Minimizing Uncertainty of Selected Composite Materials Using Various Mathematical Approaches

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***Abstract* —** Three different mathematical approaches are presented herein as procedural and numerical results for flexural rigidity characterization of Fiber Reinforced Polymer (FRP) plates retrofitted with various types of composites and fabrics. The first approach is based on theoretical and experimental findings using composite materials such as FRP and Kevlar® during simulation of heavy-ion cosmic radiation. Since it is thought that FRP and composites are non-magnetic and corrosion resistant, it is assumed that a better protection from radiation in both space and during exposure to radioactive situations would be provided. The second approach used FRP plate flexural rigidity values calculated with a central finite-difference iterative scheme while utilizing the experimental load-deflection relations based on bending tests. The tests were performed on each plate by applying a concentrated load at the center. A fourth-order partial differential equation of plate equilibrium was adopted to estimate the plate flexural rigidities and ultimately obtain the theoretical load-deflection relations. The results were verified with Navier's solution for the same type of loading. The third approach used FRP and various composites to their fullest potential with ultimate goal to minimize uncertainty by comparing probability and evidence theories. The application of these criteria or standards will be demonstrated through practical analysis and design examples. This research further expanded the FRP and composites knowledge base by identifying material strengths and weaknesses through conducting experimental versus theoretical studies. The proposed methods synthesize the study of the emerging new materials with a probability-based approach. Such an approach is considered a pre-cursor to the so-called Load and Resistance Factor Design (LRFD) philosophy based on which a currently evolving FRP design specification will be based and will subsequently become Standard of Practices and Procedures that could provide promising tools to implement various mathematical models during conceptual design and selection of appropriate composite material.

***Keywords*—** Composites, Evidence Theory, Finite-difference, Probability, Radiation shield, Uncertainty

I. Introduction

When NASA explores new material that will be operated in extreme risk environments, they rely on experts to evaluate available data, interpret the significance of risk and minimize uncertainty between known and unknown variables [1]. When a new product is introduced, scientist and engineers use analyses and experimentation in order to determine results. Various programs are used to determine quantitative and qualitative results. Some approaches are done by programs readily available (off-the-shelf), while others are based on mathematical principles and fundamentals. Matlab and Excel are useful for matrices and graphs, while Monte Carlo Simulation and @Risk are used for as enhancers of qualitative analyses. Validations of experiments are done by using laboratories such as Applied Research Center (ARC), Structures and Physics Laboratories and a local hospital's state-of-the-art attenuator to determine Half Value Layer (HVL). An expected value is assumed and validation of findings is determined using classic mathematical operations such as Finite Element Method, Finite Difference Method, and first, second and third order of differential equations.

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Even though there are numerous projects developed during the last years, the ones that made a significant effect was the experimental and theoretical study to determine if Fiber Reinforced Polymer (FRP) combined with Kevlar® were excellent space radiation shielding materials. If yes, to what extent? If no, what other options exist? Are there any other materials that can be used for space missions which could yield assurance of non-magnetic, corrosion resistant and light in weight properties? What it will take to validate the above requirements and answer all these questions? Additionally, if FRP is the material of choice; can retrofitting FRP improve the flexural rigidity of the plates? What mathematical model can we use? Is finite difference a good option, or by using a combination of probability and evidence theory can we provide a better uncertainty assessment to the solution?

By asking all these questions one might think that the focus of this research is very vague. On the contrary, this research is focused in composite materials and it is a non-stop pursuit of solutions using tools that may produce satisfactory results.

This paper is divided into three summarized research areas with common goal to better understand the high complexity of composite material matrices.

The first part will address the FRP and composite materials as radiation shield

The second part will define retrofitting and flexural rigidity characteristics of rectangular FRP plates coated with various composite fabrics and

The third part will provide options of uncertainty assessment for system safety during conceptual design: A probabilistic and evidence approach comparison.

II. FRP and Composite Material as Radiation Shield

Half Life Radiation exposure

This part of the research identifies how FRP and Kevlar material behave under half-life radiation exposure. If there is a nuclear disaster or terrorist attack that involves nuclear weapons, people need some assurance that the building structure is not only blast resistant, but it can prevent radiation from infiltrating. The International Atomic Energy Agency (IAEA) agrees that the nuclear threats have changed; Post war period dismantling and nuclear weapon ownerships resulted in the creation of weapons-grade nuclear material in storage facilities [2]. Changing the world by strengthening nuclear security is a noble idea. However, protecting people by developing structures that can block radiation and withstand impact is a higher priority given the volatile global politics and aging nuclear power plants (as evidenced in Japan after its horrific tsunami). The results of this part of the research will yield a radioactive-protective, impact-resistant shield that could easily be implemented within the infrastructure of homes, schools, industrial buildings, military vehicles, hospitals, etc.

It is true that we live in an increasingly radioactive world with radiation exposures being linked to ulcerations, gastrointestinal disease, thyroid disease, infection, blood disorders, neurological effects, cancers and death. Humans risk exposure to radiation through living and work spaces in the form of radon and industrial releases, industrial work places, military operations and even outer space [3]. Human suffering from radiation exposure can be minimized with the availability and implementation of a readily available "off the shelf", inexpensive, lightweight material capable of shielding environments from radioactive fallout. Traditional radiation shields are heavy, unwieldy, expensive, not widely available and from non-sustainable sources. This part of the research aim to:

Further explore the viable application of composite material combinations, as well as, FRP in terms of radiation shielding and

Advance the knowledge base of the mechanical properties of FRP and composite materials.

