Numerical Analysing the Slabs by Means of the Finite Difference Method and the Finite Element Method

O. Sucharda, J. Kubosek

***Abstract*—** The article is dealing with computation and optimization of computation of inner forces and deformations of slabs. Finite element method was chosen. Algorithm is written under Matlab. Variant solution of the system of equations is used for optimization. Algorithms were made for chosen numerical methods and functions of Matlab. Comparison is made afterwards. Created algorithms are illustrated using chosen square slab as example.

***Keywords*—** Finite element method, finite difference method, slab, algorithms, speed calculation, deflection.

I. Introduction

THE paper deals with the analysis of concrete structures. Much attention has been paid to slab analyses in past to experimental [1] and theoretical research [2, 3]. Slab made from concrete are often used.

Designs of concrete structures have been dealt with in recommendations [4]. This task is closely connected with non-linear analyses which have been used by the authors for 2D tasks [5, 6]. Relations and theories for analysing internal forces of slabs are listed in [7, 8]. The most frequent methods used for the slabs are the Finite Element Method [9, 10] and Finite Difference Method [11, 12]. The Finite Element Method is used in [13, 14, 15]. When analysing the slabs by means of the Finite Difference Method, orthotropic properties can be also taken into account [16]. For some tasks the Finite Difference Method was used also for the non-linear analysis [17, 18].

The goal is to develop an algorithm by means of the finite difference method (which is also referred to as the network method) and compare results with those obtained by the finite element method. Deflections, specific bending moments and torsion moments were chosen for the analysis. The program Matlab [19] was used to develop an algorithm and the program Scia Engineering 2012 [20] was used to the verify results. One of other possible approaches used for development of algorithms for such tasks is described in [21]. Another goal of this paper was to compare the methods used for calculation of the system of linear equations [22, 23, 24] depending on the density of the network.

Two directs methods and two methods integrated in the program Matlab [19] were chosen for purposes of such comparison. More details about algorithm development are described in [25].

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II. Finite difference method

Thin slabs can by analysed using the finite difference method. For theory and more details relating to this method see [7, 24]. The algorithm assumes that the structure fulfils conditions specified in Kirchhoff theory [11]. The unknown variable in the slab is w deflection which can be obtained by solving the slab equation:

$$\frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial x^2 y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p}{D} \tag{1}$$

where w is deflection of the slab, p is magnitude of the load, and D is slab rigidity. The basic principle of the finite difference method is that the structure is divided into nodes and the differential equation (1) of the slab is modified by means of difference replacement to give a system of linear equations for node values of the deflection. The system of equations was created using a difference scheme which defines environment for each point in the network, creating thus a system of linear equations. The calculation used a reduced deflection which depends on real deflection and the formula is as follows:

$$w = \frac{D}{a \cdot b} w \tag{2}$$

where a and b are distances between network nodes (the difference step), D is slab rigidity and w is real vertical dislocation (deflection) of a point in the network. When using the reduced deflection, the right side of the difference equations equals, for each point, to:

$$P = p \cdot a \cdot b + F \quad (3)$$

where F is the value of a single load applied in a node point. Because the difference scheme requires 13 points in the surrounding and such points may exceed beyond edges of the slab, boundary conditions should be used to define the outer points.

III. Finite Element method

Finite element method is numerical method. This method is used for analysis of inner forces, deformations and heat conduction in fields of civil engineering, mechanical engineering and electrotechnics. Method is considered as energy balance method and is based on discretization of structural model to small parts. These parts are called finite elements. Approximate solution of unknown values is searched within finite elements. This method leads to solution of the system of equations.

$$K \cdot u = F\,, \quad (4)$$

Where K is stiffnes matrix, u is vector of deformations and F is vector of loads. Chosen version of finite element method is based on Lagrangian variational principle. Its Ritz method. Used slab finite element is in the rectangle shape. Aproximation function of defflection is:

$$\begin{gathered} w = a_1 + a_2 x + a_3 y + a_4 x^2 + a_5 xy + a_6 y^2 + \\ + a_7 x^3 + a_8 x^2 y + a_9 xy^2 + a_{10} y^3 + a_{11} x^3 y + a_{12} xy^3 \end{gathered}. \quad (5)$$

IV. Numerical example for the Finite Difference Method and the Finite Element Method

The concrete slab, which is analysed by means of the finite difference method is simply supported along its perimeter and loaded with the even load p = 10 kNm^{-2}. Dimensions of the slab are b = 5 m and l = 5 m. The thickness is h = 180 mm. The static modulus of elasticity is E = 30 GPa and Poisson coefficient is 0.2. The calculation is carried out for 25 internal points.

