The variability of gravitational constant G in constitutive medium - based models of space: the vacuum-polarizable approach and the fluid-dynamic model of gravity as a pressure force

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Abstract**—** Gravitation is still the less understood among the fundamental forces of Nature. The ultimate physical origin of its ruling constant G could give key insights in this understanding. In a previous paper the author proposed, starting from ZPF inertia hypothesis and the Polarizable -Vacuum approach to General Relativity, a novel model of G as a function of quantum vacuum energy density, showing that, according to it, G could actually be a function of the distance r from the mass generating the gravitational field. In this paper the variability of G within a scalar fluid dynamics-like theory of gravity as a pressure force, basing on the above previous results, is outlined. An analytical expression for $G(r)$, under the hypothesis of spherically symmetric body and slowly varying gravitational potential, is also derived in this case and compared with that previously obtained within the Polarizable – Vacuum approach by starting from different hypothesis. The results strongly suggest G could be actually related to the property of physical vacuum viewed as a medium characterized by special properties and depend from the distance from massive bodies.

The proposed idea could give new interesting insights into a deeper understanding of gravitation and represent a starting theoretical point for the engineering of unimaginable solutions related, for example, to the field of gravity control and space propulsion.

Keywords**—** Gravitation; Physical Vacuum; Vacuum Polarization; ZPF Inertia Hypothesis; Pressure Force; Archimede's Thrust.

I. Introduction

GRAVITY is the most mysterious and still incompletely understood among the fundamental forces of Nature. A reason for this could probably derives from the description that General Theory of Relativity (GTR) gives of it in terms of "spacetime metric" which may hide some fundamental underlying physical details.

To this aim the study of the physical origin of the gravitational constant G ruling the strength of gravitation, through the well know Newton's law of universal gravitation

$$\vec{F}_g = G\left(m_1 \cdot m_2 / r^2\right)\vec{u}_{\vec{r}} \tag{1}$$

where m_1 and m_2 are the interacting masses and $\vec{r}$ is their relative distance vector, could give very important insights. In the commonly accepted theoretical framework G is generally assumed to be a universal constant whose currently accepted value is [1]

$$G = \left(6.67384 \pm 0.00080\right)\cdot 10^{-11}\, m^3 \cdot kg^{-1} \cdot s^{-1} \tag{2}$$

regardless of the magnitude of the mass generating gravitational field and of the distance between them. Nevertheless, since from the beginning of the past century, some interesting models, involving a variable gravitational constant G has been proposed by authoritative researchers [2,3,4].

A very interesting and intriguing suggestion, in part also coming from these studies, is that G could be somehow related to the QED Zero – Point – Field (ZPF) or the so-called Quantum Vacuum (QV).

Indeed ZPF is the only "medium" between gravitational matter and then the relation between (QV) and gravitation is a task of primary importance in order to establish a quantum theory of gravity. Several theoretical and experimental results have shown QV can be influenced by electromagnetic fields [5,6,7]. In addition to electromagnetic field, also the presence of matter is though able to modify the structure of QV. In 1967, Sakharov [8] suggested gravity could be the effect of a change in the quantum-fluctuation energy of ZPF quantum vacuum induced by the presence of matter as experimentally demonstrated by the Casimir effect [9,10,11].

Later, starting from Sakharov's results, Puthoff [12] proposed the hypothesis that ordinary matter could be ultimately composed of sub-elementary constitutive charged entities he called "partons", able to dynamically interact with the fluctuating QED quantum vacuum according to a sort of resonance mechanism. According to Puthoff's model, the inertia of a body would be the result of the interaction between partons and ZPF quantum fluctuations whose effect would result in the modification of the electromagnetic modes of ZPF at the interface between a body and its surrounding space determining the so – called Zero-Point-Field Lorentz force [13]. In this way both the inertial and gravitational masse of a body could be substantially composed of confined e. m. modes of ZPF whose presence modifies the previous state of QED QV.

On the other hand, it is a known fact, theoretically explained within the GTR and supported by strong experimental evidences, that the gravitational potential

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generated by mass, depending on the radial distance from it, affects the running rate of clocks, the measure of distances as well as the velocity of light.

All the above results and many other available in the literature [14,15,16] strongly suggest QV can be actually considered as a sort of "optical" medium equipped with a own inner structure.

On this basis, Puthoff [17] showed that, under "standard" (weak-field) astrophysical conditions, the basic principles of GTR can be coherently reformulated in terms of the changes in the permittivity ε_0 and permeability μ_0 constants of a polarizable vacuum (PV) of an optical electromagnetic medium whose properties are able to reproduce all the features of GTR under the above conditions.

Moreover, in some recent works [18-21] the author proposed a model of (QV), characterized by a Planckian metric, described in terms of the dynamics of energy density in which inertial and gravitational mass are interpreted as local stable variations of QV energy density with respect its "unperturbed" value. Within this model, gravity is interpreted as originated by the local gradients of QV energy density $\Delta\rho\left(\vec{r},t\right)$, due to presence of mass, giving an unbalanced ZPF pressure that manifest itself as gravitational force.

More recent researches [8,22,23] also suggested the possibility that G could be truly expressed as a function of more fundamental physical quantities, i.e. the so – called "Quantum Vacuum Zero Point Field Mass – Density Equivalent" ρ_{QV} giving a measure of the energy density of the QED QV and the Planck time t_P.

On the other hand many researches [24,25,26,27], including some very recent ones [28] authoritatively indicate that physical vacuum could be actually constituted by a (quantum at microscopic scales) fundamental substrate like an elastic solid-state medium or fluid or a Higgs condensate.

This view is also coherent with the framework of QFT if we remember that, although the Standard Model (SM) is usually interpreted as describing fundamental particles (leptons and quarks) and their interactions through bosons, all the elementary particles are actually QV excitations and with the picture of space-time as arising from a sort of large-scale condensate of more fundamental objects, in which matter appears as excitations of these constituents, describable by hydrodynamics techniques [18,27,28].

In some previous works, starting from the interpretation of inertial and gravitational mass as the seat of standing waves of ZPF [13,23] and from the picture of QV as a special optical medium characterized by a refraction index [17,29], a novel theoretical model describing the gravitational constant G as a function of QV energy density, has been proposed by this author.

According to the above model, the gravitational constant G, could be considered as function of physical vacuum energy density whose value is in turn determined by the presence of mass originating gravitational potential and variable with the distance from it, thus making G depending upon the radial distance as well.

An approximate analytic expression of this functional relation, for a spherically symmetric mass distribution and a weak and slowly varying (with distance from mass) gravitational potential, has been also proposed in these studies [30,31].

In this paper we present and discuss a reformulation of the above idea in the framework of a fluid - dynamics – like theory of physical vacuum in which gravity is viewed as a pressure force expressed as a function of a single scalar field associated to the density of a universal substrate medium supposed to "fill" or constitute the physical vacuum itself, comparing the result with that previously derived within the PV approach [31].

By considering this density as the macroscopic average of the local QV mass-equivalent energy density ρ_{QV} (or a definite function of it), we show, assuming the ZPF inertia hypothesis for inertial and gravitational mass and the already considered relationship between G and ρ_{QV}, the gravitational constant G, even in this case, varies as a function of the distance r.

from the mass generating a gravitational potential.

In particular, the analytical expression for $G\left(r\right)$ so obtained, under the hypothesis of spherically symmetric body and slowly varying gravitational potential, has the same functional dependence on r of that already obtained by this author by means of the PV approach [31] by starting from different hypothesis [30] but an opposite monotonic behaviour with respect to r, whose meaning is discussed in this work.

We also demonstrate that, if we assume a fluid-like constitutive model of space, characterized by specific properties [32,33], the variability of G with respect the radial distance r, emerges, under the simplified hypothesis discussed, also without a priori considering any quantum relationship between G and quantum vacuum energy density.

The result strongly suggests G could be actually related to the structure and properties of the physical vacuum considered as a medium characterized by special properties.

II. Inertial and Gravitational Mass as the Result of Stading Waves of Quantum Vacuum

In the model of Hairsh, Rueda and Puthoff (HRP) [12,13] a material body is represented, with respect to the electromagnetic interaction, as a resonant cavity in which a suitable set of oscillating modes of QV. According to this hypothesis (also known as ZPF Inertia Hypothesis), the inertial and gravitational masses m_i and m_g associated to a given material body are given by

$$m_i = m_g = \left(V_0/c^2\right)\int_0^\infty \eta\left(\omega\right)\rho\left(\omega\right)d\omega \qquad (3)$$

in which ω is the angular frequency of ZPF mode, $\rho\left(\omega\right)$ is the spectral energy density of quantum vacuum ZPF fluctuations and $\eta\left(\omega\right)$ is a function that would quantify the fraction of ZPF energy density that electromagnetically interacts with the particles contained in the "useful volume" V_0 or, in other words the "efficiency" of interaction [13]. In this way the apparent inertial mass of a given object would originate by the interaction, during the accelerated motion of the body, between the ZPF energy density fraction enclosed in the object (given by $\eta(\omega)$) and the partons contained in the volume V_0.