The definition of the concept of sustainability as described in the Common Future, states that sustainable development is achieved "…through balanced exploitation of resources, redirection of investments, reorientation of technological development, and institutional change in order to address present needs while preserving future ability to do likewise [4]". Besides having mechanically important properties and characteristics that distinguish them from conventional materials such as wood, steel, and concrete; FRP and composite products combined with metals are also recognized as sustainable due to their durability and recyclability. Among other useful properties, FRP and composites are lightweight, non-magnetic, corrosion resistant and durable [5]. Lightweight composites have been shown to be an effective space radiation shielding material [6]. However, additional exploration of the failure modes is necessary to utilize this material to its fullest potential.

While working on this research the physical and mechanical properties of FRP and composite products were tested. Radiation shielding is based on the principle of attenuation, which is the ability to reduce a wave's or ray's effect by blocking or bouncing particles through a barrier material. Charged particles may be attenuated by losing energy to reactions with electrons in the barrier, while x-ray and gamma radiation are attenuated through photoemission, scattering, or pair production. Neutrons can be made less harmful through a combination of elastic and inelastic scattering, and most neutron barriers are constructed with materials that encourage these processes. Two of the main types of radiation encountered in industrial projects include:

Gamma and X-rays: These are forms of electromagnetic radiation that occur with higher energy levels than those displayed by ultraviolet or visible light.

Neutrons: Neutrons are particles that have neither a positive nor a negative charge, and thus provide a wide range of energy and mass levels that must be blocked.

Medical accelerators are more compact and generate electron beams with energies in the Mega electron Volt (MeV) range. Table 1 indicates the readings taken during the initial test for radiation tolerances performed at the local oncology hospital.

Table 1. Energy applied to material in Mega electron Volt (MeV) range and photons.

Energy	Open/No Material		Kevlar	Kevlar/Carbon 50/50 Fiber
6x	47.7	47.7	47.75	47.68
15x	57.36	57.33	57.45	57.44
9e	36.58	36.51	36.24	36.17
20e	45.15	45.16	45.24	45.28

Energy	Open/No Material		Carbon Fiber Epoxy FRP	Fiberglass FRP Sandwich
6x	47.7	47.7	46.27	45.99
15x	57.36	57.33	56.13	56.03
9e	36.58	36.51	7.23	4.72
20e	45.15	45.16	46.62	46.77

Since no previous radiation data existed on composite materials, testing was limited to an open-face (no material) and placement of specific composite materials. The accelerator tube was first calibrated and then programmed to exert different amounts of energy to the electrons and therefore different final energies for the beam. First, the researchers used the linear accelerator that offered two different electrons (6 and 15 MeV) and two photons (9 and 20 MeV) energies. The immediate results identified that FRP composite materials and various plastics might be non-magnetic, corrosion resistant, and durable, making them viable and sustainable space materials, but were lacking the radiation blocking property.

This emerging technology was focused in areas where health monitoring and characterization of materials could be detrimental to health. For that reason, additional developments needed to enable economical, full mechanical integrity characterization of these materials.

III. Retrofitting and Flexural Rigidity

The second research presented the outcomes of the study testing for flexural rigidity when retrofitting of FRP is imperative during space missions. The experiments consisted of four FRP plates 24 by 48 by 1/4 (thin) inches of Pultex® 1625 series were custom cut from the same piece of material [7]:

- FRP Pultex® 1625 series with no additional composite material
- FRP plate epoxied with one layer of Kevlar® 49 (Aramid),
- FRP with one layer of Carbon Fiber (Harness-Satin H5), and
- FRP with two layers perpendicular one to another of Unidirectional Carbon Fiber (T700 Aerospace Grade).

The first FRP plate was the control plate, meaning that this piece will be used as set of reference data to compare with the data or specimens being tested. The second FRP plate was epoxied with a single layer of composite fabric, called Kevlar® 49 (Aramid). The third FRP plate was epoxied with a single layer of composite fabric which is 100% carbon fiber (Harness-Satin H5). The fourth FRP plate was epoxied with Super High Quality, T-700 Aerospace grade fiber fabric. Kevlar® is the trade name for a polymer known as polyarylamide [8]. It has certain stiffening properties that could be beneficial if used correctly. Typically the material is spun into ropes or fabric sheets to be used in racing tires, bicycle tires, racing sails and body armor. When woven it becomes more suitable for mooring lines and other underwater applications. Since the fabrics initial design is only uniaxial and unidirectional, while the other fabrics are bi-axial, the final plate was epoxied with two layers perpendicular to each other. The strain gauges were installed in a predetermined position in the center of the plates; there was an initial concern that the strain gauges could develop unrealistic results due to the epoxied method of mounting and lamination method. This concern was disregarded upon completion of the experimental results. Three major questions needed to be answered for this project:

1) Can finite-difference method be used to accurately verify experimental results?
2) Is there a way to compare two different materials laminated together to show the overall stiffness increase or decrease with respect to one material?
3) Which of the three epoxied fabrics increases the stiffness of the total system the best?

Experimental Summary of Test Results

The blank FRP plate is being used as the base control test. The results are the average values from multiple tests. The FRP plate has a deflection-to-thickness ratio of 15.8%. As the different fabrics were tested the results show a lowered ratio signifying the increase in stiffness and a more capable retrofitting material. The FRP with Kevlar® coating had a ratio of 12.5%, the Carbon fiber had a ratio of 13.7% and the Aerospace Carbon fiber had a ratio of 9.2%. The Aerospace Carbon fiber increased the stiffness dramatically in comparison to the FRP. The Carbon fiber produced better results than Kevlar® experimentally.