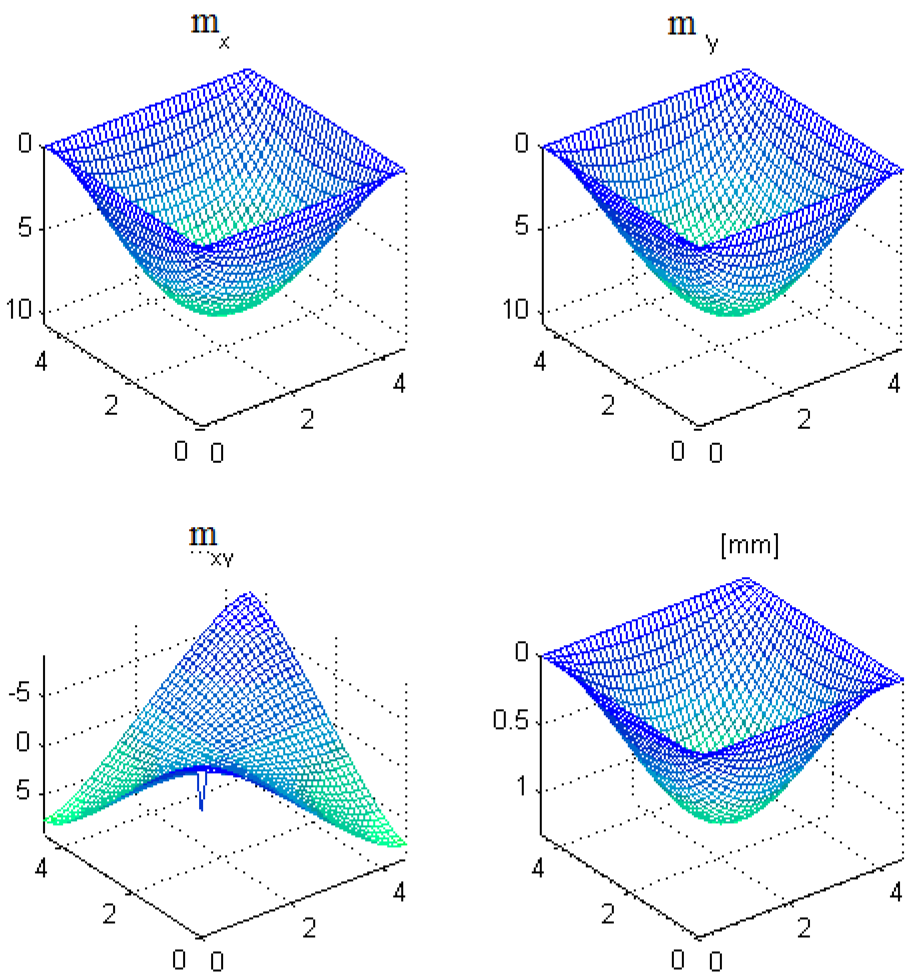

Fig. 1. Internal forces and deflections calculated using the finite difference method in Matlab [19]

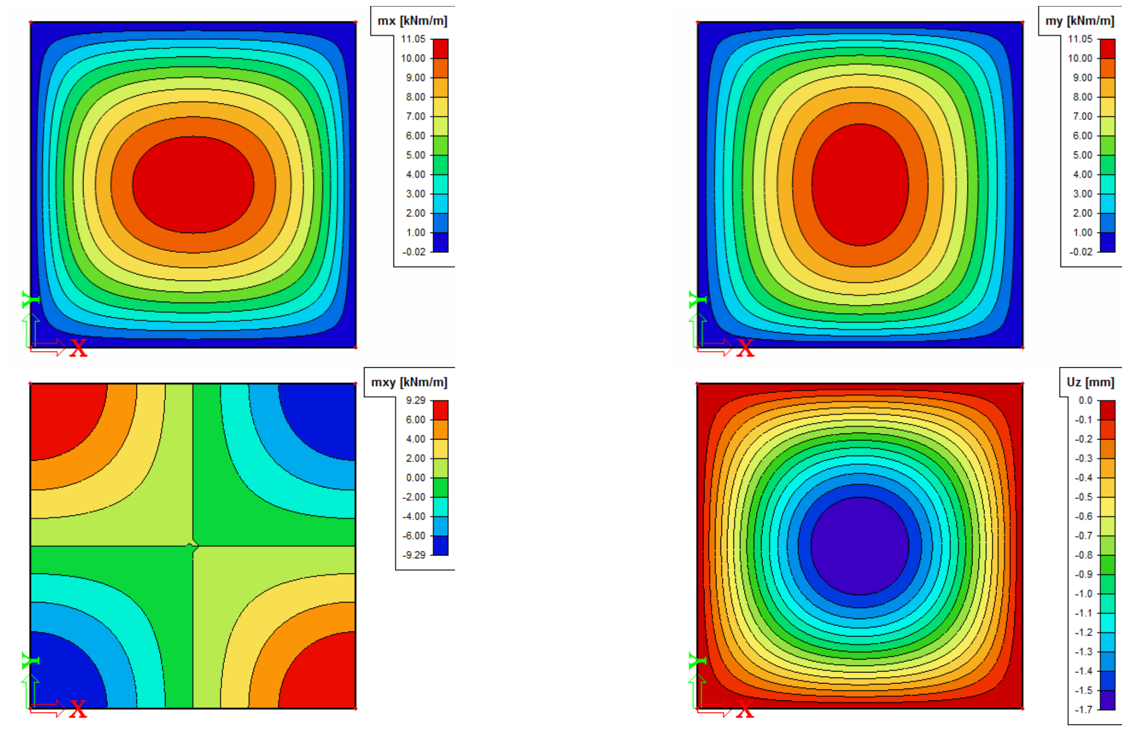

Fig. 2. Deflections and internal forces calculated in Scia Engineer [20]

