Forecasts of dynamic response for structural systems with low robustness

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***Abstract*—** In the paper one addresses the problem of forecasting the dynamic response of structures behaving according to rocking modes. Purely rotational motions do often occur in structures and facilities subject to seismic solicitations, turning those in monolithic rigid bodies or assemblies. Rocking dynamics, thus, usually interests a wide variety of structures and objects, also including historical and monumental constructions or ancient art items, usually made of large-dimension monolithic stone blocks. Nevertheless, the high non-linearity and complexity affecting the rocking dynamics of rigid bodies make hard to perform some reliable forecasts about their response, and push towards the adoption of alternative strategies.

***Keywords*—**Modeling, Structures, Dynamics, Response forecast, Monolithic bodies, Earthquakes.

I. INTRODUCTION

IN mathematical models of structural phenomena a number of approximations or simplified hypotheses are usually necessary and admitted, either because of difficulties in reproducing the complexity of the real conditions and because of the need of making the models manageable for practical purposes and further investigations.

This results in some uncertainty embedded in the model itself, which superposes to the uncertainty affecting the real modeled phenomena, which turns into some uncertainty in both the input and output data.

In the following, with reference to structures behaving according to a rigid mode under seismic-type dynamic excitation, one discusses the desirability of adopting worst scenario approaches for producing reliable forecasts about the response of such structures.

II. REAL PROCESSES AND WORST-CASE ANALYSES

The prediction of real processes thus appear deeply influenced by uncertainty, which should be handled or treated somehow in order to guarantee some degree of reliability of the relevant forecasts.

The availability and amount of input data plays a central role in the choice of the approach to be adopted.

In case when one does not have at one's disposal the probability characterization of input data, stochastic methods should be discarded, rather preferring some worst scenario or fuzzy approaches.

In engineering applications where one has to check that some output variables of the problem do not exceed some given thresholds at selected locations, worst scenario approaches are aimed at searching for the input data that maximize those variables.

The state problems can stand for differential or integral equations, or a system of linear equations, thus allowing the treatment of a wide variety of problems [1]-[4] relevant to new, existing or ancient structures [5]-[20], where main sources of uncertainties are encountered.

Fig.1: Electrical transformer failure during the 1971 San Fernando earthquake, and remains of the

Usually the problem may be, then, set up by selecting a suitable criterion functional, with the objective of identifying the values of the sets of parameters it depends on that make it maximum.

Seismic analysis and design of buildings finds a powerful tool in worst scenario approaches, since the common objective of reliably forecasting the structural response of constructions or designing earthquake-resistant structures consists of ensuring their structural safety in the worst situation, that is to say against the worst possible future earthquakes.

This also in the light of future dynamic load, that may be hard to be forecasted in their properties [21]-[22].

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This approach is also conceived in the framework of the planning of strategies aimed at improving the building-earthquake resilience, also by means of the adoption of new reinforcement techniques [23]-[28] and dynamic control strategies [29]- [35].

To this regard, it has been referred to also for considering the uncertainties of structural parameters in structural control applications and investigating the worst-case scenario for dynamically passively controlled buildings.

The importance of the worst-scenario approach for improved earthquake resilience of buildings and nuclear reactor facilities has been recognized an extreme significance especially for large or important structures in particular after the recent great earthquake (March 11, 2011) in Japan, making one of the most important and challenging missions of structural engineering to narrow the range of unexpected incidents in building structural design and emphasizing the important role of redundancy, robustness, and resilience in such circumstances [36].

III. About Forecasting the response of rigidly moving structures

III.1 Response forecast of structures under rocking dynamic mode

The rocking response and the possibility of overturning of rigid bodies in earthquakes are central considerations in seismic safety problems.

A broadly similar response can be observed, during earthquakes, in a variety of structures, such as electrical equipments (Fig.1), retaining walls, liquid storage tanks, tall rigid buildings such as bell-towers or minarets, or single structural elements, like single columns or walls, or sets of elements, like colonnades or soft floors (Fig. 2).

A similar behavior may be observed in sculptures and remnants of ancient Greek and Roman stone temples, which may result in the damage of precious and ancient pieces (Figs 1 and 3).

Moreover rocking structures may be realized on purpose in order to avoid structural damage by shifting the burden of energy dissipation to non-critical, replaceable structural elements, and by preventing weak story failure (Fig. 4). Thus, by enabling structures to be serviceable after a seismic event, rocking systems are a highly sustainable approach to structural design in earthquake-prone regions [37].

