Adaptive sliding mode control of chaos in nonlinear friction-driven oscillator

E. Maani Miandoab and A. Yousefi-Koma

Abstract— In this paper, the adaptive sliding mode controller (ASMC) is used to control the chaotic vibration of a block-on-belt system subjected to harmonic external excitations and supported by a limited energy supply. To control the chaotic vibration of this system, a switching surface is defined such that it becomes easy to ensure the stability of the error dynamics in the sliding mode. Then an adaptive sliding mode controller (ASMC) is derived to ensure the occurrence of the sliding motion and the Lyapunov stability analysis is used to guarantee the stability and tracking of the controlled system. Two different desired responses are considered in this study and the effectiveness of the proposed method is shown for both cases.

Keywords—Chaos, Friction driven, Adaptive sliding mode.

I. INTRODUCTION

The friction induced vibration has been of considerable interests for researchers since many years ago, because it occurs frequently in everyday life and engineering systems. A number of works on the theory of oscillations with self excitation exists in the literature, for example: influence of the belt speed on the system response, dynamics of there-block mechanical with dry friction [1], investigation the geometry of chaotic attractors for dry friction oscillators [2, 3], numerical study of a dry friction oscillator with parametric and external excitations [4] and the dynamic behavior of friction driven oscillator with impact damper [5].

In practical situations, the system has a limited energy source such as DC motor and thus energy source dynamics is influenced by the oscillating system [6]. This increases the number of degrees of freedom, and is called a non-ideal problem.

Dynamics of friction driven oscillator with limited power supply was investigated and the power supply influence on the vibrating system was observed along with chaotic motions [7].

Control and synchronization of chaotic systems have become an important topic since the pioneering work of Pecora and Carroll [8] in order to its vast application in physics and engineering systems such as in chemical reactions, power converters, biological systems, information

This work was done in Advanced Dynamics and Control Systems Laboratory, Faculty of Mechanical Engineering, College of Engineering, University of Tehran, Tehran, Iran.

E. Maani Miandoab, PhD Student, Faculty of Mechanical Engineering, College of Engineering, University of Tehran, Tehran, Iran . Tel: +98 (912) 226 34 89.Email: e.maani@ut.ac.ir

A. Yousefi-koma, PhD, Assistant Professor, Director, Advanced Dynamic and Control Systems Laboratory (ADCSL), Faculty of Mechanical Engineering, College of Engineering, University of Tehran, Tehran, Iran .Tel: +98 (912) 536 96 24; Fax: +98 (21) 8833 65 59.Email : aykoma@ut.ac.ir processing and et al. [9–13]. Many different chaos synchronization strategies have been developed, , adaptive control [14], variable structure control [15–17], optimal control [18], digital redesign control [19], Impulsive control [20] and adaptive sliding mode control [21, 22] and Impulse damper [22].

In this paper, first, the dynamical behavior of non-linear friction-driven oscillator with limited power supply under harmonic excitation is investigated numerically. Investigating the response of the system in the wide range of excitation frequency shows that the chaotic motion of the system near the system natural frequency. To control the chaotic vibration of this system to desired response, an adaptive sliding mode control is proposed. Using the sliding mode control technique based on Lyapunov stability theory, an adaptive control law is established which stabilizes the chaotic response of the system to desired response. Numerical results have verified the effectiveness of the proposed method to control the chaotic vibration of friction driven oscillator for static and dynamic desired response. The effect of control parameters is also investigated on the effectiveness of the proposed control method and the amplitude of control input.

II. PROCEDURE FOR PAPER SUBMISSION

Fig. 1 shows the friction-driven oscillator supported by limited power DC motor. A block is connected to a fix frame by a nonlinear spring and a linear viscous damper. The non-linear spring force is given by $F_k = k_1 x - k_2 x^3$; where, x is the block displacement with respect to equilibrium position.