Theoretical equations

Navier's Solution - Using Navier's solution for the maximum deflection at the center of the plate and assigning a factor α for deflection of a centrally loaded rectangular plate which is based on a ratio of the plate geometry. The plate ratio b/a is 2.0, which gives a α factor of 0.01651. The D value can be calculated using the experimental results and the equation below [9]:

$$w_{max} = \alpha \frac{Pa^2}{D} \quad (1)$$

In Equation 1, wmax is the max deflection at the center of the plate. P indicates the concentrated point load used, a is the short side of the rectangular plate. D represents the flexural rigidity term that shall be used to compare the stiffness's.

Equation 1 was used by taking the experimental deflection values and the known concentrated load value and substituting to solve for the flexural rigidity term.

Governing Differential Equation

The fundamental assumptions of the small deflection theory of bending or so called classical or customary theory for isotropic, homogeneous, elastic, thin plates is based on geometry of deformations. The above assumptions, known as the Kirchoff hypothesis, are analogous to these associated with the simple bending theory of beams. The flexural rigidity of the plate is given by [9]:

$$D = \frac{Et^3}{12(1-\upsilon^2)} \quad (2)$$

where D is the flexural rigidity, E is the modulus of Elasticity, h is the elastic thickness and υ is Poisson's ratio. The flexural rigidity *EI* or bending stiffness is the quality that describes a filament's resistance to bending forces, just as stiffness describes a filament's resistance to elongation.
The governing differential equation of plate bending is [9]:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p}{D} \quad (3)$$

in which w is the deflection of the plate, p is an evenly distributed load on the plate, and D is given by Equation 2.

Boundary Conditions

The edges of the plate in the x and y axis are assumed to be taken parallel to the sides of the plate. A simply supported edge has a deflection at that edge as zero. At that same time that edge can rotate freely with respect to the edge line, which means that there are no moments alone the edge. Referring to Figure 1, the analytical expression for the boundary condition are [11]:

$$w = 0, \quad M_x = \frac{\partial^2 w}{\partial x^2} = 0 \quad \text{from } x = 0 \text{ to } a$$

$$w = 0, \quad M_y = \frac{\partial^2 w}{\partial y^2} = 0 \quad \text{from } x = 0 \text{ to } b \qquad (4)$$

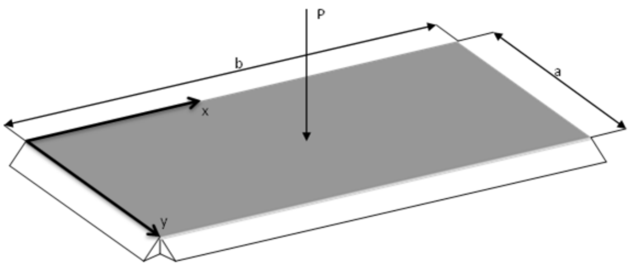

Fig. 1 - Rectangular Plate with Concentrated Load

Finite-Difference Formulation

The analysis of the problem is based on a finite-difference solution of Equation 3 in which the following finite-difference equations were substituted back into the plate deflection equation. Then using the experimental deflections and known concentrated loads, the flexural rigidity term can be calculated.

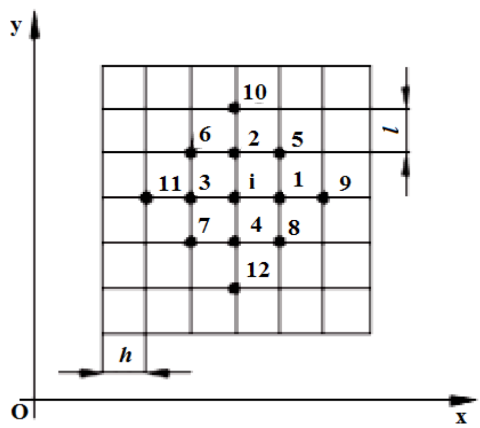

Fig. 2 - Generic Nodal Locations

Figure 2 depicts the generic nodal locations for the finite-difference method. At the node i with coordinates x and y, the first partial derivatives are given by [12]:

$$\left(\frac{\partial w}{\partial x}\right)_i = \frac{w_1 - w_3}{2h} \qquad (5)$$

$$\left(\frac{\partial w}{\partial y}\right)_i = \frac{w_4 - w_2}{2l} \qquad (6)$$

Similarly, the partial derivatives of the second, third and fourth orders are as follows [12]:

$$\left(\frac{\partial w^2}{\partial x^2}\right)_i = \frac{w_1 - 2w_i + w_3}{h^2} \qquad (7)$$

$$\left(\frac{\partial w^2}{\partial y^2}\right)_i = \frac{w_2 - 2w_i + w_4}{l^2} \qquad (8)$$

$$\left(\frac{\partial w^3}{\partial x^3}\right)_i = \frac{w_{10} - 2w_2 + 2w_4 - w_{12}}{2h^3} \qquad (9)$$

$$\left(\frac{\partial w^3}{\partial y^3}\right)_i = \frac{w_{10} - 2w_2 + 2w_4 - w_{12}}{2l^3} \qquad (10)$$

$$\left(\frac{\partial w^4}{\partial x^4}\right)_i = \frac{w_9 - 4w_1 + 6w_i + w_{11}}{h^4} \qquad (11)$$

$$\left(\frac{\partial f^4}{\partial y^4}\right)_i = \frac{w_{10} - 4w_2 + 6w_i + w_{12}}{l^4}$$