The equivalence between the apparent inertial mass m_i and passive gravitational mass m_g (namely the "weak" equivalence principle) then "naturally" arises, within the HRP model, from the consideration that a body accelerating through ZPF is identical to a body that remains fixed in a gravitational field and having the QED QV fall past on curved geodesic.

It is now important to stress the physical meaning of such volume, that must intended as an electromagnetic resonant cavity with conducting wall, being the volume V_0 the space enclosed within the cavity walls. In this way the electromagnetic modes of ZPF are trapped inside the cavity and the resulting energy is accumulated in it.

This energy cannot be however accumulated without limit, since the possible electromagnetic modes inside a resonant cavity are upper bounded by a limiting frequency ω_{up} whose value is substantially determined by the plasma frequency ω_{Pl} of the electrons in the cavity walls.

The connection between the modes inside cavity with those outside it is allowed by the conductive structure of cavity walls.

Now if we consider an ideal resonant cavity (i.e. neglecting energy dissipation of modes) at the absolute temperature $T=0$, outside the cavity there are only the ZPF quantum fluctuations while inside it there is a discrete number of possible modes possible oscillating at their exact characteristic frequencies ranging from 0 up to ω_{Pl}.

So, said N the maximum number of this modes, we have

$$E_{tot}=\sum_{k=1}^{N}\hbar\omega_k/2 \quad (4)$$

where $\omega_1\le\omega_2\le...\le\omega_N\le\omega_{Pl}$. Now, under the above assumptions, the energy given by (4) must be equal to the quantity given by (3) multiplied by c^2, namely

$$E_{tot}=mc^2=V_0\int_0^\infty\eta(\omega)\rho(\omega)d\omega=\sum_{k=1}^{N}\hbar\omega_k/2 \quad (5)$$

On the other hand we know that the density of ZPF electromagnetic oscillation modes in the frequency interval between ω and $\omega+d\omega$ is given by

$$N(\omega)d\omega=\left(\omega^2 d\omega/\pi^2c^3\right)d\omega \quad (6)$$

and, assuming an average energy per mode equal to $\hbar\omega/2$, we obtain the spectral energy density of ZPF fluctuation as

$$\rho(\omega)d\omega=\left(\hbar\omega^3/2\pi^2c^3\right)d\omega \quad (7)$$

that substituted into (5) gives

$$\eta(\omega)=\sum_{k=1}^{N}\left(\pi^2c^3/V_0\right)\left[\delta(\omega-\omega_k)/\omega_k\right] \quad (8)$$

Equation (8) states that the spectrum of electromagnetic field inside the cavity is composed by a sum of N lines placed at $\omega=\omega_k$ whose amplitude diminishes with the increase of frequency.

Equation (8) holds under the simplification that no dissipation occurs. Nevertheless it can be shown [13] that, if the dissipation is small, a more accurate expression for the line-shaped functions $\delta(\omega-\omega_k)/\omega_k$ is given by the so-called Lorentzian –lineshape function, whose expression is

$$l(\omega)=\left(\Delta\omega/2\pi\right)\left[\left(\omega-\omega_0\right)^2+\left(\Delta\omega/2\right)\right]^{-1} \quad (9)$$

where the quantity $\Delta\omega$ is the lineshape broadening parameter and describes the various types of dissipation and broadening effects.

By discretizing (9) and using it in (8), we have

$$\eta(\omega)=\left(\pi^2c^3/2\pi\omega V_0\right)\sum_{k=1}^{N}\Delta\omega_k\left[\left(\omega-\omega_k\right)^2+\left(\Delta\omega_k/2\right)\right]^{-1} \quad (10)$$

where, as above, ω_k is the proper frequency of the k-th mode and $\Delta\omega_k>0$ its frequency broadening.

Finally, the mass associated to a resonant cavity (not including the overall mass of the walls) is given by (3) using the result of (10), namely

$$\begin{aligned}m&=\left(V_0/c^2\right)\int_0^\infty\eta(\omega)\rho(\omega)d\omega=\\&=\int_0^\infty\left(\pi^2c^5/2\pi\omega\right)\rho(\omega)\times\\&\times\sum_{k=1}^{N}\Delta\omega_k\left[\left(\omega-\omega_k\right)^2+\left(\Delta\omega_k/2\right)\right]^{-1}d\omega\end{aligned} \quad (11)$$

now using (7) in (11) and recalling the definition of energy given by (5), we can write

$$m = \int_0^\infty \left(\pi^2 c^5 / 2\pi\omega_0\right) \rho(\omega) \times \sum_{k=1}^{N} \Delta\omega_k \left[\left(\omega - \omega_k\right)^2 + \left(\Delta\omega_k/2\right)\right]^{-1} d\omega = \int_0^\infty \left(\pi^2 c^5 / 2\pi\omega_0\right)\left(\hbar\omega^3 / 2\pi^2 c^3\right) \times \sum_{k=1}^{N} \Delta\omega_k \left[\left(\omega - \omega_k\right)^2 + \left(\Delta\omega_k/2\right)\right]^{-1} d\omega = \left(1/c^2\right) \sum_{k=1}^{N} \int_0^\infty \left(\Delta\omega_k / 2\pi\right)\left[\left(\omega - \omega_k\right)^2 + \left(\Delta\omega_k/2\right)\right]^{-1} \times \left(\hbar\omega/2\right) d\omega \tag{12}$$

or, in a more compact form,

$$m = \sum_{k=1}^{N} \int_0^\infty A_k(\omega)\left(\hbar\omega/2\right) d\omega \tag{13}$$

where we have posed

$$A_k(\omega) = c^{-2} \int_0^\infty \left(\Delta\omega_k/2\pi\right)\left[\left(\omega - \omega_k\right)^2 + \left(\Delta\omega_k/2\right)\right]^{-1} \tag{14}$$

Equation (13) is very meaningful since it shows the total mass inside the resonant cavity associated to a body can be expressed, even in the presence of dissipation, as the overlapping of the zero point energies of all the electromagnetic modes of QV, each of them broadened by a suitable factor given by (14). Furthermore, it is expected that the most part of modes are not overlapping as long as the cavity size remains small, since their frequency separation will become comparable with the broadening $\Delta\omega$ only at the highest frequencies [13].

III. The Effect of Gravitational Field in the Polarizable-Vacuum Approach to General Relativity

In the Polarizable – Vacuum (PV) formulation of GTR [17], the metric changes in terms of variations of permittivity and permeability of a polarizable QV and the Maxwell's equations in curved space are treated in the isomorphism of a polarizable medium characterized by a variable refractive index in flat space. In this way, as already suggested by Eddington [34] almost a century ago, the bending of a light ray around a massive object could be considered as a refraction effect of the space (actually the vacuum) in flat space. In this model the reduction of light velocity (as well as all the other effects on time and length intervals) are interpreted as the effect of an effective increase of the refractive index of QV.

The basic assumption of PV approach is to consider that the presence of a mass induces vacuum polarization effects so that the polarizability of vacuum around a mass differs from its asymptotic value (in the far-field condition). Formally it is done by assuming, for the vacuum, the electric flux density $\vec{D}$ is given by the following expression

$$\vec{D} = \varepsilon \vec{E} = K\varepsilon_0 \vec{E} \tag{15}$$

where $\vec{E}$ is the electric field, ε_0 is the permittivity of vacuum interpreted as the polarizability of the vacuum per unit of volume, and K is the modified (by the presence of mass) dielectric constant of the vacuum (considered as a general function of position) due to the vacuum polarizability changes. The quantity K then represents, within PV model, the fundamental variable since it rules the variations of all fundamental physical quantities due to the altered properties of medium (QV) in the presence of the mass. Some theoretical cosmological considerations about fine structure constant [15] require

$$\varepsilon(K) = K\varepsilon_0, \quad \mu(K) = K\mu_0 \tag{16}$$

then, accordingly, the light velocity will be a function of K

$$c(K) = c/K \tag{17}$$

where c is the asymptotic light velocity in flat space ($K = 1$). Equation (17) has a very important meaning since it shows the dielectric constant of vacuum is a sort of refractive index of the PV in which vacuum polarizability changes in response to GTR-induced effects. Equation (17) implies a "rescaling" of the other fundamental physical quantities as energy, mass, time and length intervals. In particular, as observed by Dicke [15] by using a limited principle of equivalence, if E_0 is the energy of a system in a flat space ($K = 1$), we have, in general

$$E = E_0 / \sqrt{K} \tag{18}$$

in a region of space in which $K \neq 1$. The combined use of (17) and (18) gives

$$m = K^{3/2} m_0 \tag{19}$$

for the rest mass m_0 of a particle. As a consequence of a change in energy due to the variation of QV polarizability we have, starting from $E = \hbar\omega$,

$$\omega = \omega_0 / \sqrt{K} \tag{20}$$

and the correspondent equations for time interval

$$\Delta t = \Delta t_0 \sqrt{K} \tag{21}$$

and for length interval

$$\Delta l = \Delta l_0 / \sqrt{K} \tag{22}$$

According to (22), the dimension of a material object varies with the local changes in QV polarizability so reproducing, from a different standpoint, the variable metric of GTR. Furthermore, according to (21) and (22), the "natural" measurement of light velocity by rods and clocks returns the unperturbed value c so maintaining the invariance of the locally measured velocity of light assumed by Einstein's Theory of Relativity.