The need of understanding, forecasting the rigid response and predicting associated failures have motivated a number of studies on the rocking response of rigid blocks [38]-[39].

As concerns the vulnerability of systems behaving like rigid blocks, one should first emphasize the poor robustness of analysis of rocking blocks, which usually also couple the rocking mode to other rigid modes (Figs 5 and 6).

The block is shown to posses extremely complicated dynamics, with many different types of response being revealed, sensitivity to initial conditions, and big uncertainty in the prediction of the asymptotic dynamics.

Actually low robustness of dynamic behaviour of rigid blocks may be proven, since the superposition of a null distribution such that the time-displacement of the system is not formally altered produces non-null effects on the response of the system, with the impact obeying to a strongly non-linear equation, as it will be shown in the following Sect.III.2; the additional null term is able to produce an effect similar to the one of the restitution coefficient, thus pointing out its very aleatory nature. This is somehow confirmed by the tracing of overturning domains, which have been shown to exhibit significantly irregular shapes.

Moreover, one should consider all the problems of practical nature that deeply influence the dynamics of rigid blocks, such as imperfections in the geometry of the base section with non perfect angles of the parallelepiped, imperfections in the level surface of the block basis, and imperfections in the orthogonality of the excitation with respect to the middle plane of the block.

Fig.2: Collapse of soft floors or collonades.

Fig.3:Historical monuments and artistic objects undergoing rocking mode: Remains of the amphitheatre in S. Maria Capua Vetere, the Pisa tower, Milo Venus.

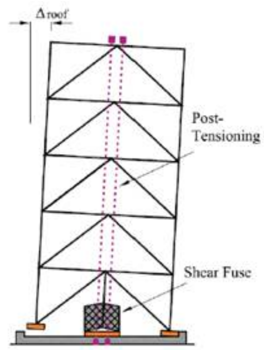


Fig.4: A rocking frame structures [37].

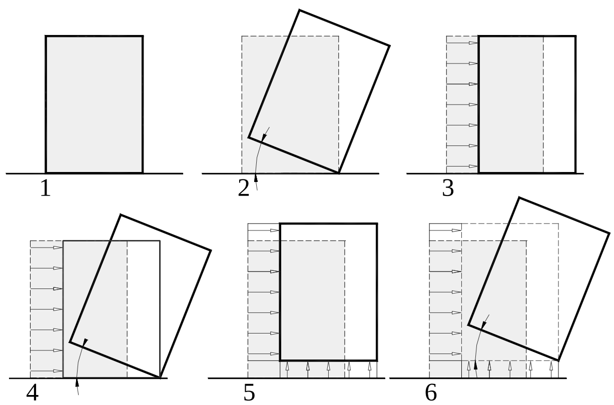


Fig. 5: Response modes of rigid block moving in a plane: (1) rest, (2) rotation, (3) slide, (4) slide and rotation, (5) slide and bounce, (6) rotation, slide and bounce.

Without considering the problems related to the analysis of blocks with non-rectangular basis and other problems possibly occurring during the motion also depending on the material composing the block itself and other phenomena, such as the blunting of the base edges, which has been shown to be able to deeply change the dynamic properties of the block, significantly affecting its response.

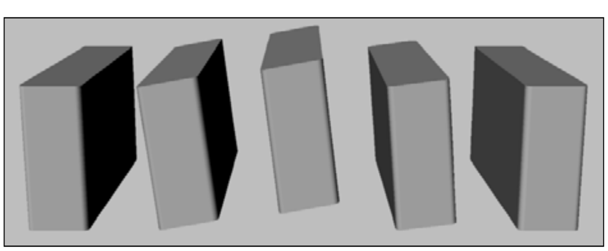


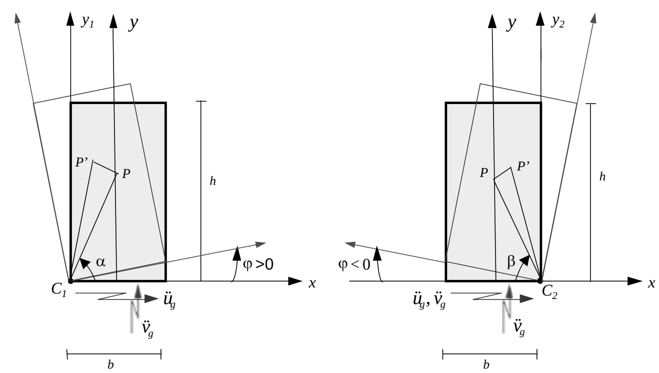
Fig.6: The 3-D rigid parallelepiped model and dynamics' simulation.