Similarly, [12]:

$$\left(\frac{\partial^2 w}{\partial x \partial y}\right)_i = \frac{1}{4h^2}\left(-w_5 + w_6 - w_7 + w_8\right) \qquad (12)$$

Substituting the above finite-difference equations into Equation 12, and setting l = h, results in [13]:

$$20w_i - 8(w_1 + w_2 + w_3 + w_4)$$

$$+2(w_5 + w_6 + w_7 + w_8) + w_9 + w_{10} + w_{11} + w_{12} = \frac{ph^4}{D} \qquad (13)$$

To convert this point load to a distributed load, the point load is assumed to be distributed 2.54-cm by 2.54-cm. area over the node. The concentrated load is divided by the area [12]:

$$p = \frac{P}{lh} \qquad (14)$$

Applying Equation 3 at each of the interior nodes of the plate, and incorporating the boundary conditions in the finite-difference form, a system of linear equations is first formulated in the following matrix form:

$$[K]\{\Delta\} = \{q\} \qquad (15)$$

where [K] is the coefficient matrix, {Δ} is the unknown deflection vector, and {q} is the load vector. In this study, a total of 288 interior nodes were used, thus creating 288 simultaneous linear equations.

Figure 3 shows the location of the nodes used in the finite-difference method. This is also a quarter of the actual plate. Node 1 starts at the top left corner and the numbers increase across the plate left to right to node 12, the next row starts node 13 all the way down to the center point of the plate at node 288. This is also where the location of the point load is at the center of the plate. The strain gauges are on top and bottom, which are offset from the center four inches length wise and two inches width wise at node 238.

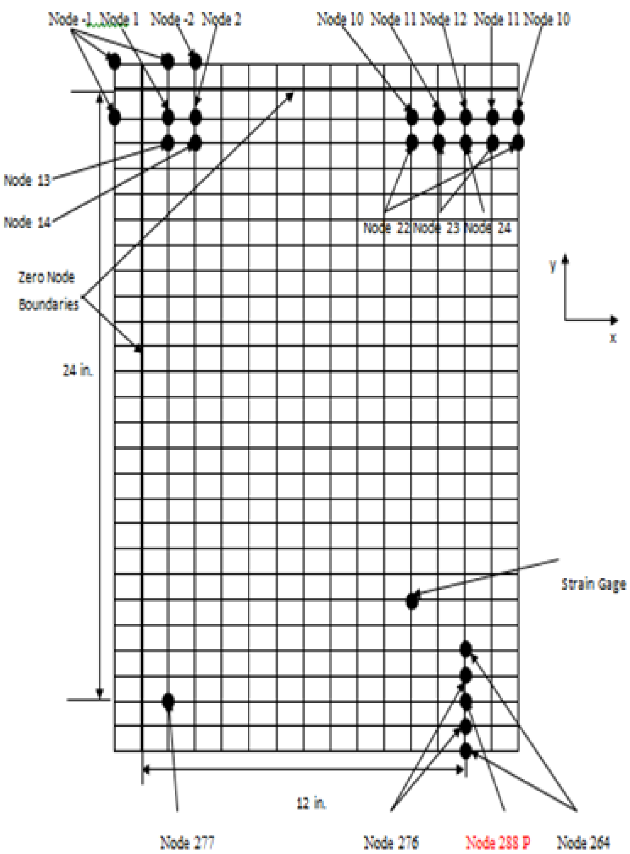

Fig 3. FRP – Deflection Nodal Diagram

This is the nodal location system that was used in the finite-difference Matlab® program. Since the edges are simply supported there is a zero node around all of the edges. Also, a phantom point that extends beyond the plate is used to accurately compute the deflections. This point, due to the plate being simply supported, is equal in magnitude and opposite in direction to the point adjacent to the real plate side.

Thickness Flexural Rigidity Ratio

Figure 4 indicates the cross section of a plate, "t" is the total thickness of the plate, h1 is the constant thickness of the FRP and h2 is the variable thickness of the multiple fabrics. It also allows for the experimental results to be used in the theoretical calculations and to compute the Young's Modulus values for each of the plates.

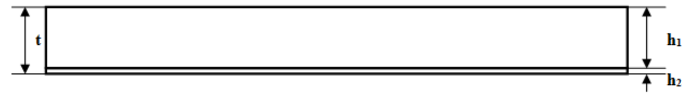

Fig 4. Cross Section Diagram of a Generic Plate

In equation 16, h1 is the constant thickness of the FRP plate which was 0.25 inches. The Do term is the flexural rigidity of the FRP plate with no fabric epoxied. D is the flexural rigidity term from a FRP plate and a fabric combination. This ratio of flexural rigidities allows for a ratio of the stiffness's to correlate into the fabric thickness h2.

Problem solution

Table 2 summarizes the plate flexural rigidity (D), equivalent thickness (h), and Young's modulus (E) values for the baseline FRP plate as well as the retrofitted plates. In this table, the first column identifies the plate type. The second column list the D values calculated using the finite-difference method combined with experimental load-deflection relations. The third column presents the h values computed from Equation 2 using the experimental D values. The last column presents the respective E values also calculated by using Equation 2.