The key point, for our following treatment, is the explicit expression of K as a function of the mass and the distance from it. It can be obtained by following a lagrangian approach [35]. The starting point is the lagrangian of a free particle

$$L = -mc^2 \sqrt{1 - \left(v/c\right)^2} \tag{23}$$

that, using (17), becomes

$$L = -\left(m_0 c^2 / \sqrt{K} \right) \sqrt{1 - \left(Kv/c \right)^2} \tag{24}$$

whose density $\bar{L}$ is

$$\bar{L} = -\left(m_0 c^2 / \sqrt{K} \right) \sqrt{1 - \left(Kv/c \right)^2} \delta^3 \left(\vec{r} - \vec{r}_0 \right) \tag{25}$$

being $\vec{r}$ the generic position and $\vec{r}_0$ is the position of the particle with respect a given frame or reference. The interaction between a particle of charge q and an electromagnetic field given by the four-potential $\left(\phi, \vec{A} \right)$ is described by the lagrangian density $\bar{L}_p$

$$\begin{aligned} \bar{L}_p = &-\left[\left(m_0 c^2 / \sqrt{K} \right) \sqrt{1 - \left(Kv/c \right)^2} + q\phi - q\vec{v} \cdot \vec{A} \right] \times \\ &\times \delta^3 \left(\vec{r} - \vec{r}_0 \right) \end{aligned} \tag{26}$$

On the other hand the lagrangian density of the electromagnetic field itself is, in PV formulation

$$\bar{L}_{em} = -\left(1/2 \right) \left(B^2 / K\mu_0 - K\varepsilon_0 E^2 \right) \tag{27}$$

and we must also specify a lagrangian density for the quantity K, being considered as a scalar variable, that can be obtained, by imposing the standard Lorentz-invariant form for the propagational disturbance of a scalar and following [15], in the form

$$\begin{aligned} \bar{L}_K = &-\left(c^4 / 32\pi G K^2 \right) \times \\ &\times \left[\left(\nabla K \right)^2 - \left(K/c \right)^2 \left(\partial K / \partial t \right)^2 \right] \end{aligned} \tag{28}$$

so that the total lagrangian density for a matter-field interaction in a PV with a variable dielectric constant, is

$$\bar{L} = \bar{L}_p + \bar{L}_{em} + \bar{L}_K \tag{29}$$

The equation of motion in a dielectric vacuum is then obtained, using the principle of least action, by the variation of the lagrangian density $\delta \int \bar{L}\, dV dt$ with respect the particle variables $\left(x, y, z, t \right)$ while the equation, in K, describing the effect of vacuum polarization due to the presence of matter and field, is get by the variation of $\bar{L}$ with regard to the K variable

$$\begin{aligned} &\nabla^2 \sqrt{K} - \left(K/c \right)^2 \left(\partial^2 \sqrt{K} / \partial t^2 \right) = \\ &= -\left(8\pi G \sqrt{K} / c^4 \right) \times \\ &\times \left\{ \left(m_0 c^2 / 2\sqrt{K} \right) \left[\left(1 + \left(Kv/c \right)^2 \right) \Big/ 2\sqrt{1 - \left(Kv/c \right)^2} \right] \delta^3 \left(\vec{r} - \vec{r}_0 \right) \right\} + \\ &- \left(4\pi G \sqrt{K} / c^4 \right) \left(B^2 / K\mu_0 + K\varepsilon_0 E^2 \right) + \\ &+ \left(\sqrt{K} / 4K^2 \right) \left[\left(\nabla K \right)^2 + \left(K/c \right)^2 \left(\partial K / \partial t \right)^2 \right] \end{aligned} \tag{30}$$

We are now interested in the simplified case of a static field, for which $\partial K / \partial t = 0$, with spherical symmetry. In this case we have, from (30), after some simple manipulations

$$d^2 \sqrt{K} / dr^2 + \left(2/r \right) \left(d\sqrt{K} / dr \right) = \left(1/\sqrt{K} \right) \left(d\sqrt{K} / dr \right)^2 \tag{31}$$

whose solution, satisfying the Newtonian limit, is [17]

$$K = \left(\sqrt{K} \right)^2 = e^{2GM/rc^2} \tag{32}$$

As shown in [17] this solution correctly reproduces the usual GTR Schwarzschild metric in weak-field conditions as those occurring in the Solar System and is in agreement with the scaling factor used in the previous analysis proposed by this author [25].

IV. Gravitational Field in a Fluid Dyamics Theory of Gravity

A certain number of models, aimed to describe a theory of gravity based on the idea that physical space could be really "filled" with a constitutive continuum medium characterized by specific properties, has been proposed [24,25,26,27] some time ago besides the PV model previously discussed. Nevertheless a lot of these attempts remain in a early developments, while some of them don't give a complete dynamical explanation of gravitation (since, for example, don't include a celestial mechanics or a complete treatment of the Newtonian limit).

Nevertheless among these, there is one proposed some years ago by Arminjon [33,36] that appears quite interesting in connection with our picture of gravity as originating from physical vacuum density gradients.

In this model the physical vacuum is supposed to be a sort of perfect barotropic fluid or "micro-ether" characterized by its own pressure p_e and density ρ_e related by an equation of state $p_e = f\left(\rho_e \right)$. As previously observed by Romani [32] such type of fluid should not interfere with the motion of material bodies being able to exert pressure forces only. Furthermore, as a "perfect" fluid, this micro-ether should be a neither entropic nor dissipative system and then resulting continuous at any scale.

By reducing the gravity to a pressure force the considered model then interprets any interaction at a distance ultimately as a contact action.

In few words, the gravitational force is viewed, within this picture, as an Archimedes "thrust, occurring at the scale of elementary particles, due to the vacuum fluid. Such interpretation was been already proposed by Euler in the XVIII century [37] according to which the particles of matter (all supposed to have the same density), called "molecules", are pushed by the fluid (characterized by a density less than the matter one) filling the "empty space" each with a force proportional to the extension or the volume of the considered molecule, due to fluid pressure gradients.

In this way Euler was able to reproduce the correct dependence of the weight of a body from r^{-2} (being r the distance from Earth centre) in the Earth's gravitational field.

In the Euler's model the vacuum fluid is considered as having zero compressibility so implying the instantaneous "propagation" of the gravitational interaction and then reproducing the classic Newtonian Gravity (NG) behaviour.

On the other hand in the fluid-like model here considered [33] the fluid compressibility is different than zero allowing for a finite time of propagation of interaction within the fluid medium, able to give a physical explanation of gravitational waves in terms of classical mechanics concepts (and then without necessarily introducing the concept of graviton).

The detailed theoretical treatment (like, in particular, the equivalence principle between gravitation and metrical effects of the motion and space-time metric) of the above fluid-pressure model of gravity is reported in the cited references and will not be considered here unless necessary. In this paper our goal is, in fact, to generalize the model already proposed in [33] to the case of a variable gravitational constant G, comparing the results with those obtained within the PV model previously discussed. This represents a completely new approach not previously considered in the cited references.

In order to develop a consistent model of gravity as a pressure force exerted by a fluid, we must assume the fluid itself defines the preferred inertial frame [33] in which we can relate the gravitational effect to the local state of this fluid. The possibility to define and use, for the motion description of any body in the Universe, a preferred inertial frame has been shown to be not incompatible with the experimental previsions of Einstein's Special Theory of Relativity (STR) that on the other hand, as known, is based on the equivalence of all the possible inertial frames and the Lorentz invariance requirement.

One of the first and most important examples of preferred-frame theory of relative motion is represented by the Lorentz – Poincare ether theory (LPET) [38,39] in which the relative effects of motion (the metrical effects associated to the length contractions and time dilatation) are considered as "true" effects due to the motion of a given inertial frame (in which the motion itself is described) with respect the preferred inertial frame E in which the first synchronization of clock is made (according to the Einstein's procedure) and respect to which the simultaneity is absolute.

This ether-preferred frame of reference, where the Maxwell's equations hold, also constitutes, within the LPET, the support (elastic) medium for the propagation of electromagnetic waves, sometimes interpreted, by Lorentz himself, as a fluid [39].