Fig. 1 Friction-driven oscillator

Because of the interaction between the motor and the oscillating system, angular velocity of the motor isn't constant. Characteristic curves of the energy source (DC motor) are assumed to be straight line [23]:

$$T(\dot{\theta}) = I(u_1 - u_2\dot{\theta}) \tag{1}$$

Where, T is the mechanical torque, $\hat{\theta}$ is the motor's speed, I is the mass moment of inertia of the motor, u_1 is a control parameter depending on voltage and u_2 is the rotational damping coefficient of the motor.

The equation of the motion of the block and motor can be written as:

$$M\ddot{x} - k_1 x + k_2 x^3 + C\dot{x} = F_{fr} + F_t \cos(\omega t)$$
(2)

And the equation of motion of motor angular position can be written as:

$$I\ddot{\theta} = T(\dot{\theta}) - rF_{fr} = I(u_1 - u_2\dot{\theta}) - rF_{fr}$$
⁽³⁾

Where, F_t , ω , C and F_{fr} are the amplitude and frequency of external harmonic excitation, the coefficient of linear viscous damper, the friction force between the belt and the block respectively. Friction force follows Coulomb's friction law. If the velocity of belt doesn't equate the velocity of block, the friction force can be modeled as [24, 25]:

If
$$\dot{x} \neq r\dot{\theta} \text{ or } (\dot{x} = r\dot{\theta} \& \ddot{x}r \neq \dot{\theta})$$

 $F_{fr} = \mu_k Mg \operatorname{sgn}(r\dot{\theta} - \dot{x})$
(4)

And

If
$$(\dot{x} = r\dot{\theta} \& \ddot{x} = r\ddot{\theta})$$

$$F_{fr} = \frac{IM(u_1 - u_2\dot{\theta}) - k_1x + k_2x^3 + C\dot{x} - F_r\cos(\omega t)}{M}$$
(5)

Where, μ_k is the kinetic friction coefficient.

The non-dimensional form of the equations of the block, liquid and the motor rotation can be written as:

$$x'' - x + \delta x^{3} + \mu x' = \frac{F_{fr}}{M \omega_{n}^{2}} + \frac{F_{t}}{M \omega_{n}^{2}} \cos(\Omega \tau)$$
$$\ddot{\theta} = (E_{1} - E_{2}\dot{\theta}) - \frac{r}{I \omega_{n}^{2}} F_{f}$$
(6)

The prime denotes differentiation with respect to τ , and

$$\omega = \sqrt{\frac{k_1}{M}}; \quad \tau = \omega t; \quad \mu = \frac{C}{\sqrt{k_1 M}}; \quad E_1 = \frac{M u_1}{k_1};$$
$$E_2 = u_2 \sqrt{\frac{M}{k_1}}; \quad x' = \frac{dx}{d\tau}; \quad \Omega = \frac{\omega}{\omega_n}$$

The system parameters are fixed as:

$$E_1 = 2.5, E_2 = 1.5, \delta = 0.1, \mu = 0.01,$$

Frequency response and bifurcation diagram of this system were plotted in [24, 25], and it was shown that the system has chaotic vibration for $\Omega=1.2$. Time response of the block, excited by frequency $\Omega=1.2$ is plotted in Fig. 2 and the phase plane of the system is plotted in Fig. 3. As seen in this figures, time response and phase plane of the system are irregular which indicates the chaotic motion.

To see the chaos clearly, Poincare map is plotted in Fig. 4. As seen in this figure, the Poincare´ section takes on an irregular pattern indicating the chaotic vibration.