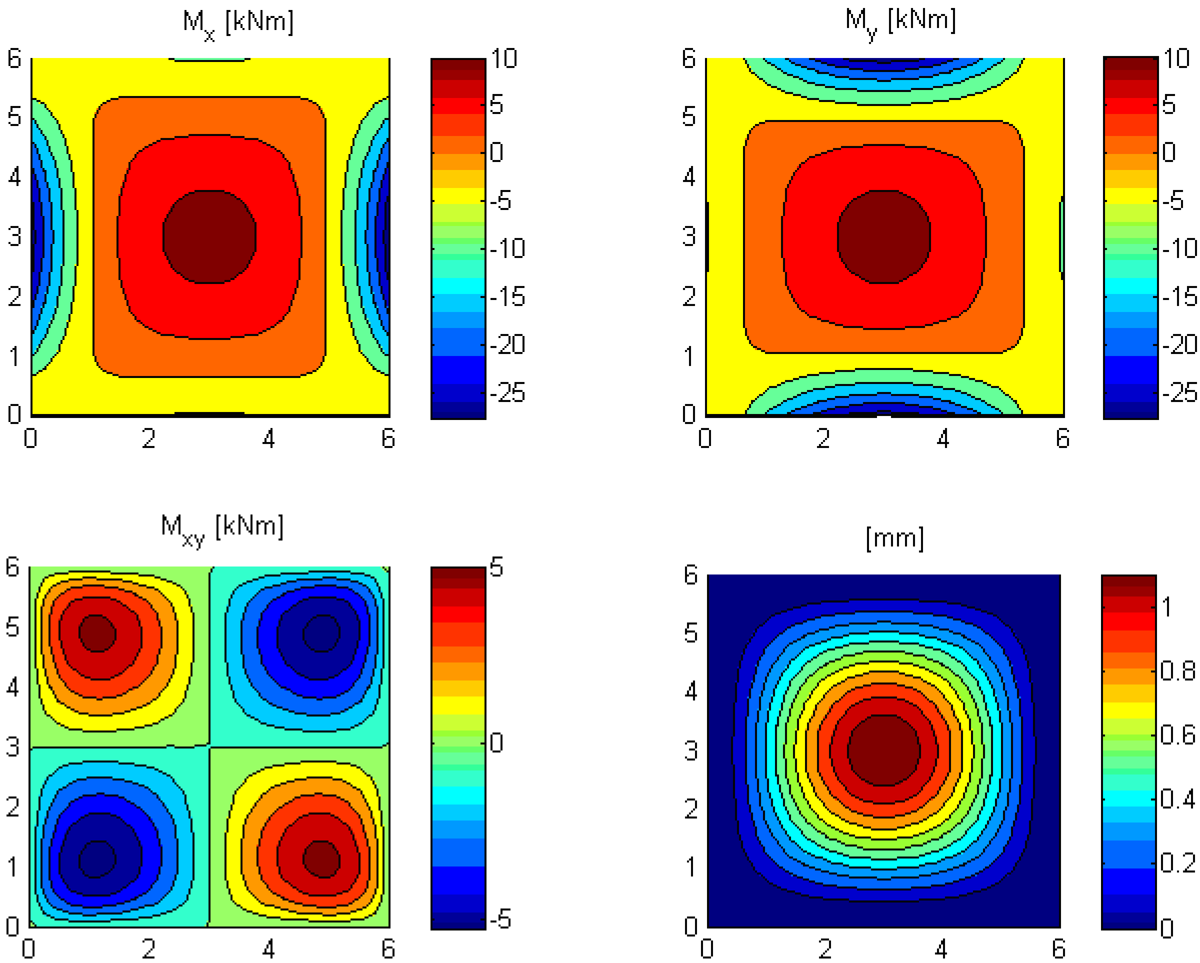

Fig. 3. Internal forces and deflections calculated using the finite difference method in Matlab [19]

The maximum calculated values were as follows: deflection w = 1.6715 mm, bending moments $m_x = m_y$ = 11.038 kNm/m and torsion moments m_{xy} = ±9.177 kNm/m. Results obtained in the program Matlab [19] were checked by calculations made in the program Scia Engineer [20]. The program Scia Engineer [20] is based on the finite element method [9].

The calculated values were slightly different: deflection w = 1.792 mm, bending moment's $m_x = m_y$ = 54.51 kNm/m and torsion moments m_{xy} = ±43.63 kNm/m. Fig. 1 shows 3D results obtained for the finite difference method. Fig. 2 shows results obtained in Scia Engineer [20].

V. Comparing the Methods Used for Calculation of the System of Linear Equations

The system of linear equations is solved using matrix calculation where the system can be briefly described as follows:

$$\boldsymbol{A} \cdot \boldsymbol{x} = \boldsymbol{b} \qquad (4)$$

where: A is the matrix of the system (the left side of the equations), b is the vector of the right sides and x is the vector of system roots which should be determined.

Four methods were used for purposes of comparison:

- direct methods: Gauss-Jordan method and LU decomposition method,
- Matlab functions: division ($x = b/A$) and inversion ($x = b^{-1} \cdot A$).

Variability of the network size and, in turn, the number of linear equations were also taken into account. The modified algorithm changes gradually the network size and measures the time needed for calculations. The basis for comparison was a slab which is fixed at all sides and loaded with the even load of 15 kNm^{-2}. Dimensions of the slab are $b = l = 6$ m. The thickness is h = 200 mm. The static modulus of elasticity is E = 30 GPa and Poisson coefficient is 0.2.