The LPET was proved to be fully compatible with the results of STR by a lot of authors [40,41,42], so showing the assumption of the existence of a fluid-constitutive medium of physical vacuum would be consistent with the relativity principle.

Nevertheless, both LPET and STR don't provide for gravitation and, furthermore, almost all the theoretical models of gravity, somehow related to the assumption of classical or quantum constitutive media of physical vacuum, so far proposed, introduce some kind of Lorentz symmetry violation.

On the other hand, more recent researches [19,21,43] have shown it is possible to build different versions of STR basing on space and time transformations different than the Lorentz's ones as, in particular, the Inertial Transformations (IT) suggested by Selleri [44] that exactly define a preferred inertial systems compatible with that assumed by LPET.

All the above considerations then suggest that, in absence of gravity, the "equivalence" between the Einstein's Special Theory of Relativity and the LPET and the consequent "indetectability" of physical vacuum (fluid) medium, corresponds to a uniform field of density and pressure within it while, in the presence of gravity, the emergence of the gradients of such quantities would make the fluid-medium "detectable" just through its gravitational effects. This also would determine the "revelation" of the preferred fluid medium reference system also determining the equivalence between the metrical effect of motion and gravitation [36].

More specifically, as proposed in [36], the definition of the preferred inertial frame S can start by considering a flat space-time as a Lorentz manifold Γ equipped with a flat metric γ^0, coinciding with the medium filling the physical vacuum in LPET. We then consider an application (coordinate chart) Ω from the set P of the points of the manifold V onto R^4, $\Omega : \mathrm{P} \to \left(x^\mu\right)$, such that $\gamma^0_{\mu\nu} = \eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowski tensor of the space-time and the invariant infinitesimal distance is expressed in the usual way as $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$. In this way we can define a reference body as a manifold B in such a way that every point of B is a vector

$$\vec{x} \equiv x^j \left(j = 1,2,3\right) \tag{33}$$

and the "absolute" time

$$t \equiv x^0/c \tag{34}$$

where c is the (constant) velocity of light in the preferred frame. Since we want the fluid itself somehow defines the preferred system of reference (that coincides with the Lorentz – Poincarè rigid ether) it is obvious that its "microscopic" motion must not be relevant for the mechanics we would describe with respect to it (otherwise we should define another more "fundamental" system of reference independent from the fluid motion with respect to which describe the fluid motion itself). A solution, proposed in [36], is to suppose the reference body B to "follow" the average motion of the fluid (or micro-ether). In particular, if the velocity

$$\vec{v}^{\,'}_f \equiv d\vec{x}/dt \tag{35}$$

(with $\vec{x}$ and t respectively given by (33) and (34)), the density $\rho^{'}_f$ and the pressure $p^{'}_f$ of fluid (micro-ether) are defined, we can introduce the corresponding macroscopic quantities $\vec{v}_f = \left\langle \vec{v}^{\,'}_f \right\rangle_{V^0}$, $\rho_f = \left\langle \rho^{'}_f \right\rangle_{V^0}$ and $p_f = \left\langle p^{'}_f \right\rangle_{V^0}$, obtained as volume-averaging (using the volume measure $V^0 = x^1 x^2 x^3$ associated to the Euclidean metric $g^0_{ij} = \delta_{ij}$ in B according the given chart Ω) in macroscopic finite domains of the space B. Then the preferred inertial frame S is defined as the set of the point of space whose local velocity field is given by the time and space averaged microscopic velocity field

$$\vec{v}\left(\mathrm{t}, \vec{x}\right) \equiv \left\langle\left\langle \vec{v}^{\,'}_f \right\rangle\right\rangle_{\mathrm{t},\vec{x}} \tag{36}$$

$\forall \vec{x} \in B$ and for a $t \in T$, being a T some fixed time interval.

According to the definition given by (36), the requirement that the reference body follows the average motion of fluid "particles" (micro-ether) can be expressed as

$$\vec{v}\left(\mathrm{T},\vec{x}\right)=0 \quad (37)$$

for any space-time point $\left(t,\vec{x}\right)$ and then the (37) can be considered as the "operative" definition of the preferred inertial frame S.

Within this picture, gravity is then interpreted as the effect of the macroscopic portion of the constitutive fluid medium pressure, whereas the microscopic fields $\vec{v}_f^{'}$, $\rho_f^{'}$ and $p_f^{'}$ would account for the microscopic behaviour of Newton's gravity whose variations are negligible on very small distances.

Since the fluid is assumed to be not viscous we can write, using the divergence theorem, the pressure force acting on a massive body M as

$$\vec{F}=-\int_{\partial S(M)} p_f\vec{n}\cdot dS=-\int_{V(M)}\nabla p_f\,dV \quad (38)$$

where $V\left(M\right)$ is the overall volume associated to the body M and $\partial S\left(M\right)$ the total boundary surface delimiting the volume $V\left(M\right)$.

The connection between the pressure force and the gravity force is then established by assuming that

$$\vec{F}=-\int_{V(M)}\nabla p_f\,dV=\int_{V(M)}\vec{g}\rho_M\,dV \quad (39)$$

where $\vec{g}$ is the gravitational acceleration and ρ_M is the matter density of the considered macroscopic body. Actually the mass of every macroscopic body can be considered as the sum of the masses of its constitutive particles that are in turn subjected to gravity so implying the force of gravity is due to the pressure of the fluid filling the void between the particles and then acting on a small fraction of the macroscopic volume V associated to a body. Consequently the force of gravity on a single particle can be written, assuming ∇p_f substantially constant in the neighbourhood of the particle, as

$$\vec{F}_i=-\nabla p_f V_i \quad (40)$$

where V_i is the volume associated to a particle i. This also means that if we consider $\vec{g}$ uniform in the macroscopic domain associated to the body M (a condition usually verified for NG) we can write the (39) as

$$-\nabla p_f\sum_i V_i=\vec{g}\sum_i\rho_i V_i \quad (41)$$

from which we obtain

$$\vec{g}=-\nabla p_f\big/\overline{\rho}_p \quad (42)$$

having defined

$$\overline{\rho}_p=\sum_i\rho_i V_i\Big/\sum_i V_i \quad (43)$$

Equation (43) represents the average mass density of the particles within a macroscopic domain enclosed in the volume V and it is supposed to be a function of p_f only, namely $\overline{\rho}_p=f\left(p_f\right)$, in order to ensure the independence of (42) from the particular kind of matter composing the body M. This also implies the average particle density $\overline{\rho}_p$ is the same for every massive macroscopic body, being a function of the pressure field only.

This last assumption is crucial and, as we'll also discuss in the following, it is strongly related to the dynamical mechanism supposed to rule the formation of matter and how the latter locally (around a body) influences the fluid density field.

In particular, this assumption somehow requires the matter particles itself to be made of the same constitutive fluid filling the physical vacuum. In [33,36] this hypothesis is assumed as a postulate following the suggestion given, several years ago by Romani [32] in a purely classical context considering particles as organized flow or "vortex" in the constitutive fluid.

On the other hand, in a more modern conception, also considering quantum aspects as those related to ZPF features, as previously discussed, we can consider inertial and gravitational mass as the manifestation of the electromagnetic QV energy whose density is "altered" by the presence of a massive body interacting with its e.m. fluctuating modes. We can then ultimately picture every matter particle as a "structured" form of a constitutive fluid-like substratum of physical vacuum, a view also confirmed by a more sophisticated model, involving coherent excitations of ZPF modes, elaborated by this author [45].

So without discussing more about this aspect, which would be beyond the scope of this paper, we can assume, from (43)

$$\overline{\rho}_p=\rho_f \quad (44)$$

so that (42) becomes

$$\vec{g}=-\nabla p_f\big/\rho_f \quad (45)$$

The use of the macroscopic averaged quantities ρ_f and p_f in (45) ensures [33], since gravity mainly varies over large distances, ∇p_f to be uniform at macroscopic distance (even if $\rho_f^{'}$ and $p_f^{'}$ are not so at microscopic scale) and also allows the possibility that other interactions (as, for example, the electroweak and nuclear ones) acting on smaller scales, could also be explained by the constitutive fluid dynamics at these scales. This also establishes a further connection between the model proposed in [33] and our present and previous [18,20,23,30,31] results.

The dynamic connection between the source of gravity (massive bodies) and the gravitational field $\vec{g}$ can be then established by setting up a dynamic equation relating the matter density field with the fluid density and pressure fields.

As well known, in classical mechanics, the relationship between matter and gravitational field is given, in the stationary case, by the Poisson equation

$$\nabla^2\phi=4\pi G\rho_M \quad (46)$$

where ρ_M is the mass density, ϕ the gravitational potential and $\nabla^2=\partial^2\big/\partial x^2+\partial^2\big/\partial y^2+\partial^2\big/\partial z^2$.