Fig. 2 Time history response of the system



Fig. 3 Phase plane of the system



Fig. 4 Poincare Map

III. CONTROL METHOD

A. Adaptive Sliding Mode Control (ASMC)

In this section, the adaptive sliding mode control is used to control the chaotic vibration. The controlled chaotic system can be rewritten as follows:

$$\ddot{x} - x + \delta x^{3} + \mu \dot{x} = \frac{F_{fr}}{M \omega_{n}^{2}} + \frac{F_{t}}{M \omega_{n}^{2}} \cos(\Omega \tau) + u$$
$$\ddot{\theta} = (E_{1} - E_{2}\dot{\theta}) - \frac{r}{I \omega^{2}} F_{f}$$
(7)

For tracking control purpose, the error states are defined as

$$e_1 = x - \varphi(t)$$

$$\dot{e}_2 = \dot{e}_1 = \dot{x} - \dot{\varphi}(t)$$
(8)

Where $\varphi(t)$ is the desired system displacement and assume it to be twice differentiable with respect to time and its derivations are bounded as:

$$\begin{aligned} |\varphi(t)| &\leq M \\ |\dot{\varphi}(t)| &\leq N \qquad M, N \text{ and } P \in R^+ \\ |\ddot{\varphi}(t)| &\leq P \end{aligned} \tag{9}$$

To ensure stability of the sliding mode, a switching surface s(t) is defined as follows:

$$s(t) = e_1(t) + e_2(t) \tag{10}$$

When the system operates in the sliding mode, it satisfies the following equations [26, 27].

$$s(t) = \dot{s}(t) = 0$$
 (11)

Sliding mode dynamics can be written as:

$$\dot{e}_{1} = e_{2} = -e_{1}$$

$$\dot{e}_{2} = \alpha(e_{1} + \varphi(t)) - \delta(e_{1} + \varphi(t))^{3} - \mu(\dot{e}_{2} + \dot{\varphi}(t))...$$

$$-\ddot{\varphi}(t) + \frac{F_{fr}}{M\omega_{n}^{2}} + \frac{F_{t}}{M\omega_{n}^{2}}\cos(\frac{\omega}{\omega_{n}}\tau) + u$$
(12)

Following functions are defined:

$$f(e_1, e_1, \varphi) = \alpha e_1 - \delta e_1^3 - 3\varphi \delta e_1^2 - 3\varphi^2 \delta e_1 - \mu e_2$$

$$g(\varphi, \dot{\varphi}, \ddot{\varphi}) = \alpha \varphi - \delta \varphi^3 - \mu \dot{\varphi} - \ddot{\varphi} + \frac{F_{fr}}{M \omega_n^2} + \frac{F_t}{M \omega_n^2} \cos(\frac{\omega}{\omega_n} \tau)$$
(13)

Using (13), (12) can be rewritten as:

$$\dot{e}_2 = f(e_1, e_2, \phi) + g(\phi, \dot{\phi}, \ddot{\phi}) + u$$
 (14)

Using (9), we have

$$g(\varphi, \dot{\varphi}, t) \leq |\alpha \varphi| + |\delta \varphi^{3}| + |\mu \dot{\varphi}| + |\ddot{\varphi}| + ...$$

$$|\frac{F_{fr}}{M \omega_{n}^{2}}| + |\frac{F_{t}}{M \omega_{n}^{2}} \cos(\frac{\omega}{\omega_{n}}\tau)|$$

$$\leq |\alpha M| + |\delta M^{3}| + |\mu N| + |P| + |\frac{F_{fr}}{M \omega_{n}^{2}}| + |\frac{F_{t}}{M \omega_{n}^{2}}| = \beta$$
(15)

The controller is designed as [22]:

$$u = -\gamma \eta \operatorname{sgn}(s(t)) \tag{16}$$

Where, $\eta = |f(e_1, e_2, \varphi)| + \beta + |e_2|$ and the adaptive law for parameter γ is proposed as:

$$\dot{\gamma} = -\gamma - \xi, \quad \xi > 1 \tag{17}$$

The proposed adaptive control scheme will guarantee the globally asymptotical stability for the error, and is proven in the following.