Table 2 - Plate Flexural Rigidity and Young's modulus

Plate Type	Calculated Properties		
	Flexural Rigidity (D) (MPa-cm^3)	t (cm)	Young's Modulus (E MPa)
FRP only	295.0	0.635	1.24x 10^4
FRP Retrofitted with Kevlar®	351.0	0.653	1.36x 10^4
FRP Retrofitted with Carbon Fiber	338.0	0.670	1.21x 10^4
FRP Retrofitted with Aerospace Grade Carbon Fiber	488.0	0.753	1.23x 10^4

Figure 5, presents a comparison of the experimental and theoretical load vs. deflection relations. The theoretical values of the slope of the load-deflection curve from this figure are 788.4 N/cm, 545.7 N/cm, and 567.6 N/cm, respectively, for the FRP plate with Aerospace Grade Carbon Fiber fabric, Carbon Fiber fabric, and Kevlar. The corresponding value for the baseline plate FRP plate is 476.5 N/cm.

The FRP plate has a maximum deflection-to-thickness ratio of 0.158. The FRP plate with Kevlar® fabric has a ratio of 0.125; the one with Carbon fiber fabric a ratio of 0.137; and that with the Aerospace Carbon fiber fabric a ratio of 0.092. The Aerospace Carbon fiber fabric increased the stiffness dramatically in comparison to the FRP. The Carbon fiber produced better results than Kevlar® experimentally.

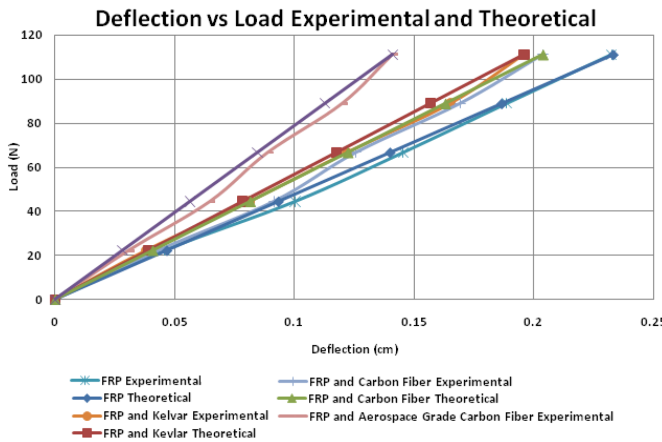

Fig. 5. Deflection vs. Load comparing Experimental and Theoretical Results

Remarks

The findings of this research are summarized as follows:

- The epoxied fabrics increased the stiffness of the plates.
- The Aerospace Grade Carbon fiber was the best retrofitting material to reduce the deflections and stresses.
- After calculating the flexural rigidity values with the finite-difference method and taking the ratio between the control FRP plate and the plate coated with Kevlar®, the stiffness of the plate coated with Kevlar®, increased by 19 percent, the Carbon fiber plate stiffness increased by 15 percent and the Aerospace Grade Carbon fiber stiffness increased by 65 percent.
- Comparing overall deflections, the Aerospace Grade Carbon fiber reduced the deflection the most, the Kevlar® was the next best and the Carbon fiber was the fabric that provided the least increase in stiffness.
- The plate flexural rigidity values were validated with a direct comparison to Navier's flexural rigidity solution.
- The finite-difference method together with experimental load-deflection relations can be used to accurately compute the plate flexural rigidity.

IV. UNCERTAINTY ASSESSMENT

In conceptual design of one-of-a-kind systems such as selecting the most appropriate composite material for space missions, quantifying operational uncertainty and performing risk analysis is a challenging task mainly due to lack of data. Asking disciplinary experts for their "best expert judgment" may sometimes be the only option available. Expert judgment (EJ) methodologies were utilized in prior studies for quantifying input parameter uncertainty as probability distributions so that probabilistic risk analysis studies can be conducted [14]. Data obtained utilizing EJ can in many cases provide a basis for analysis and interpretation of significance of risk [1]. Through the use of EJ, prior studies introduced an approach to quantify critical system design parameter uncertainty as probability distributions [15]. However, there is significant uncertainty in these judgments themselves and a probabilistic assessment alone may not be sufficient [34].

During this part of the study a combined probabilistic and non-probabilistic approach for uncertainty assessment in conceptual design was explored. The extension of the efforts to define the development of a more robust approach for uncertainty assessment is explored through evidence theory [15-28]. Evidence theory provides a promising addition to current probabilistic uncertainty assessment practices, and the combination of the approaches may allow for a more realistic representation of the implications of uncertainty given the complex nature of real world problems.

Figure 6 illustrates comparisons of the origination and development of the two theories. The ultimate goal was to assess the level of uncertainty using expert judgment elicitation and the combined methods of probabilistic and non-probabilistic approach.

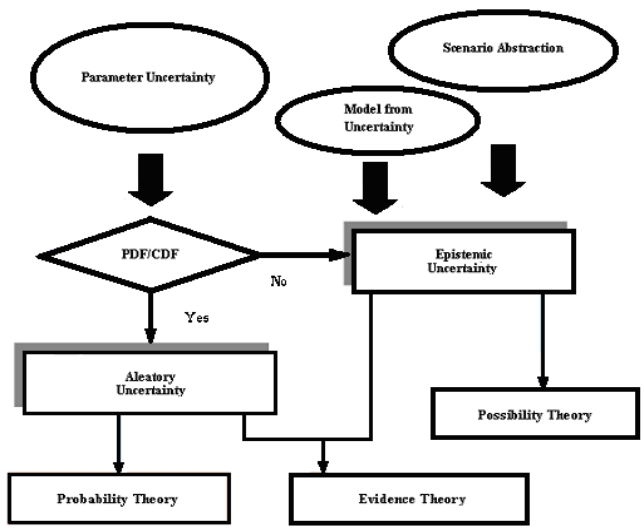

Fig. 6 - Uncertainty quantification strategy [22]