A Poisson – like equation can also be obtained [33] within the described fluid model by considering the (45) and the divergence theorem applied to $\vec{g}$ namely

$$\int_{\partial V} \vec{g} \cdot \vec{n}\, dA = \int_{V} \nabla \cdot \vec{g}\, dV = -\int_{V} \nabla^2 p_f \big/ \rho_f dV \tag{47}$$

if ρ_f is uniform within the volume V and the by Gauss theorem

$$\int_{\partial V} \vec{g} \cdot \vec{n}\, dA = -\,4\pi G \int_{V} \rho_M dV \tag{48}$$

so that, equating the (47) and (48), we finally have

$$\nabla^2 p_f = 4\pi G \rho_M \rho_f \tag{49}$$

that can be considered as the field equation for the fluid medium pressure p_f.

As shown in [33] the (49) can be assumed to hold even if the fluid density ρ_f is not strictly uniform. This must be the case since, in general, due to the non-zero macroscopic compressibility of the fluid $K \equiv d\rho_f \big/ dp_f$, the density ρ_f cannot be considered uniform but $\rho_f = f\left(p_f\right)$.

It is then evident that, within this picture, the case of NG will correspond to the condition of incompressible fluid ($K = 0$) determining an infinite velocity of interaction that can be considered as the limiting case of the behaviour of a compressible fluid as occurring in the static case.

The general case of non zero compressibility then will give raise to the propagation of pressure wave (like acoustic ones) within the medium with a finite "velocity" of propagation u_f given by

$$u_f = \left(dp_f \big/ d\rho_f\right)^{1/2} \tag{50}$$

We can then interpret these waves just as "gravitational-pressure" waves by also assuming the conservation of fluid and that its average motion, with respect to the preferred frame E, obeys the second Newton's law.

We must now observe that, in all the previous equations of this section, we have not considered the effect of the spatial variability of G, as supposed in the previous sections. In order to consider now this possibility we note the "classic" gravitational Poisson equation (46) as well as its "equivalent" version (49) in the considered fluid model are obtained by using the Gauss theorem applied to the vectorial flow $\Phi\left(\vec{g}\right)$ of vector $\vec{g}$ across a closed surface ∂V delimiting the volume V containing the matter distribution of total mass M, namely

$$\Phi\left(\vec{g}\right) \equiv \int_{\partial V} \vec{g} \cdot \vec{n}\, dS = -4\pi G M \tag{51}$$

Nevertheless, as known, this important result holds provided that the function describing $\vec{g}$ has a spatial dependence like r^{-2} where r is the radial distance from the centre of mass of the considered massive body.

In our model (see previous sections) we just suppose G could actually vary as a function of physical vacuum density and then, ultimately, as a function of the distance from the mass generating the gravitational field (or, equivalently, the fluid density perturbation in the fluid model). In this case, for a point-like mass m, we can assume a generalized Newton's law and write the gravitational field at a point P, as

$$\vec{g}\left(r\right) = -G\left(r\right)\left(m/r^2\right)\vec{u}_r \tag{52}$$

where $\vec{u}_r$ is the unitary vector from the mass to the point P and r its distance form the mass.

Equation (52) doesn't in general satisfy the Gauss theorem (51) due to the presence of the function $G\left(r\right)$ and, in the most general case of a mass distribution characterized by a density function $\rho_M\left(\vec{x}\right)$, we have

$$\Phi\left(\vec{g}\right) = \int_{\partial A} \vec{g}\left(\vec{x}\right) \cdot \vec{n}\, dS \tag{53}$$

with

$$g\left(\vec{x}\right) = = -\int_{V} G\left(\left|\vec{x} - \vec{x}'\right|\right)\left(\vec{x} - \vec{x}'\right)\rho_M\left(\vec{x}'\right) \Big/ \left|\vec{x} - \vec{x}'\right|^3 d^3x' \tag{54}$$

where $\vec{x}$ is a point of space outside the distribution, $\vec{x}'$ a point inside it and ∂A a generic closed surface surrounding the mass distribution.

We can then write

$$\int_{V} \nabla^2 p_f \big/ \rho_f d^3x' = = \int_{\partial A}\left(\int_{V} G\left(\left|\vec{x} - \vec{x}'\right|\right)\left(\vec{x} - \vec{x}'\right)\rho_M\left(\vec{x}'\right) \Big/ \left|\vec{x} - \vec{x}'\right|^3 d^3x'\right) \cdot \vec{n}\, dS \tag{55}$$

and, exchanging the order of integration, gives

$$\nabla^2 p_f = = \rho_f \int_{\partial A} G\left(\left|\vec{x} - \vec{x}'\right|\right)\left(\vec{x} - \vec{x}'\right)\rho_M\left(\vec{x}'\right) \Big/ \left|\vec{x} - \vec{x}'\right|^3 \cdot \vec{n} dS \tag{56}$$

Equation (56) represents our generalization of (49) and of the model described in [32,33,36] to include the possibility of a variable gravitational constant $G\left(r\right)$ based on the idea already developed in our previous works.

V. The Constant G as a Function of Physical Vacuum Energy Density in the Scalar Model of Gravity

A. The relation between gravitational constant G and QV energy density

As well-known, the physical vacuum cannot be considered, due to Heisenberg uncertainty principle, as a void deprived by any physical dynamics but as physical entity manifesting a complex and fundamental background activity in which, even in absence of matter, processes like virtual particle pair creation – annihilation and electromagnetic fields fluctuations, known as zero point fluctuations (ZPF) continuously occur.

The maximum amount of "virtual" energy density $\rho_{QV,\max}$ stored in the "unperturbed" ZPF fluctuations of QV can be estimated by considering the Planck's constants. Planck showed, basing on dimensional arguments, that the values of gravitational constant G, velocity of light c and Planck's constant $\hbar$, it was possible to derive some natural units for length, time and mass, i.e. respectively the co-called Planck's length (l_P), time (t_P) and mass (m_P).

Then he (and we with him) reversed the point of view by considering these quantities as the most elementary ones, from which the "fundamental" constants (as G, c and so on) can be derived.

In order to assume GTR to remain valid up the Planck scale, we must have

$$\rho_{QV,\max} = m_P c^2 \big/ l_P^3 \tag{57}$$

where $l_P = 1.616 \times 10^{-35}\, m$, $m_P = 2.177 \times 10^{-8}\, kg$ when G has the currently accepted constant value $6.67384 \times 10^{-11}\, m^3 kg^{-1} s^{-2}$ and $c = 299792458\, m\, s^{-1}$.

The value of $\rho_{QV} \approx 10^{113}\, Jm^{-3}$ so obtained by (57), can be considered as the maximum possible value $\rho_{QV,\max}$ of QV energy density, since it would represent, within the currently accepted picture, the maximum energy density can exist "without being unstable to collapsing space-time fluctuations" [13] associated to the value given in [1].

As already shown [22,23,25,30,31] the relationship between the gravitational "constant" G and QV energy density ρ_{QV} can be expressed in a "natural" way by noting that, dimensionally

$$\left[G\right] = \left[L\right]^3 \left[M\right]^{-1} \left[T\right]^{-2} \tag{58}$$

and

$$\left[\rho_{QV}\right] = \left[M\right]\left[L\right]^{-3} \tag{59}$$

where we'll indicate from now on, for simplicity, with ρ_{QV} the so-called Mass – Density – Equivalent (MDE) of QV energy density (equal to $\rho_{QV} \big/ c^2$ where ρ_{QV} is the originally defined QV energy density function) referring to it simply as QV energy density, so we can write

$$G = 1 \big/ \left(\rho_{QV} t_P^2\right) \tag{60}$$

where t_P is the Planck's time whose value is $t_P = 5.391 \times 10^{-44}\, s$ in correspondence to $G = 6.67384 \times 10^{-11}\, m^3 kg^{-1} s^{-2}$.

We can then assume that also G is a function of ZPF energy density defining a fundamental property of space itself, originated from QV and related to the most elementary units for time, length and mass by the equation

$$G = l_P^3 \big/ \left(m_P c^2 t_P^2\right) \tag{61}$$

Equation (60) can be naturally generalized to the case of a variable QV energy density by formally assuming

$$G\left(\rho_{QV}\right) = 1 \big/ \left(\rho_{QV} t_P^2\right) \tag{62}$$

Equation (62) also means that, far from any mass, where the quantum vacuum energy density reaches its "unperturbed" value given by (57), the gravitational constant G, given by (62), takes the value given by (61) while, in the proximity of a mass its value varies according to (62). We'll see in the following that its value, at a given point of space will, depend upon its radial distance from the center of mass of the body (or the system of bodies) generating the gravitational field itself.

B. Quantum Vacuum energy density in a gravitational potential according to the Polarizable-Vacuum approach

The results so far obtained allow us to interpret the mass of a body as the place of occurrence of electromagnetic standing – waves of ZPF that determines a storing of electromagnetic energy density within the body itself.