B. Stability analysis

In this section, we analyze the stability of the sliding mode dynamics based on the Lyapunov stability theory. For this purpose, the Lyapunov function is defined as 1 + 2 + 2

$$V(t) = \frac{1}{2}(s^2 + \gamma^2)$$

which yields to:

$$\dot{V} = s\dot{s} + \gamma\dot{\gamma} = s(\dot{e}_1 + \dot{e}_2) + \gamma\dot{\gamma} =$$

$$s(e_2 + f(e_1, e_2, \varphi) + g(\varphi, \dot{\varphi}, \ddot{\varphi}) + u) + \gamma\dot{\gamma}$$

$$\leq |s|(|e_2| + |f(e_1, e_2, \varphi)| + \beta) + su + \gamma\dot{\gamma}$$
(18)

By solving (17) we have: $\gamma = \alpha e^{-t} + \xi$, where α is positive constant. By $\gamma = \alpha e^{-t} + \xi$ substituting in (18) we have:

$$\dot{V} \leq |s| \eta (1 - \alpha e^{-t} - \xi) + (-\alpha^2 e^{-2t} - \xi \alpha e^{-t}) < 0$$
⁽¹⁹⁾

If we define

$$w(t) = |s| \eta(1 - \alpha e^{-x} - \xi) + (-\alpha^2 e^{-2x} - \xi \alpha e^{-x})$$

And integrating the above equation from zero to t, it leads to following equation.

$$V(t) \le V(0) - \int_{0}^{t} w(\lambda) d\lambda \implies V(0) \ge V(t) + \int_{0}^{t} w(\lambda) d\lambda$$

$$\int_{0}^{t} w(\lambda) d\lambda \le V(0) < \infty$$
(20)

Taking the limit as $t \rightarrow \infty$ on both side of (20) gives

$$\lim_{t \to \infty} \int_{0}^{t} w(\lambda) d\lambda \le V(0) < \infty$$
(21)

Now the Barbalat's lemma is given below.

Lemma. (Barbalat's lemma [28]). If $w: R \to R$ is a uniformly continuous function for $t \ge 0$ and if

 $\lim_{t \to \infty} \int_{0}^{t} |w(t)| dt \le V(0) < \infty \text{ exists and is finite, then}$ $\lim_{t \to \infty} w(t) \to 0.$

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Thus according to Barbalat's lemma (see Lemma 1), it is
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obtained that

$$\lim_{t \to \infty} w(t) = \lim_{t \to \infty} (|s| \eta (1 - \alpha e^{-x} - \xi) + (-\alpha^2 e^{-2x} - \xi \alpha e^{-x})) \to 0$$
(22)

Since $\lim_{t \to \infty} e^{-t} = 0$, $\eta > 0$ and $\xi > 1$ implies $s(t) \to 0$ as $t \to \infty$.

Thus the system works in the sliding mode and the stability of e_1 and e_2 is surely guaranteed using (12), therefore $(e_1(t), e_2(t))$ converge to zero.

IV. NUMERICAL SIMULATIONS

Here the numerical results are given to confirm the validity of the proposed method. In the numerical simulations the desired response is set as $\varphi(t) = 0$ and the control parameters, ξ and α are set as: $\xi = 1.1$ and $\alpha = 1$.

Fig. 5 and Fig. 6 depict the synchronization error of the state variables.



Fig. 5 Time response of first error state, $\varphi = 0$.



Fig. 6 Time response of second error state, $\varphi = 0$.

Fig. 7 shows time response of the switching function s(t).



Fig. 7 Time response of the switching function, $\varphi = 0$.

As seen in this figures, state variables converge to desired values and the switching function converges to zero in a short time.

The control effort is plotted in Fig. 8.



Fig. 8 Time response of control effort for $\alpha = 1$ and $\xi = 1.1$.

For investigation the effect of control parameters on the control effectiveness, we plot the time history of switching function for three values of ξ and α .

Time history of switching function is plotted in Fig. 9 for $\xi = 1.1,5$ and 10.



Fig. 9 Time response of the switching function for $\xi = 1.1, 5$ and 10

As seen in Fig. 9, by increasing ξ , control effectiveness increases and the convergence time of switching function decreases.

To show the effect of α on the effectiveness of proposed method, switching function is plotted for $\alpha = 0.1, 5 \text{ and } 10$ in Fig. 10.