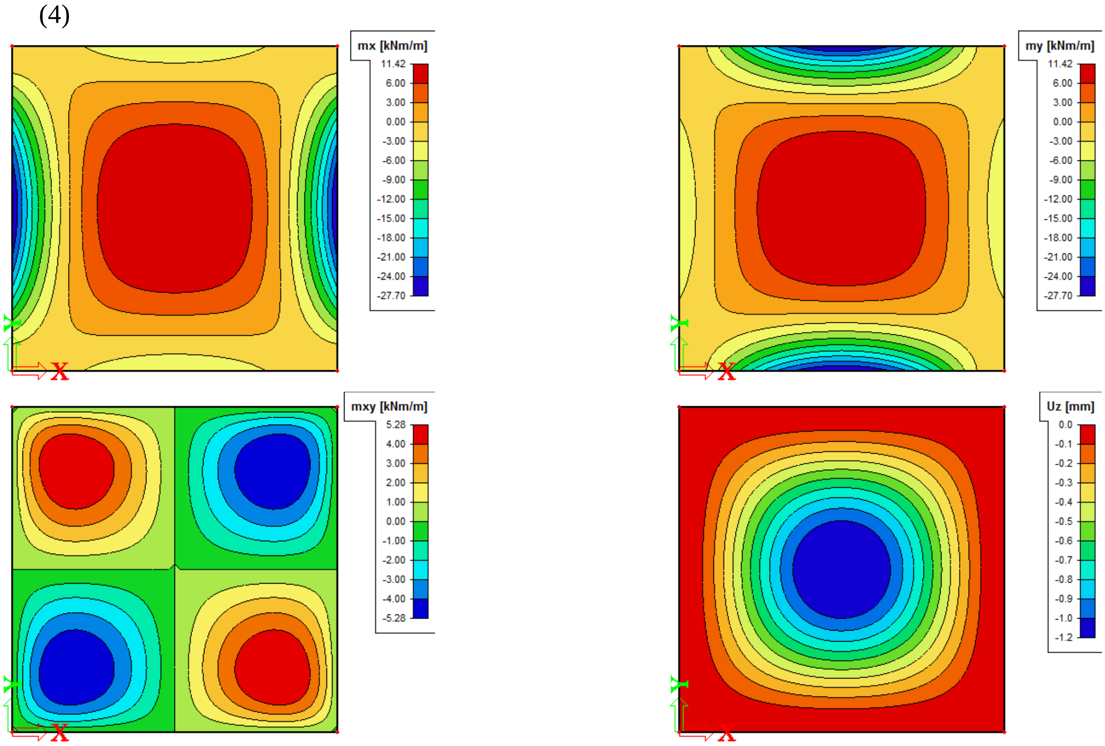

Fig. 4. Deflections and internal forces calculated in Scia Engineer [20]

The final deflection in this case (for 59 internal points where the distance between the points in the network is 100 mm) is w = 1.1835 mm. The final bending moments are $m_{x+} = m_{y+}$ = 11.423 kNm/m, $m_{x-} = m_{y-}$ = –27,680 kNm/m and the torsion moment is m_{xy} = ±5.269 kNm/m. Fig. 3 shows 2D results for the finite difference method, while Fig. 4 shows results obtained in the program Scia Engeneer [9] (w = 1.181 mm, $m_{x+} = m_{y+}$ = 11.416 kNm/m, $m_{x-} = m_{y-}$ = –27.698 kNm and m_{xy} = ±5.284 kNm/m). The graphic comparison in Fig. 6 was based on 5 calculation cycles where the density of network varied from 5 to 50 internal points. An average was made for the values from each cycle, and the average values were plotted in the chart. The maximum time was set at 60 seconds. In order to compare functions in the program Matlab [19] in detail, the function were compared for the network density from 50 to 100 internal points in Fig. 7. After the calculation is completed, the dump print in the command window in Fig. 5 is as follows (the first 16 cycles are shown there):

1. kolo

počet	dělení	inverze	LU rozklad	Gauss Jor.	
5	0.001154	0.105075	0.032429	0.038576	25
6	0.000057	0.007961	0.012424	0.002881	36
7	0.000078	0.000360	0.023237	0.006084	49
8	0.000191	0.000414	0.038634	0.012756	64
9	0.000289	0.024167	0.058457	0.027751	81
10	0.000382	0.006779	0.090045	0.045590	100
11	0.000436	0.001948	0.137238	0.081048	121
12	0.000633	0.001878	0.193176	0.139073	144
13	0.000792	0.002883	0.271684	0.225606	169
14	0.000997	0.004089	0.369483	0.351565	196
15	0.001469	0.005564	0.505367	0.527605	225
16	0.001542	0.014369	0.652290	0.813571	256
17	0.002259	0.011073	0.906777	1.139325	289
18	0.004909	0.008916	1.106082	1.607688	324
19	0.003441	0.012345	1.449597	2.287747	361
20	0.003716	0.016753	1.837868	3.089810	400

Fig. 5. Comparing duration of the methods depending on the size of the matrix (density of the network: 50 – 100 points)

If the maximum preset time is exceeded in the calculation (60 minutes, in this case), the method is not measured anymore. Average times from each cycle are plotted in the chart.