III.2 Some considerations on robustness of pure rocking dynamics

As emphasized in the above, rocking motion (RM) is a dynamical process that frequently occurs in nature. When a rigid body instantaneously changes its centre of rotation from one point to another the resulting motion is usually described in terms of differential equations for certain domains of the dynamical variables involved.
For simple planar motions, these domains can be assigned to the sign of the angular coordinates.

In spite of the apparently simple nature of the motion, this process hides both a great richness in dynamical behavior and a wide range of practical relevance.

After denoting by u_g and v_g respectively the horizontal and vertical ground displacement components and marking by one or two superimposed dots their first and second order time derivatives, one assumes that the rigid block model depicted in Fig.7 is subject to the soil acceleration with components $\ddot{u}_g, \ddot{v}_g$, which induces rotations φ of the block (the dependence of variables on the time variable ″t″ is implicitly meant).



(a) (b)

Fig.7: Rigid block under the action of the mass and surface forces for:
a) counter-clockwise rotations and b) clockwise rotations.

Because of the unilateral nature of the hinges that are activated alternatively during the motion at the basis corners of the block, uniquely counter-clockwise rotations are allowed around the rotation centre C_1 on the right side of the block and clockwise rotations around the rotation centre C_2 on the left side of the block.

By considering positive rotations around the rotation centre $C_1(x_{C1}, y_{C1})$ with respect to the axes system <x,y> (Fig.7.a), under the hypothesis of large displacements, the generic point P(x,y) of the block assumes the updated position P′(x′,y′) with coordinates

Because of the unilateral nature of the hinges that are activated alternatively during the motion at the basis corners of the block, uniquely counter-clockwise rotations φ are allowed around the rotation centre C_1 on the right side of the block and clockwise rotations around the rotation centre C_2 on the left side of the block.

By considering positive rotations φ_{C1} around the rotation centre $C_1(x_{C1}, y_{C1})$ with respect to the axes system $\langle x,y \rangle$ (Fig.7.a), under the hypothesis of large displacements, the generic point $P(x,y)$ of the block undergoes the displacement $s_P(s_{Px}, y_{Py})$ assuming the updated position $P'(x',y')$.

Rotations around $C_2(x_{C2}, y_{C2})$ are then denoted by φ_{C2} . Displacements around the two corners are denoted by $\mathbf{s}_{\mathbf{P1}}(s_{P1x}, s_{P1y})$ and $\mathbf{s}_{\mathbf{P2}}(s_{P2x}, s_{P2y})$ respectively.

After introducing the Dirac function $\delta(t)$ and the related functions

$$H_1(x) = \int_{-\infty}^{x} \delta(t)dt, \quad H_2(x) = 1 - \int_{-\infty}^{x} \delta(t)dt \tag{1}$$

one may express the displacement vector, as follows

$$\mathbf{s}_P = H_1(\varphi) \cdot \mathbf{s}_{P1} + H_2(\varphi) \cdot \mathbf{s}_{P2} \tag{2}$$

After some developments, one may get the rotational acceleration (denoted by double superimposed dots of the rotational variable φ) in the form

$$\ddot{\varphi} = H_1(\varphi)\ddot{\varphi}_{C_1} + H_2(\varphi)\ddot{\varphi}_{C_2} \tag{3}$$

with

$$\ddot{\varphi}_{C_1} = -2\frac{G(\varphi,\dot{\varphi}) \cdot (V_y \cdot s_{P2x} - V_x \cdot s_{P2y}) + (\ddot{v}_g \cdot V_x - \ddot{u}_g \cdot V_y)}{V_y \cdot [2F(\varphi) \cdot s_{P2x} + \cdot s_{P1x}] - V_x \cdot [2F(\varphi) \cdot s_{P2y} + \cdot s_{P1y}]}$$