Results of uncertainty assessment

A design study for material was selected to incorporate an uncertainty assessment using expert judgment elicitation through a combined probabilistic and non-probabilistic evidence theory approach [34]. Three variables were chosen that would lead to a critical subsystem failure of the composite material during its lifecycle. It was thought that critical subsystem failures may be a function of Construction, Installation and Operations. These failures were:

A. Construction anomalies that can occur production.

B. Installation anomalies that may lead to the possibility of radiation penetration into the cabin.

C. Operations anomalies that may result from debris damage or impact.

D. All combinations of the three anomalies.

A combined probabilistic and evidence approach was utilized in an effort to enhance uncertainty assessments during the selection of critical material that will be used in space missions. Uncertainty estimates obtained from a panel of experts were presented bound by belief and plausibility functions as well as probability distributions. The results suggest that this combined probabilistic and evidence approach may provide additional information to the decision maker in critical system safety and uncertainty assessments.

To elicit inputs, a questionnaire was utilized for uncertainty assessments for using composite materials in a space mission that can lead to a critical system failure. The resulting data was utilized to conduct a probabilistic and evidence theory based analysis. Using a graphical approach this study provided various visual representations of the experts' uncertainty assessments. The methodology demonstrated in this study enabled the capturing of expert confidence in uncertainty assessments. A probabilistic analysis alone may lead to conclusions that may be misleading without further investigation, while the Evidence approach does not provide a concrete non-probabilistic assessment; rather it provides an enhancement of probabilistic analysis. Figure 7 shows the process that was followed in analyzing the material.

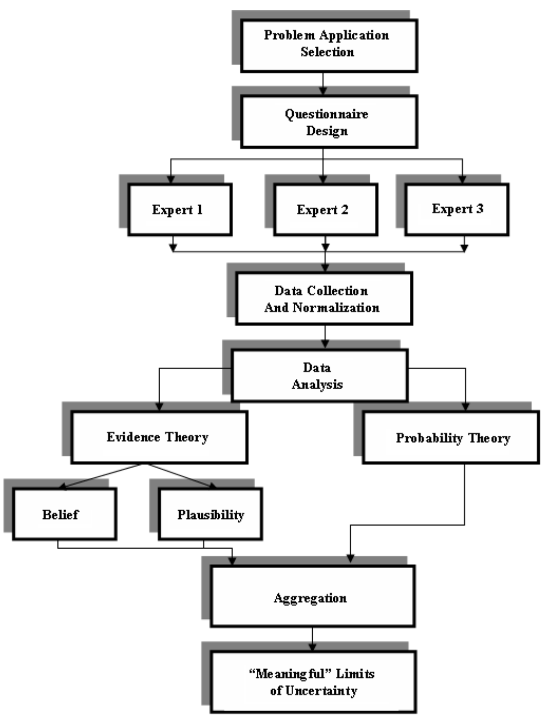

Fig. 7. Combined approach for uncertainty assessment

Probabilistic Approach

Probability theory provides a mathematical structure traditionally used in the representation of aleatory (i.e., random) uncertainty for well-known systems. Aleatory uncertainties are typically modeled as random variables described by probability distributions. A probability in this case refers to the number of times an event occurs divided by the total number of trials. For instance, the flipping of a truly fair coin would have a probability of landing on heads of 0.5, indicating that for every N trials, the coin would land heads up, 0.5*N times. In order to attain the actual probability for an event, an experiment would have to be repeated an infinite number of times.

Since this is impossible, decision makers typically make assumptions about the characteristics of the probabilities (i.e. the mean and variances). Given the lack of operational data in conceptual design for one-of-a-kind systems, one may have to rely on expert judgment data obtained by a probability elicitation method to quantify CDFs in representing uncertainty. The use of expert judgment or opinion to aid in decision-making is well known [34].

Based on Baenen, Bayesian belief networks are rooted in traditional subjective probability theory, which builds on the foundation of Pascalian calculus. In subjective probability theory, the probability of a proposition represents the degree of confidence an individual has about that proposition's truth. This matches quite well to our knowledge base of information from a human expert in addition to his or her subjective beliefs about the accuracy of that information [29]. Before Bayesian belief networks are described, we must begin with the fundamentals of probability theory. Let A be some event within the context of all possible events E, within some domain, such that A $\in$ E and E is the event space.

The probability of *A* occurring is denoted by P(*A*). P(*A*) is the probability assigned to *A* prior to the observation of any evidence and is also called the *apriori* probability. This probability must conform to certain laws [29]. First, the probability must be non-negative and must also be less than one; therefore,

$$\forall A \in E, \ 0 \leq P(A) \leq 1 \qquad (1)$$

A probability of 0 means the event will not occur while a probability of 1 means the event will always occur. Second, the total probability of the event space is 1 or in other words the sum of the probabilities of all of the events A_i in *E* must equal 1[29].

$$\forall A \in E, \sum A_i = 1 \qquad (2)$$

Finally, we consider the compliment of *A*, $\neg A$, which is all events in *E* except for *A*.