Fig. 10 Time response of the switching function for $\alpha = 0.1, 5$ and 10

From Fig. 10, it is obvious that the effect of α is the same as ξ . And the large values of α leads to small time convergence than small values of α .

From (16), it is clear that the large values of ξ and α lead to more control input. To show this numerically, control input is plotted in Fig. 11 for $\alpha = 1$ and $\xi = 5$ and $\alpha = 0.1$ and $\xi = 1.1$ in Fig. 12.



Fig. 11 Time response of control effort for $\alpha = 1$ and $\xi = 5$.



Fig. 12 Time response of control effort for $\alpha = 0.1$ and $\xi = 1.1$.

Comparison of Fig. 11 and Fig. 12 with Fig. 8 shows that increasing control parameters increase the control input.

For evaluation the proposed control law in tracking the dynamic desired response, desired response is considered as $\varphi(t) = 0.3\sin(5\tau)$. Time responses of errors are plotted in Fig. 13 and Fig. 14. The control parameters, ξ and α are set as: $\xi = 1.1$ and $\alpha = 1$.



Fig. 13 Time response of first error state, $\varphi = 0.3 \sin(5\tau)$



Fig. 14 Time response of first error state, $\varphi = 0.3 \sin(5\tau)$

Time response of state variables plotted in Fig. 15 and Fig. 16.



Fig. 15 Time response of first error state, $\varphi = 0.3 \sin(5\tau)$



Fig. 16 Time response of first error state, $\varphi = 0.3 \sin(5\tau)$

Fig. 17 shows the Time response of the switching function which indicates that the system has been controlled to harmonic desired response in short time.



Fig. 17 Time response of block displacement, $\varphi = 0.3 \sin(5\tau)$.

From the simulation results, it shows the proposed ASMC works well and the system error states are regulated to zero asymptotically.

V. CONCLUSION

In this paper, the chaotic vibration of a block-on-belt system subjected to an external excitation and supported by a limited energy supply was controlled using adaptive sliding mode control. Using the sliding mode control technique based on Lyapunov stability theory, an adaptive control law was established which stabilizes the chaotic response of the system to desired response. Numerical results have verified the effectiveness of the proposed method to control the chaotic vibration of friction driven oscillator for two different static and dynamic desired responses. The effect of control parameters which defined in control law is analyzed and it shown that increasing the control parameters decreases the switching function settling time but it needs more control inputs.

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Ehsan Maani Miandoab, was Born in 1985, IRAN. Ph.D student at Mechanical Engineering Department, University of Tehran. M.S. and B.S. also from Mechanical Engineering Department, University of Tehran. He is the Research Assistant in Advanced Dynamic and Control Systems Laboratory, Faculty Mechanical Engineering, University of Tehran, Tehran, Iran. Mr. Maani Miandoab is a member of National Talent Foundation.

Aghil Yousefi-Koma, PhD, was born in 1963 in Fouman, Iran. Dr. Yousefi-Koma got his PhD from Carleton University, Aerospace Engineering in 1997 and his MSc and BSc from University of Tehran, Mechanical Engineering in 1989 and 1986 respectively. He has been an assistant professor at the Faculty of Mechanical Engineering in University of Tehran since 2005. He is also serving as the director of University of Tehran Technology Incubator, the director of the Advanced Dynamic and Control Systems Laboratory (ADCSL), and the director of Centre for Advanced Vehicles (CAV). Dr. Yousefi-Koma' s main field of interest includes system dynamics, control, smart structures, micro-robotics, micro-underwater vehicles, and micro-aerial vehicles. He is the author of more than 100 refereed journal and conference papers and technical reports. He is also a co-author of the "Intelligent Materials" book by The Royal Society of Chemistry, 2008. Before joining University of Tehran Dr. Yousefi-Koma had been working in the Institute for Aerospace Research of National Research Council Canada (NRC), SNECMA Motors, and Canadian Space Agency (CSA) between 1997-2005.