This dynamics of ZPF together with the consideration of other theoretical elements [13] show that, inside the portion of space associated to "electromagnetically useful" volume V_0, the energy density of ZPF reduces giving rise to a standing wave structure in which this energy is "stored". Outside this structure, on the contrary, the QV energy density is higher and determines the gravitational potential associated to that mass.

This view is also coherent with the model already developed in previous works [23,30,31,46] in which the inertial mass of a body or particle is interpreted as the result of the reduction of the local QV energy density determining, in its neighbourhoods (where the QV energy density is higher), an energy density gradient $\Delta\rho\left(\vec{r},t\right)$ which originates the gravitational potential.

The decrease of energy density inside the massive body can be mathematically proved within the model of PV by considering the correspondence between the parameter K and a refraction index n of QV and calculating the expression of vacuum refraction index inside and outside the massive body for a not too strong gravitational field.

In the model so far discussed the energy spectrum related to ZPF modes of standing waves inside the resonant cavity originating the inertia of a body is substantially discrete and includes a finite number of modes whose frequencies are in the interval $\omega_1 \le \omega \le \omega_N$, with $\omega_N \le \omega_{Pl}$. Nevertheless, when the size of cavity increases so do the number of modes and, due to broadening of frequencies, we obtain a continuous-like frequency spectrum. Physically this must be the case since, when the maximum size of the cavity $L_{\max} \to \infty$, all the modes are possible and we obtain the limit of continuum.

We can then interpret the standing waves inside the resonant cavity associated to a massive body like those generated within a continuum elastic fluid medium [13,23,30,31,46] whose properties characterize the QV behaviour.

Actually, the observation that light propagation in vacuum can be modified by the interaction with an electromagnetic field strongly indicates vacuum itself is a special kind of optical medium. Furthermore, the results of the PV approach to GR, showing a deep analogy QV and a dielectric medium, further indicate this vacuum must have an inner structure that can change by the interaction with matter and electromagnetic fields.

This general view is also consolidated within the QFT by the consideration that all the elementary particles are actually QV excitations and by some recent studies, based on this assumption, picturing space-time as arising from a sort of large-scale condensate of more fundamental objects, in which matter is a collective excitations of these constituents, describable by hydrodynamics techniques [18,28].

For such a medium the relation between the (longitudinal) wave propagation velocity v and the medium density ρ can be written as

$$v = \Gamma / \sqrt{\rho} \tag{63}$$

where Γ is a constant related to the specific medium characteristics.

Applying our analogy to (63) we have

$$c = \Gamma_{QV} / \sqrt{\rho_{QV}} \tag{64}$$

where c is the velocity of light, ρ_{QV} is the ZPF energy density and Γ_{QV} a constant related to QV structure. Inserting (64) in (17) and squaring both the members we obtain

$$\rho_{QV} = K^2 \rho_{QV,0} \tag{65}$$

where ρ_{QV} is the QV density in a generic point of the space around the mass, $\rho_{QV,0}$ the asymptotic density in the flat space and K the QV "refraction" index given by (32). It shows that QV energy density in the space surrounding the body is multiplied by a factor K^2 with respect its "unperturbed" asymptotic value. We can finally write the relation between G and QV density by inserting (32) in (65), namely

$$\rho_{QV} = \rho_{QV,0} e^{4GM/rc^2} \tag{66}$$

that also coincides with the expression already found in a previous work [30] by considering the gauge of light velocity introduced in GTR by the presence of a gravitational field.

C. *The relation between G and density in the fluid dynamics theory of gravitation*

As we have seen from the above discussion the (60) allows us to consider G as a function of QV energy density ρ_{QV}. We now make the assumption that $\rho_{QV} = \rho_f$ since both of them refers to the mass density of the fundamental medium we think to "fill" the physical vacuum itself.

This means we can formally write the (56) as

$$\nabla^2 p_f = \rho_f \int_{\partial A} G(\rho_f)(\vec{x} - \vec{x}')\rho_M / |\vec{x} - \vec{x}'|^3 \cdot \vec{n} dS \tag{67}$$

in which at the right side appear the same variables as in (49).

Equation (56) leads, in general, to a field equation different than (49) due to the presence of the function $G(\vec{x})$ nevertheless, if we consider the case of weak and slowly-varying gravitational fields we can also suppose in (67) the function $G(\vec{x})$ to be practically uniform within the volume V associated to the considered massive body and on its delimiting surface ∂A (but varies at the scale $\Delta x \gg R$, being R the "radius" of the greater sphere containing the mass distribution M). With these assumptions the (67) formally reduces to the (49), although it now represents an approximate field equation valid under more specific conditions. From the fluid dynamic standpoint this corresponds to assume a low value of the fluid compressibility K.

We can then linearize the (50) around some reference value $[\rho_{f,0}, \mathrm{p}_{f,0} = f(\rho_{f,0})]$ such as

$$p_f - p_{f,0} = u_f^2(\rho_{f,0}) \cdot (\rho_f - \rho_{f,0}) \tag{68}$$

where, for convenience, we have assumed

$$p_{f,0} = \lim_{|\vec{x}| \to \infty} p_f(\vec{x}); \quad \rho_{f,0} = \lim_{|\vec{x}| \to \infty} \rho_f(\vec{x}) \tag{69}$$

namely the asymptotic values of p_f and ρ_f far from any sources of gravitational fields. By substituting the (68) in (45) we obtain, after some simple algebraic manipulations

$$\vec{g} = -u_{f,0}^2 (\nabla \rho_f / \rho_f) \tag{70}$$

where $u_{f,0} = u_f(\rho_{f,0})$. Now if we assume, as in the case of NG, the field given by (70) is related to a suitable potential function U, namely

$$\vec{g} = -\nabla U \tag{71}$$

we have, using (70)

$$u_{f,0}^2 (\nabla \rho_f / \rho_f) = \nabla U \tag{72}$$

For a spherically symmetric dependence of the functions ρ_f and U upon r such that

$$\rho_f = \rho_f(r); \quad U = U(r) \tag{73}$$

we have, solving the first order differential equation (72)

$$u_{f,0}^2 \ln(\rho_f / \rho_{f,0}) = U(\rho_f) \tag{74}$$

with the boundary condition $U(\rho_{f,0}) = 0$.

VI. The constant G as a function of distance from mass generating gravitational potential

A. *Polarizable – Vacuum approach*

We are now in position to obtain the dependence of gravitational constant G on the distance from the mass originating the gravitational potential.

Equation (66) can be formally rewritten as

$$\rho_{QV}(r) = e^{4G(r)M/rc^2} \rho_{QV,0} \tag{75}$$

where we have just explicitly expressed the functional dependence of ρ_{QV} and G on r and assumed the value of $\rho_{QV,0}$ as constant. Multiplying the (75) side by side by t_P^2, taking the reciprocals and using (62), we have

$$G(r) = G_0 e^{-4G(r)M/rc^2} \tag{76}$$

where $G_0 \equiv 1/\rho_{QV,0}t_P^2$. A similar expression for ZPF density can be also obtained by using the (62) in (75), namely

$$\rho_{QV}(r) = e^{4M/r\rho_{QV}(r)c^2 t_P}\rho_{QV,0} \quad (77)$$

Equations (76) and (77) respectively describe the dependence of G and ρ_{QV} on the radial distance r from the mass M generating the gravitational potential.

They are transcendent equations and cannot be solved analytically but their qualitative behaviour can be discussed in the case of weak and slowly-varying gravitational fields. In this case we can expand the exponential factor in (76) obtaining, at the first order in G

$$e^{-4GM/rc^2} = 1 - 4GM/rc^2 + ... \quad (78)$$

Using this result in (43) we find

$$G(r) = G_0\left[1 - 4MG(r)/rc^2\right] \quad (79)$$

Equation (79) is a first order approximate equation for $G(r)$ that can be immediately solved to give the solution

$$G(r) = G_0/\left(1 + 4G_0M/rc^2\right) \quad (80)$$

The asymptotic behaviour of this function appears to be coherent with general physical assumptions since we have

$$\lim_{r\to+\infty} G(r) = G_0$$
$$\lim_{r\to 0} G(r) = 0 \quad (81)$$

the case $r \to +\infty$ corresponding to a point far from gravitational source (in which G assumes its "unperturbed" value G_0), while the case $r \to 0$ to point at the center of spherically symmetric mass in which gravitational field is zero.

B. Fluid dynamics model of gravitation

Using the condition (72), the solution of (73) is

$$\rho_f(r) = \rho_{f,0}e^{U(r)/u_{f,0}^2} \quad (82)$$

now, recalling that $u_{f,0}$ represents the asymptotic value of the gravitational perturbation propagation speed (i.e. when there are no gravitational fields), we must assume [33]

$$u_{f,0} = c \quad (83)$$

where c is, as usual, the velocity of light in the vacuum.