From equation (2) we then get:

$$P(A) + P(\neg A) = 1 \qquad (3)$$

Now consider another event in *E*, *B* such that *E* 0 *B*. The probability that event *A* will occur given that event *B* has occurred is called the conditional probability of *A* given *B* and is represented by P(*A* | *B*) [29]. The probability that both *A* and *B* will occur is called the joint probability and is defined by P(*A* | *B*) . P(*A* | *B*) is defined in terms of the joint probability of *A* and *B* by:

$$P(A|B) = \frac{P(A \cap B)}{P(A|B)} \qquad (4)$$

Equation (4) can be further manipulated to yield Bayes Rule:

$$P(A|B) = \frac{P(B|A) \ X \ P(A)}{P(B)} \qquad (5)$$

If these two events are independent, in that the occurrence of one event has no effect on the occurrence of the other, then

P(*A* | *B*) = P(*A*) and P(*B* | *A*) = P(*B*) [29]. If we derive equation 5 still further we get:

$$P(A|B) = \frac{P(B|A)\ X\ P(A)}{[P(B|A)\ X\ P(A)] + [P(B|A)\ X\ P(\neg A)]} \quad (6)$$

This lays the foundation for managing and deriving uncertainty using probability theory in expert systems. It allows us to turn a rule around and calculate the conditional probability of *A* given *B* from the conditional probability of *B* given *A*. Some of the advantages of Bayesian belief networks are that the representation is visual and easy to understand. It is also relatively straightforward to implement as the methodology for combining uncertainty follows set rules and procedures. Probability theory is a well-refined method for dealing with knowledge of unknown certainty [30].

The CDF describes the probability distribution of a random variable X. For every real number x, the distribution function of X is defined by:

$$F(x) = P(X \leq x) \quad (7)$$

where the right of x represents the probability that X takes on a value *less* than or equal to x and the left of x represents the probability that X takes on a value *greater* than x. The probability that X lies in the interval [a, b] is, therefore, F(b) − F(a) if a < b [31].

In this research, the analysis of how often the random variable is above a particular level. This is referred to "the exceedance question" and is necessary for the correlation with Evidence theory [32]. This graphical analysis called the complementary cumulative distribution function (CCDF), which can be defined by:

$$F_c(x) = P(X > x) = 1 - F(x) \quad (8)$$

The knowledge of subject matter experts (SMEs) has been "mined" in many disciplines (such as medicine, weather forecasting, and military tactics) to provide estimates for parameters associated with yet-to-be-developed systems [1][15].

A probability elicitation method may be any aid that is used to acquire a probability from an expert [31]. Generally, a distinction is made between direct and indirect methods. With direct methods, experts are asked to directly express their degree of belief as a number, be it a probability, a frequency or an odds ratio. For expressing probabilities, however, people prefer to express their beliefs linguistically rather than numerically. This is likely because the ambiguity of words captures the uncertainty they feel about their probability assessment; the use of numerical probabilities can produce considerable discomfort and resistance among those not used to it [33]. In addition, since directly assessed numbers tend to be biased, various indirect elicitation methods have been developed to quantify parameters of a CDF for uncertainty [23].

CCDF curve is typically obtained by sampling based techniques and are, therefore, approximate. "These distributions mathematically describe a degree of belief, based on all of the available evidence (e.g., data, background knowledge, analysis, experiments, expert judgment), of the range and weight, in terms of likelihood, of the input values used in the analysis", shown in Figure 8 [32]. The complementary nature of the CCDF results in the right of x representing the probability that X takes on a value *greater* than or equal to x and the left of x representing the probability that X takes on a value *less* than x.

However, probabilistic approaches to uncertainty assessment have been criticized for lacking the capability of capturing epistemic uncertainty [24]. Klir notes that as a consequence of this criticism, supporting theories have been developed and categorized into the "fuzzy measure theory" [25]. One such approach, evidence theory, takes into account aleatory and epistemic uncertainty that is bounded by the belief and plausibility functions [Bel(Ai) , Pl(Aj)] and is found without any assumptions made on the information obtained from the experts [25]. Evidence theory is discussed in further detail in the following section.

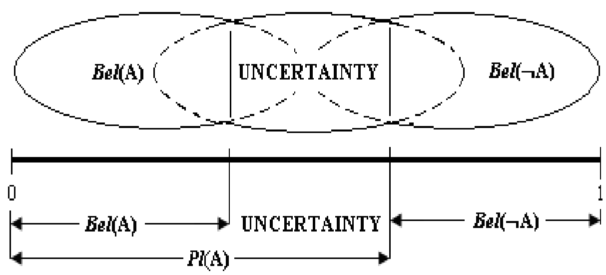

Fig. 8. Belief (Bel) and plausibility (PL) relationship

Evidence Theory

Evidence theory originated with Arthur Dempster in the 1960's and was expanded by Glen Shafer in the 1970'[16][17][18]. In evidence theory, uncertainty is separated in Belief (Bel) and Plausibility (Pl), whereas traditional probability theory uses only the probability of an event to analyze uncertainty [23]. Belief and plausibility provide bounds on probability. In special cases they converge on a single value, probability. In other cases, such as in the evidence theory representation of uncertainty, they represent a range of potential values for a given parameter without specifying that any value within the range is more or less likely than any other. The Dempster-Shafer evidence theory has three important functions; the basic probability assignment function (BPA or m), the Belief function (Bel), and the Plausibility function (Pl) [23]. The three functions can be viewed as alternate representations of uncertainty regarding the same parameter x [23].