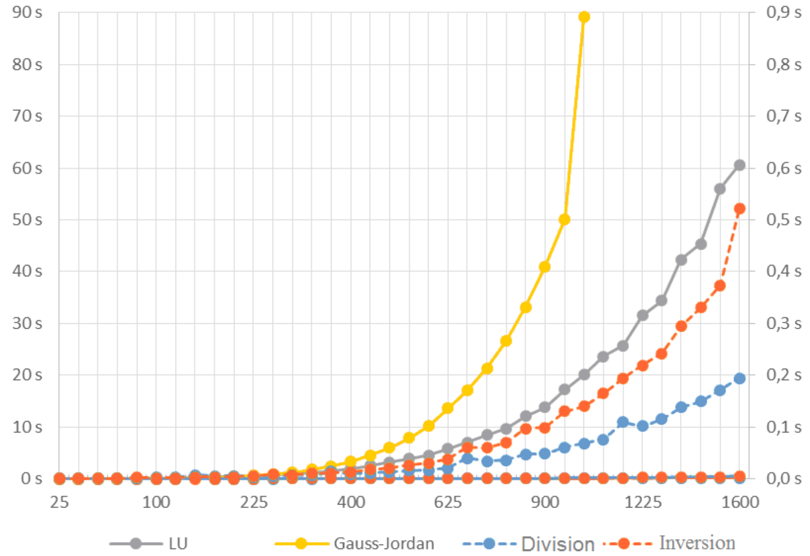

Fig. 6. Comparing the duration of each method versus the size of the matrix (the dashed connecting line belongs to the right vertical axis)

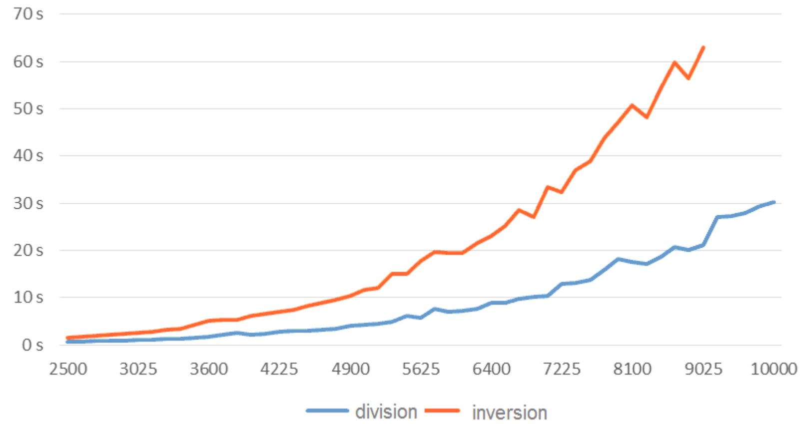

Fig. 7. Comparing duration of the methods depending on the size of the matrix (density of the network: 50 – 100 points)

VI. Numerical Example 1 for the Finite Element Method

Finite element method was used for solving example 1. Slab scheme is pictured in fig. 8, slab thickness 180 mm. Slab is loaded with uniformly distributed loading: 10 kN/m^2. Slab dimensions: 4.5 × 4.5m. Supports are shown in scheme. (two sides are fixed, one simply supported and one is without any support). Slab is made of concrete. Young´s modulus E = 30GPa and Poisson´s ratio ν = 0.2.