The great experimental accuracy of NG imposes that every "alternative" theory of gravity must "reduce" to NG under the appropriate conditions so we assume, in the weak and low-varying gravitational field case, $U(r) = -GM/r$ and obtain

$$\rho_f(r) = \rho_{f,0}e^{-GM/rc^2} \quad (84)$$

then, using the (62) as in the previous section

$$G(r) = G_0e^{G(r)M/rc^2} \quad (85)$$

We note that (84) is similar, as regards its functional dependence, to (76), obtained within the PV approach in the previous section, although different in its mathematical behaviour and physical meaning as we'll see in the following.

The same reasoning adopted in the last section allows us to write, in this case

$$G(r) = G_0/\left(1 - G_0M/rc^2\right) \quad (86)$$

that differs from (80) in the sign and a multiplicative factor in front of G as expected by (84).

It is very interesting to note that, within the fluid pressure model of gravity, it is possible to derive the (85) also without using the quantum "definition" of G given by (62), in the static linearized and spherically symmetric case of the gravitational field generated by a spherical mass M of radius R. This can be easily viewed by considering that, for $r > R$, the (49) is just the Laplace equation, because $\rho_M(r) = 0$, whose general solution, in spherical coordinates, is given by

$$p_f(\vec{r}) = p_f(r) = A + C/r \quad (87)$$

where A and C are two constants depending on boundary conditions for p_f. In particular we have, from our assumptions

$$p_f(r \to \infty) = A = p_{f.0} \quad (88)$$

and, since by (84) $p_f(r)$ is a decreasing function of r, it must be $C < 0$ so, by inserting (87) in (45) and using (68), we finally have

$$g = C/\left(r^2\rho_{f,0} + Cr/c^2\right) \quad (89)$$

For large values of r the (89) must give the corresponding NG expression $g = -GM/r^2$ so we have the equation

$$C/\left(r^2\rho_{f,0} + Cr/c^2\right) = -GM/r^2 \quad (90)$$

which gives

$$C = -\rho_{f,0}GM \quad (91)$$

that, substituted in (89), gives

$$g = -\left[G/\left(1 - GM/rc^2\right)\right]\left(M/r^2\right) \quad (92)$$

This result, already found in [33], is here interpreted in completely different way, not previously recognised, by assuming a variable value of gravitational constant G given by the expression

$$G(r) = G/\left(1 - GM/rc^2\right) \quad (93)$$

where G indicates the "unperturbed" value of gravitational constant, so that the field produced by a massive body M at a distance r from it in the spherically symmetric case and for a weak and low-varying gravitational field is given by

$$g = -G(r)M/r^2 \quad (94)$$

that is formally identical to the known Newtonian expression provided that G varies according to the (93). It is also remarkable to note that the (93), with $G \equiv G_0$, is identical to (86), obtained from the same model but assuming the relation (62) between G and QV energy density.

VII. Discussion

Both the models of gravitation, alternative to NG, discussed in this paper are based on the assumption that physical vacuum is not really empty but "filled" with a constitutive medium, respectively a fluid in one case and an elastic medium characterized by optical properties in the other, somehow related to an underlying dynamics of QV or ZPF whose large-scale manifestation is the emergence of a classical variable of state, the medium density, governing its dynamics. On this point, in a previous paper [18], the author has in fact shown that the space-time described by GR can be in principle considered as the low-energy long-wavelength geometro - hydrodynamic limit of a more fundamental dynamics of the physical vacuum, considered like a Bose – Einstein condensate. Within this picture, in fact, it has been shown that, whatever micro – scale QV dynamics we could consider, the most readily decohered variables, namely those having the highest probability to become classical (and then representing the Universe at large scales), are just density and pressure (ρ, p), i.e. precisely the hydrodynamic variables of state characterizing the models of gravitation above here discussed.

In both of these, in fact, gravitation results from the occurrence of pressure gradients in the medium constituting physical vacuum that determine a "pushing" contact force towards the regions of space characterized by lower pressure and density so actually acting as an "Archimede's thrust".

Nevertheless, although the final effect is the same in both the models, namely the generation of pressure and density gradients, the dynamics by which they are generated is very different in the two cases.

As we have seen, in the PV approach the physical vacuum is pictured as on optical – like medium characterized by a variable polarizability K whose changes are induced by the presence of a massive body.

This implies the velocity of light and other fundamental quantities, as the energy and mass of a free particle and the length and time intervals, are "gauged" as functions of K. If we now assume that mass, in this medium, is due to the local formation of stationary elastic waves, due to the ZPF interactions, we can relate the medium density around a body to its asymptotic value far from gravitational filed by means of the light velocity gauge also expressing it as a function of medium polarizability. As a result the physical vacuum density is expected to decrease in the space with the distance from a massive body according to (77).

This behaviour is also coherent with the picture of the physical vacuum as optical medium in which the bending of light around a massive body (explained, within GR, as the curvature of space-time) is due to the local increase of refraction index (in turn related to a medium density rise) around the massive body.

According to this model, gravity is then originated by a ZPF energy density gradient due to the presence, at a given point of space, of a massive body. Inside the body, the ZPF energy density is decreased, giving rise to the standing waves structure described by (15), while correspondingly increases outside it.

When two massive bodies are close each other, this ZPF energy density rise in the space between them is necessarily smaller than that occurring when the body is isolated, so giving rise to a "depressure" manifesting itself as gravitational attraction. We could then think of the mechanism generating pressure gradients as an indirect process (or, in other words, as a "two – step" process) in which the first step consists in the increase of the vacuum density outside a single massive body while the second one in the emergence of the pressure gradient due to the presence of two or more massive bodies in a given region of space from which gravity originates as a pressure force.

On the contrary, in the fluid – like model of physical vacuum the pressure gradient originating the gravitational force of attraction due to a massive body is directly given by (45), according to which gravitational field is directed along the direction in which density decreases, namely towards the massive body.

Nevertheless it must be stressed that, within this model as it is, no physical explanation of such reduction of density around the body is given but it is a priori assumed as the consequences of some other preceding fluid-dynamic process (as, for example, the formation of vortices [32]).

An important remark concerns the physical meaning of the quantity G_0: it represents the value of G at a point "infinitely" far from mass M in which the ZPF is unperturbed. Its value should be determined by experimental measurements (far from any mass) or extrapolated by means of the know value of gravitational field at a given distance from a mass M.

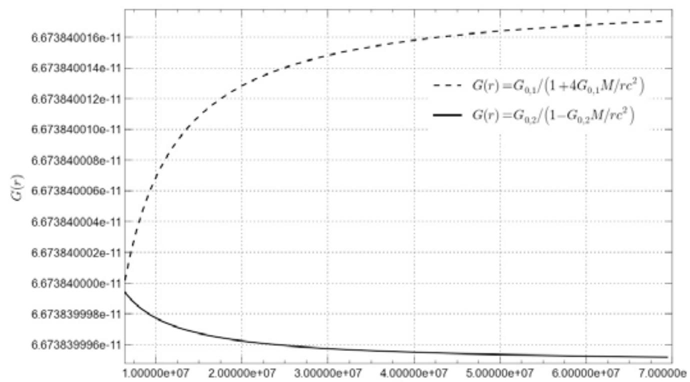

Fig. 1. Plot of G vs distance from Earth center for $r \geq R_{Earth}$. The values of G and r are expressed in standard units.

Contrary to what one could think, the value of G_0, within both the discussed models, is not equal, in principle, to the quantity $l_P^3 / m_P c^2 t_P^2$ with the Planck units given by the commonly accepted values because the latter in turn are calculated by assuming the value of G given in [1] (namely measured at Earth's surface or deduced by astronomical observations [47] in the presence of massive bodies). Furthermore, we should also consider the contributions to ZPF, and then eventually to G, coming from strong and weak interactions, at this stage not still included in our model of the function describing $G(r)$ (and not considered in the HPR model of inertia as well [12,13]).

It is remarkable to note that (80) and (86), like the more general (76) and (85), doesn't contain any Planck's unit, allowing, in principle, the calculation of G using only the value of G_0, experimentally determined.

A possible estimation of G_0 could be obtained by using the know value of G at Earth's surface, as given by (1), in the (76) and (85) with $r = R_{Earth}$ and $M = M_{Earth}$. Following this procedure we obtain, from (76)

$$\begin{aligned} G_0 &= G\left(R_{Earth}\right)\times \\ &\times \exp\left[4G\left(R_{Earth}\right)M_{Earth}\big/R_{Earth}c^2\right] = \\ &= 6.673840019\times 10^{-11}\, m^3\cdot kg^{-1}\cdot s^{-1} \end{aligned} \quad (95)$$

and, from (85)

$$\begin{aligned} G_0 &= G\left(R_{Earth}\right)\times \\ &\times \exp\left[-G\left(R_{Earth}\right)M_{Earth}\big/R_{Earth}c^2\right] = \\ &= 6.673839995\times 10^{-11}\, m^3\cdot kg^{-1}\cdot s^{-1} \end{aligned} \quad (96)$$

where we have assumed $c = 299792458\, m\cdot s^{-1}$, $R_{Earth} = 6372.7955\times 10^3\, m$ and $M_{Earth} = 5.9736\times 10^{24}\, kg$.