The value of the BPA for a given set A (represented as m(A)), expresses the proportion of all relevant and available evidence that supports the claim that a particular element of X (the universal set) belongs to the set A but to no particular subset of A [16][17][18][19]. From the basic probability assignment the upper and lower bounds of an interval can be defined [19]. This interval contains the precise probability of a set of interest (in the classical sense) and is bounded by two non additive continuous measures called Belief and Plausibility. In addition to deriving these measures from the basic probability assignment (m), these two measures can be derived from each other. For example, Plausibility can be derived from Belief in the following way:

$$Pl(A) = 1 - Bel(\bar{A}) \quad (1)$$

Where A is the classical complement of subset A [16][17][18].

The Dempster-Shafer's combination rule is the first of its kind and the foundation for the other rules. The combination of basic assignments from two sources of information can be defined as [31]:

$$m_{1,2}(A) = \frac{\sum_{\text{all } A_i \cap A_j = A} m_1(A_i)\, m_2(A_j)}{1 - \sum_{\text{all } A_i \cap A_k = \varnothing} m_1(A_i)\, m_2(A_j)} \qquad A = \varnothing \quad (2)$$

The combination of independent sources of information is the basis of this rule, and it is characterized by the product combination rule. Shaffer explains this in his own statements as "Mathematically, Dempster's rule is simply a rule for computing, from two or more belief functions over the same set Θ, a new belief function called their orthogonal sum. The burden of our theory is that this rule corresponds to the pooling of evidence: if the belief functions being combined are based on entirely distinct bodies of evidence and the set Θ discerns the relevant interaction between those bodies of evidence, then the orthogonal sum gives degrees of belief that are appropriate on the basis of the combined evidence" [13].

Dempster-Shafer [16][17][18] methods of Evidence Theory may be applied by identifying the upper limit of uncertainty called Cumulative Plausibility Function (CPF) and lower limit of uncertainty called Cumulative Belief Function (CBF).

According to Belief and Plausibility Functions, the likelihood for Event A lies in the interval [Bel(A), Pl(A)] as shown in Figure 9 [17].

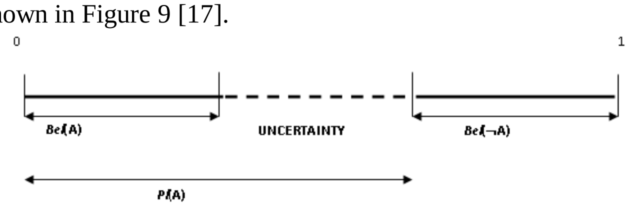

Fig. 9. Belief (Bel) and plausibility (Pl) relationship (Bae, 2003)

Dempster-Shaffer methods of Evidence Theory is applied by identifying the upper limit of uncertainty called Cumulative Plausibility Function (CPF) and lower limit of uncertainty called Cumulative Belief Function (CBF). Figure 10 is the graphical representation of the CPF and CBF.

The literature seems to be in concurrence that the use of Evidence theory is not fully developed and is yet to have widespread applications in the engineering field. Probability theory was utilized to address the probability of the occurrence of a critical system failure, while Evidence theory is used to addresses the degree of uncertainty of the results.

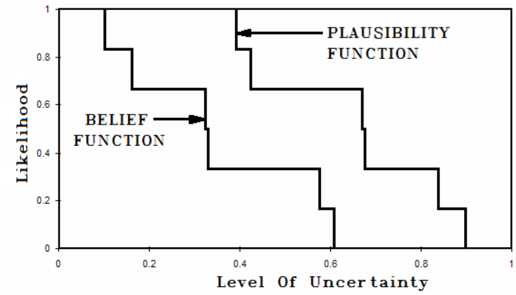

Fig. 10. Graphical representation of CPF and CBF

The results suggest that the assessment of uncertainty of experts in high-risk environments may be better conveyed to decision makers by using both probabilistic and non-probabilistic theories.

- Retrofitting and flexural rigidity
- FRP and Composite Material as Radiation Shield

This time a combined probabilistic and evidence approach was utilized. This research provided more exploration into the failure modes necessary to utilize FRP and composites to their fullest potential in an effort to enhance uncertainty assessments in critical safety assessments for composite materials during conceptual design. Uncertainty estimates obtained from a panel of experts were presented bounded by belief and plausibility functions as well as probability distributions. The results suggest that this combined probabilistic and evidence approach may provide additional information to the decision maker in critical system safety and uncertainty assessments.

V. Conclusion

The above mentioned examples of research assisted several students to have a better understanding of engineering research and applied problems in composite materials. These projects promoted the advancement of engineering knowledge, both by its creation and dissemination, and provided successful graduate students. These studies also contributed to "a continuously improving learning environment for its constituents while maintaining ethical, multicultural and global standards." [35]

The different mathematical models used further expanded the FRP and composites knowledge base by identifying material strengths and weaknesses through conducting experimental versus theoretical studies. The proposed methods synthesized the study of the emerging new materials with a probability-based approach. Such an approach is considered a pre-cursor to the so-called Load and Resistance Factor Design (LRFD) philosophy based on which a currently evolving FRP design specification will be based and will subsequently become Standard of Practices and Procedures that could provide promising tools to implement various mathematical models during conceptual design and selection of appropriate composite material.

The work performed investigated various mathematical, physical and mechanical interactive properties of materials for development of risk and reliability-based problems related to composite materials. It was part of a development of new techniques and guidelines for efficient, effective and widely applicable methodologies that enhanced real-world applications of complex radiation shielding. Elements of the project enhanced current knowledge in the field, resulting in incorporation of findings into materials courses. The research findings will develop well informed students in the technical aspect of education and training of engineering technology.

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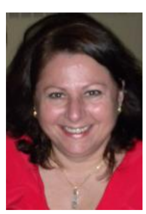
BIOGRAPHIES



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