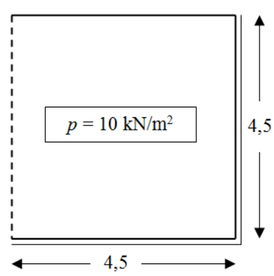

Fig. 8.Scheme slab

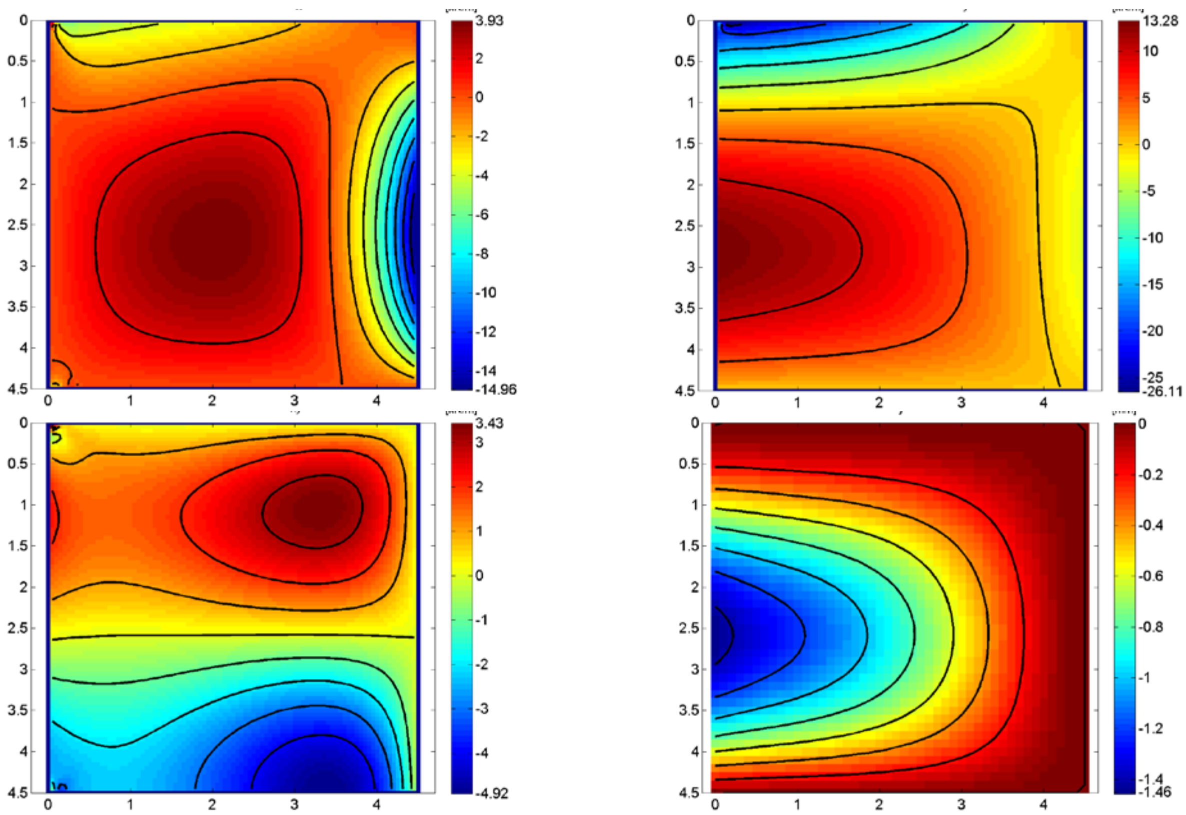

Fig. 9. Deflections and internal forces calculated in Matlab [19]

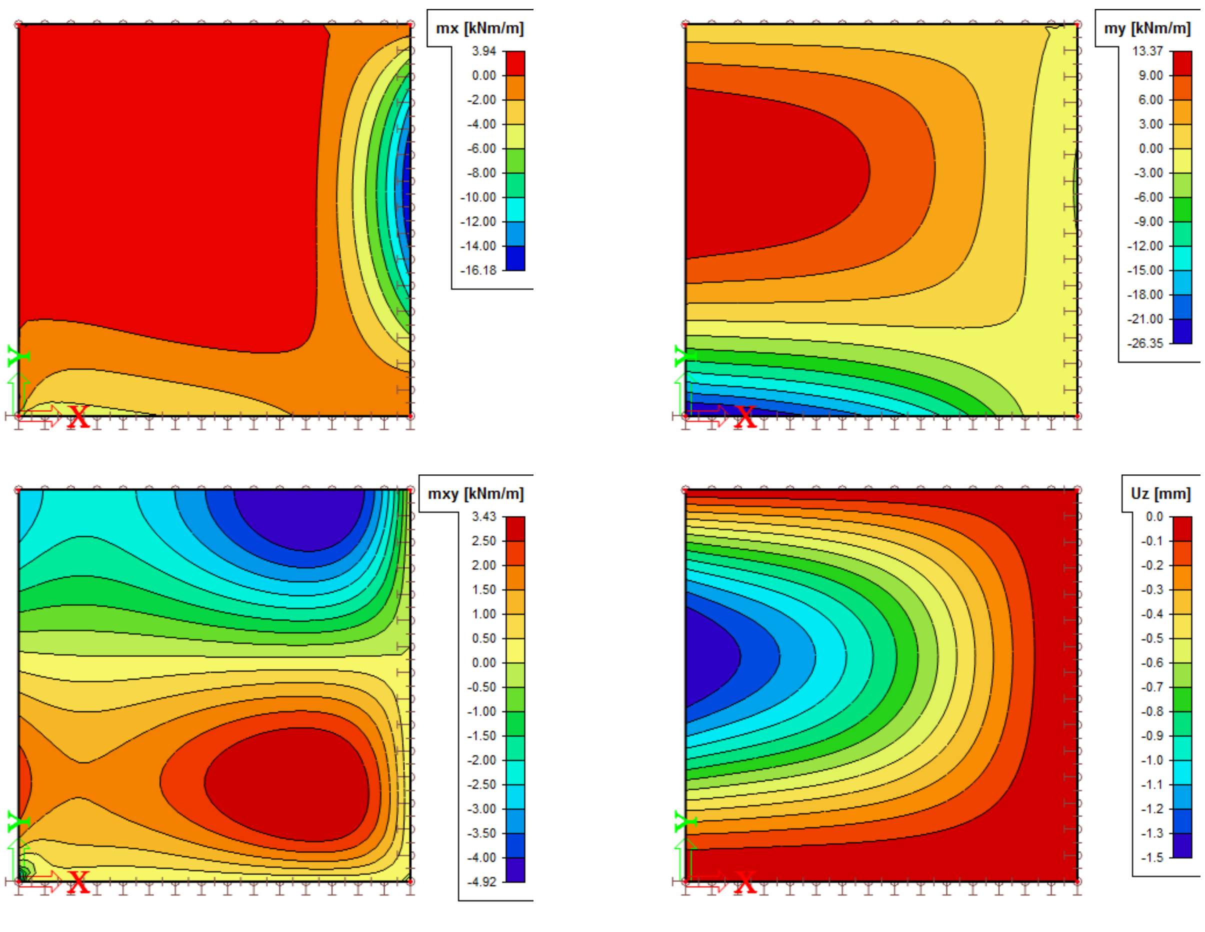

Fig. 10. Deflections and internal forces calculated in Scia Engineer [20]

Table 1. Results – Example 1

	w [mm]	m_x^+ [kNm/m]	m_y^+ [kNm/m]	m_{xy}^+ [kNm/m]	m_x^- [kNm/m]	m_y^- [kNm/m]	m_{xy}^- [kNm/m]
Matlab – MKP 250 × 250	1.447	3.927	13.108	3.410	-13.239	-21.555	-4.882
100 × 100	1.452	3.940	13.274	3.434	-14.955	-24.247	-4.915
50 × 50	1.453	3.941	13.326	3.436	-15.571	-25.323	-4.922
Scia - Kirchhoff 250 × 250	1.448	3.934	13.362	3.427	-16.023	-25.732	-4.905
100 × 100	1.453	3.940	13.371	3.434	-16.180	-26.352	-4.924
50 × 50	1.453	3.940	13.371	3.437	-16.194	-26.510	-4.923
Scia – Mindlin 250 × 250	1.474	3.913	13.414	3.397	-16.067	-25.584	-4.836
100 × 100	1.488	3.910	13.347	3.406	-16.117	-25.622	-4.734
50 × 50	1.491	3.909	13.315	3.410	-16.119	-25.681	-4.968