The values of G_0, given by (95) and (96) represent the asymptotic value of gravitational constant calculated by considering the gravitational field generated by the Earth as if it was far from all the other masses of Universe.

By using these values of G_0 we can plot, by way of qualitative example, the functions $G(r)$ respectively given by (80) and (86) as a function of the radial distance r (Fig. 1) from the Earth center.

In evaluating these graphics we must remember that (80) and (86) just represent an approximation, at first order, of the value of $G(r)$ that is valid when $\Delta G \to 0$, so they don't necessarily give the actual numerical values of the "whole" functions $G(r)$ in particular in the slope-region of the curves, since it is just here the contribution of the higher order terms could be more important; nevertheless they give correct indications about their qualitative behaviour (under the assumed hypotheses) for $r > R_{Earth}$ and the asymptotic behaviour for $r \to +\infty$.

We just note the numerical values given by (95) and (96) are respectively slightly higher and lower of that commonly assumed [1] in agreement with the predictions of the two models. However the quantity $\Delta = \left|G_0 - G\left(R_{Earth}\right)\right|$ is always very exiguous, appearing at eight decimal digit in the first case (PV approach) and ad the fifth decimal digit in the second one (fluid model) so also making it hard to be experimentally revealed by a direct measurement.

This difficulty is in part also due to the need for performing such possible measurement far from any mass able to influence the results or, equivalently, within a region characterized by a distribution of masses able to nullify, with a very high precision, the gravitational field at the measurement point.

Nevertheless, to this regard, we should recall the estimations given by (80) and (86) are respectively based on the simplifying assumptions (68) and (78) so that the inclusion of higher order terms in these expressions could modify the value previously calculated, so making the difference $\left|G_0 - G\left(R_{Earth}\right)\right|$ larger and then more easy to reveal.

Furthermore, even if the value of Δ was so small for a giving mass, this wouldn't imply the resultant physical effects is negligible in the proximity of a system of massive bodies, because of the summation of contribution of each of them to the overall value of G.

Both the models here considered, although starting from different dynamical hypotheses, finally allow for a variable value of gravitational constant $G(r)$, whose expression is respectively given by (80) and (86) in the simplified case of weak and low-varying gravitational field and spherical symmetry.

Nevertheless, although characterized by a nearly identical functional form, they show an opposite behaviour as function of r, namely diminishing with the distance from mass in PV approach and increasing in the fluid model.

This discrepancy is direct consequence of the different starting dynamic model assumed in the two cases: an elastic medium allowing for the generation of transversal waves in the PV approach and a fluid medium determining the propagation of longitudinal compressive waves (like the acoustical ones) in the fluid model.

This deep difference naturally pones a further critical question about the propagation of light in the latter model, since an ideal classical fluid is not be able to carry transversal waves.

It is then clear that only a complete definition of a starting dynamical model describing the interaction between matter and physical vacuum [45] (also in term of quantum processes) and how the former modifies the vacuum density could be able to give unambiguous indications about the actual variability of G over large distances.

A possible and advisable indication in this direction could came, on the other hand, by experimental tests of the above results.

In principle, a possible experimental test of (76) and (85) could be performed by measuring (for example on a satellite, orbiting around the Earth), with very high precision, at specified distances $r_{SAT,i}$ from the Earth center, the values of $G\left(r_{SAT,i}\right)$, comparing the latter with that measured at the Earth's surface $G\left(R_{Earth}\right)$. More specifically we can write $\forall i$, using (76)

$$G\left(R_{SAT,i}\right) = G_0 e^{-4G\left(r_{SAT,i}\right)M/r_{SAT,i}c^2} \quad (97)$$

and

$$G\left(R_{Earth}\right) = G_0 e^{-4G\left(R_{Earth}\right)M/R_{Earth}c^2} \quad (98)$$

Dividing side by side (97) by (98) we obtain the equation

$$G\left(r_{SAT,i}\right)/G\left(R_{Earth}\right)= = Exp\left\{-\left(4M/c^2\right)\left[G\left(r_{SAT,i}\right)/r_{SAT,i}-G\left(R_{Earth}\right)/R_{Earth}\right]\right\} \quad (99)$$

in which only measurable quantities and known constants appear.

A similar procedure applied to (85) gives instead

$$G\left(r_{SAT,i}\right)/G\left(R_{Earth}\right)= = Exp\left\{\left(M/c^2\right)\left[G\left(r_{SAT,i}\right)/r_{SAT,i}-G\left(R_{Earth}\right)/R_{Earth}\right]\right\} \quad (100)$$

In any case, in a realistic set-up, very high measurement precisions would be required in order to reveal the minimal variations between the values of $G(r)$ given by (99) and (100) at different distances from Earth.

Furthermore, the influence of other celestial bodies (firstly the Sun and the Moon) on the ZPF energy density at the measurement points should be taken into account. This would introduce additional terms into (68) and (78) able in principle to modify the form and behaviour of solutions and represents an important theoretical question to be addressed in the future developments of the model, already in progress.

VIII. Conclusions

In this paper two models of gravitation, different than NG but in principle reducible to it, allowing the possibility to admit the variability of gravitational constant G have been presented and discussed.

Both these models start from the fundamental hypothesis that physical vacuum is not really empty but "filled" with medium (*de facto* representing the space itself), a large-scale manifestation of some quantum substratum [18] related to ZPF dynamics, and assume a functional relation between G and QV energy density through the Planck's units.

However these two models are based on different dynamical approaches concerning the kind and features the above medium: in fact the first one treats it as a polarizable elastic medium while the second one as a compressible fluid medium.

In the first model the PV approach to Einstein's GTR is considered, according to which the physical vacuum can be pictured as a dielectric polarizable medium, characterized by a refraction index as function of the gravitational field, in which all the modifications induced by the presence of mass, as described by GTR, can be viewed as due to the altered value of the above refraction index. Then starting from some previous theoretical results, the inertial and gravitational mass of a body have been interpreted as the result of the seat of standing waves of ZPF analogous to longitudinal waves generated inside a continuum elastic medium.

These waves are able to alter the local QV energy density determining a decrease of ZPF energy density within the massive bodies and, correspondingly, a ZPF energy density increment in their surrounding space so generating QV energy density gradients (unbalanced ZPF pressure) giving rise to the gravitational force.

The second model pictures the physical vacuum as a compressible fluid medium, interpreting gravity as an "Archimede's thrust" due to the decrease of fluid density associated to the presence of a massive body.

In both the models gravity emerges as a "pressure" force generated to the density gradients induced in the physical vacuum by the presence of the massive bodies pushing them towards the regions with lower density and pressure.

In this paper we have shown these models imply physical vacuum density around a massive body depends on the distance from it and, consequently, that gravitational constant G also depends on ZPF energy density and on the distance from the mass generating the gravitational field.

Furthermore, in the simplified case of a spherically symmetric massive body, an approximate analytical expression of the function $G(r)$, describing the radial dependence of gravitational "constant", has been obtained for each of the two considered models.

Nevertheless, as we have seen, they describe an opposite behaviour of $G(r)$ with increasing of r from mass, namely an increasing of $G(r)$ in the first model and its decreasing in the second one.

This is a direct consequence of the different mechanism by which matter is supposed to interact with physical vacuum, altering its energy density in the two cases.

Despite this dichotomy, which could be solved only within a more general model of the interaction between physical vacuum and matter whose attempt in under progress [45], the obtained results appear very interesting since they show that the gravitational "constant" G could be actually variable with the distance from massive objects, depending on the physical vacuum density.

Furthermore, as we have also shown in this paper, the variability of G seems to naturally appear in compressible fluid model, even without firstly assuming the quantum relation between G and QV energy density, so suggesting a deep connection between this variability and the model of gravity as a pressure force due to vacuum density gradients.

Although the theoretical model assuming a functional connection between gravitational constant and physical vacuum energy density is still in a preliminary phase and involves some simplifying assumptions to be addressed in its future developments, its theoretical, experimental and applicative consequences could be very deep. They will be discussed in details in future and forthcoming publications.

Finally, important insights about the actual dependence of G on r could be given by precision measurements of gravitational constant performed at different distances from a massive celestial body like, for example, those possibly performed on an artificial satellite in orbit around the Earth.

A variable gravitational constant G could have very deep consequences on the current framework of theoretical physics with from GTR to Quantum Field Theory (QFT) and cosmology and unthinkable possible applications in intriguing fields as, for example, the gravity modification and space propulsion.

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