computed in Fig.17. In the neighboring region of machining point M, two surface intersecting curves are much closed, i.e., scallop heights are approximate. The computational values of Scallop heights are listed in Table 1 to provide accurate comparison. It is obvious that the scallop height of FC are smaller than DCC.

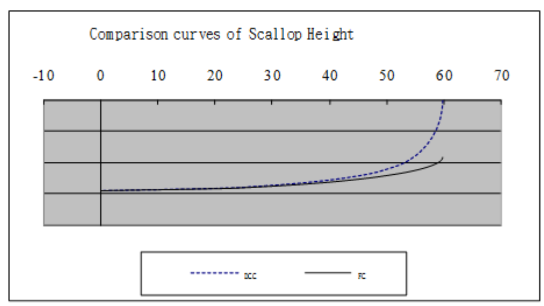


Fig.17 Scallop Height Comparison between DCC and FC

TABLE I
COORDINATE OF INTESECTING CURVES AND SCALLOP HEIGHT

θ_1	ϕ_1	n direction coordinate	g direction coordinate	$\overline{\theta}$	Scallop Height
0	0	60	0	0	9.1284425
8.5	-0.01307	59.88503	-1.54071	-0.06191	7.9112824
17	-0.02675	59.54173	-3.05144	-0.12367	6.7220752
25.5	-0.04174	58.97443	-4.50340	-0.18515	5.5877654
34	-0.05904	58.18879	-5.87045	-0.24633	4.5331628
42.5	-0.08025	57.18810	-7.13106	-0.30736	3.5794877
51	-0.10825	55.96369	-8.27195	-0.36889	2.7420811
59.5	-0.14907	54.46451	-9.29645	-0.43292	2.0258545

68	-0.21827	52.47084	-10.2508	-0.50636	1.4137751
76.5	-0.37300	48.73573	-11.3488	-0.62277	0.8297883
85	-1.5708	-0.00018	-14.3577	-1.57079	1.24367E-0

TABLE II
COORDINATE OF INTESECTING CURVES AND SCALLOP HEIGHT BETWEEN θ_1=76.5~85 (INTERVAL THAT DIFFERENCE OF SCALLOP HEIGHT ARE REMARKABLE)

θ_1	ϕ_1	**n** direction coordinate	**g** direction coordinate	$\overline{\theta}$	Scallop Height
76.5	-0.37300	48.73573	-11.3488	-0.62277	0.8297883
77	-0.38866	48.35248	-11.4323	-0.63364	0.7923391
77.5	-0.40567	47.93060	-11.5199	-0.64542	0.7541049
78	-0.42421	47.46302	-11.6125	-0.65827	0.7149868
78.5	-0.44451	46.98752	-11.7106	-0.67236	0.6748784
79	-0.46686	46.35304	-11.8151	-0.68794	0.6336648
79.5	-0.49161	45.68522	-11.9271	-0.70529	0.5912231
80	-0.51918	44.91884	-12.0479	-0.72477	0.5474214
80.5	-0.55014	44.02935	-12.1788	-0.74686	0.5021190
81	-0.58521	42.98358	-12.3217	-0.77217	0.4551663
81.5	-0.62537	41.73557	-12.4789	-0.80155	0.4064052
82	-0.67195	40.21960	-12.6532	-0.83614	0.3556699
82.5	-0.72688	38.33788	-12.8482	-0.87764	0.3027882
83	-0.79310	35.93675	-13.0684	-0.92861	0.2475827
83.5	-0.87552	32.75497	-13.3199	-0.99331	0.1898745
84	-0.98342	28.28810	-13.6106	-1.07984	0.1294863
84.5	-1.13993	21.28318	-13.9514	-1.20818	0.0662472
85	-1.5708	-0.00018	-14.3577	-1.57079	1.24367E-0

VII. Conclusion

In this study, geometry model for efficiency evaluation and demonstrate qualitative and quantitative analysis are established by utilizing differential geometry. By utilizing computer modeling, the analyzed parameters such as efficiency, scallop height for different machined surfaces have been deduced to provide reliable selection principles of FC and DCC in order to improve NC machining. Propeller blade is taken as an example to verify the proposed models of machining parameters. The comparison of surface intersecting curve (scallop height) between FC and DCC are computed and shown in Fig.17 and table. The characteristics of NC machining that influence efficiency and accuracy are compounded. The characteristics, such as path-interval, step length and tool wear will be discussed and established models to provide the reliable criteria of tool selection. Therefore, the models discussed in this paper do provide a reliable approach on improving NC machining efficiency and machining accuracy. The tool selection criteria are also proposed.

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