VII. Numerical Example 2 for the Finite Element Method

Finite element method was used also for solving example 2. Slab scheme is pictured in fig. 11, slab thickness 180 mm. Slab is loaded with uniformly distributed loading: 10 kN/m^2. Slab dimensions: 4.5 × 4.5m. Supports are shown in scheme. (three sides are fixed and one simply supported). Slab is made of concrete. Young´s modulus E=30 GPa and Poisson´s ratio ν = 0.2.

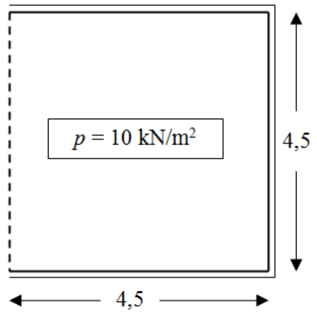

Fig. 11.Sheme slab

Table 2. Results – Example 2

	w [mm]	m_x^+ [kNm/m]	m_y^+ [kNm/m]	m_{xy}^+ [kNm/m]	m_x^- [kNm/m]	m_y^- [kNm/m]	m_{xy}^- [kNm/m]
Matlab – MKP 250 × 250	0.756	2.858	8.499	2.382	-9.007	-14.673	-2.382
100 × 100	0.757	2.881	8.673	2.404	-10.432	-16.700	-2.404
50 × 50	0.758	2.881	8.700	2.411	-10.921	-17.538	-2.411
Scia - Kirchhoff 250 × 250	0.758	2.878	8.731	2.395	-11.325	-17.842	-2.395
100 × 100	0.757	2.878	8.721	2.413	-11.407	-18.326	-2.413
50 × 50	0.758	2.881	8.733	2.413	-11.429	-18.403	-2.413
Scia – Mindlin 250 × 250	0.770	2.869	8.733	2.362	-11.350	-17.808	-2.362
100 × 100	0.775	2.870	8.654	2.532	-11.340	-17.853	-2.532
50 × 50	0.777	2.873	8.641	3.429	-11.350	-17.888	-3.429

VIII. Conclusion

Thin slabs can be analysed in an algorithm created in the program Matlab [19] which is based on the finite difference method. The variables which should be identified might be the deflection of the slab or internal forces. Correctness of the calculation was verified by comparing these results with those obtained in the program Scia Engineer [20]. The comparison of times measured for the methods used for the systems of linear equations and evaluation of the charts with the values suggest that the LU decomposition method is better, from among the direct methods, than the Gauss-Jordan method.

In case of rather small matrixes it is, however, recommended to use the Gauss-Jordan matrix. The fastest method is the direct division of matrices in the program Matlab [19]. Direct methods can be used for rather small systems of equations – for the matrices which are less than 1600.

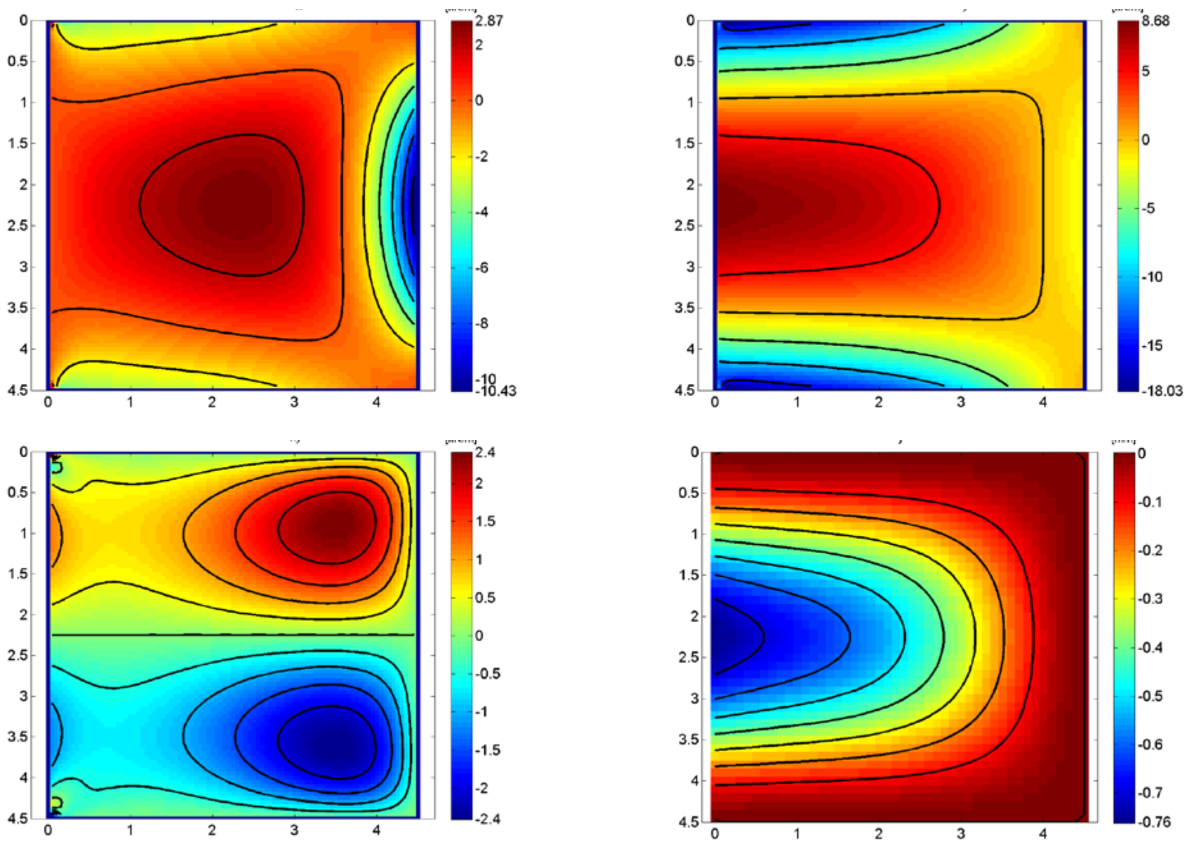

Fig. 12. Deflections and internal forces calculated in Matlab [19]

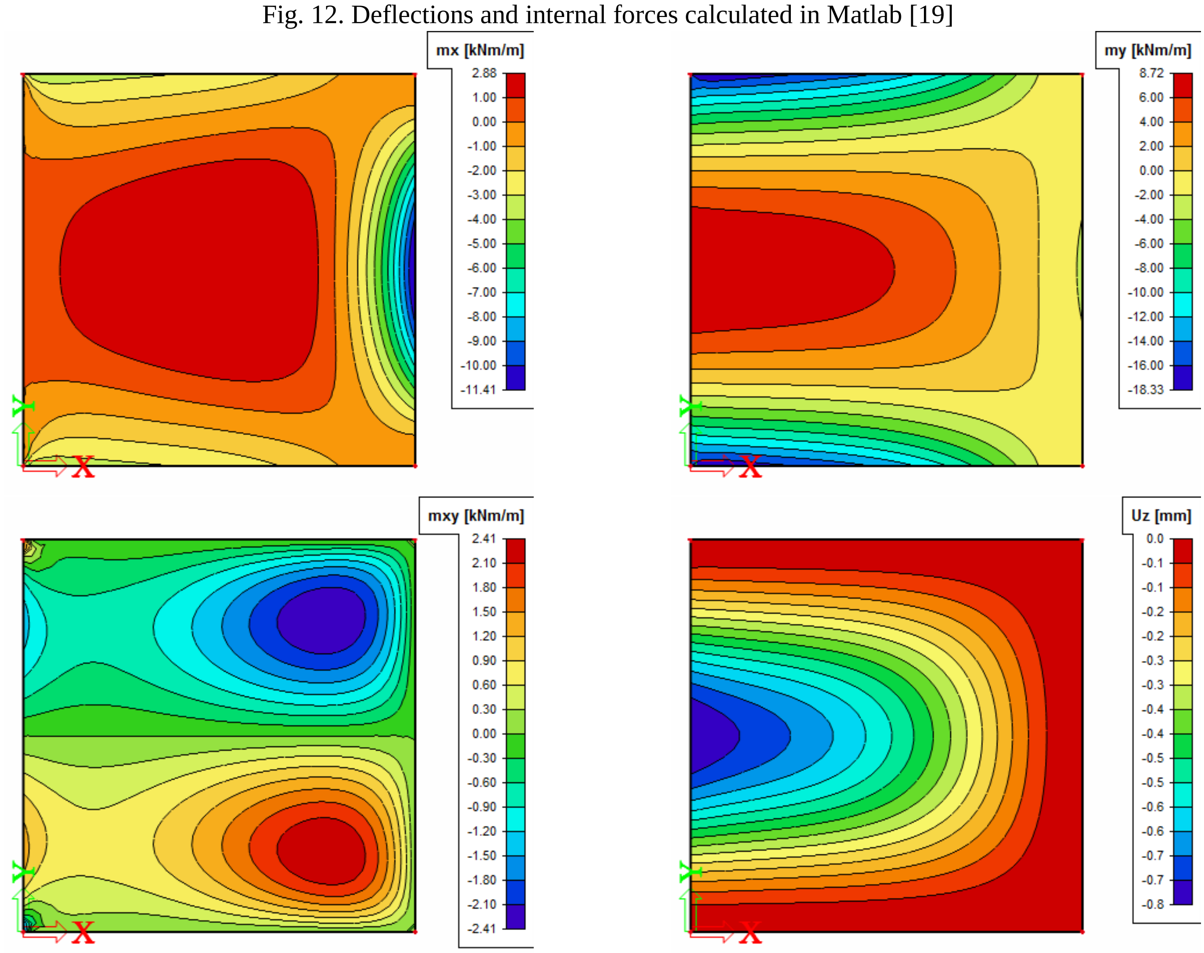

Fig. 13. Deflections and internal forces calculated in Scia Engineer [20]

Those methods are not optimised enough for rather big matrices and do not provide necessary results in a real time.

Because the matrix of the system is sparse, each row in the matrix contains not more than 13 values. It is advisable then to use this feature when optimising the calculation. Another objective of the research is the using of the Finite Difference Method for analyses of fibre reinforced concrete structures [26].

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