$$\ddot{\varphi}_{C_2} = -2\frac{G(\varphi,\dot{\varphi}) \cdot (S_y \cdot s_{P2x} - S_x \cdot s_{P2y}) + (\ddot{v}_g \cdot S_x - \ddot{u}_g \cdot S_y)}{S_y \cdot [2F(\varphi) \cdot s_{P2x} + \cdot s_{P1x}] - S_x \cdot [2F(\varphi) \cdot s_{P2y} + \cdot s_{P1y}]} \tag{4}$$

where

$$V = \begin{pmatrix} V_x \\ V_y \end{pmatrix} = \begin{pmatrix} s_{Px} + b/2 \\ h + s_{Py} \end{pmatrix}; \quad S = \begin{pmatrix} S_x \\ S_y \end{pmatrix} = \begin{pmatrix} s_{Px} - b/2 \\ h + s_{Py} \end{pmatrix}; \tag{5}$$

and $F(\varphi)$, $G(\varphi,\dot{\varphi})$ two suitably defined functions depending on the rotation and its time derivative.

It is interesting to observe some anomaly in the description of the dynamic behaviour of the unilateral rigid model.

Let superpose an additional term corresponding to a null distribution to Eq.(2) , as follows

$$\mathbf{s}_P = H_1(\varphi) \cdot \mathbf{s}_{P1} + H_2(\varphi) \cdot \mathbf{s}_{P2} + r\,\delta(\varphi)\varphi^n \cdot (\mathbf{s}_{P1} - \mathbf{s}_{P2}) \tag{6}$$

Thereafter, one realizes that it is possible to change Eq. (2) in way that the time-displacement of the system is not formally altered, by adding a null distribution, like in Eq. (6).

This produces some formal change in $F(\varphi)$, $G(\varphi,\dot{\varphi})$ in Eq.(4).

The impact obeys now to a strongly non-linear equation, and numerical solutions must be sought.

So this aim, the distribution $\delta(x)$ and its derivatives can be substituted by functions $\alpha_k(x)$ and their derivatives, that in the limit uniformly converge to $\delta(x)$ and its derivatives.

The results prove that increasing the coefficient r of the null distribution produces an increasing damping and that this effect is rather uniform when k tends to become larger and larger and functions $\alpha_k(x)$ tend to $\delta(x)$ and henceforth it is expected that the numerical solutions of the equilibrium for the displacement, velocity and acceleration fields converge to the solutions of the theoretical case.

So it is expected that the superposition of a null term to the displacement of the model produces a non-null consequence on its motion, an effect that is one more reason to assess that rocking motion can be strongly affected by the way one approaches the problem.

III.3 Towards worst-case forecasts

The low robustness of rocking response of rigid bodies, its randomness and sensitivity to imperfections, make their seismic assessment a big deal especially because of the high degree of uncertainty of the seismic action.

Results may be stabilized through a worst scenario procedure, which is particularly effective for not sharply defined forcing functions, making the response substantially independent on the details of the excitation.

The method was set forth by Drenick in the 70's [41]-[44] and by Shinozuka in the early 80's [45] with particular reference to linear structures, and more recently by Elishakoff and Pletner [46] and by Baratta et al. [47].

Worst-scenario results are likewise more severe than probabilistic evaluations based on the statistics of the response process, possibly evaluated by MonteCarlo methods, since the probability measure of the worst-scenario may most of the times be zero, despite of the fact that it is anyway one (15) possible realization and also includes possible instability of the mathematical-numerical model.

As in Drenick's original approach, the worst scenario results in a regular forcing function that overpasses many problems due to the desultory character of the forcing function.

In the specific case, a possible worst scenario approach may be set up aimed at defining the theoretical bounds on the maximum value of the structural response under dynamic loading, for non linear structures behaving like rigid blocks rocking around their base edges under (18) dynamic shaking.

Generally speaking, basic characters of the excitation can be identified in the average power spectrum, in its maximum value, its duration and so on.

So a suitable *class* of forcing functions can be reliably defined, by building up a functional space where it is assumed it must be contained.

All the accelerograms in the class differ from each other by random quantities that cannot be predicted in details. In other words the accelerograms in the class form a *stochastic process* that could be treated by means of the methods and procedures of the relevant theory to produce the probability distribution of the maximum response.

Apart from the difficulty to treat the response of a highly nonlinear system in a stochastic context, probabilistic results may be quite illusory depending on the acquisition of statistical data for the parameters of the stochastic process.

By a worst scenario approach one may search for the worst response produced by accelerograms fitting the basic properties that might be defined on the basis of the investigation of the source of the disturbance, but subject to random realizations of the details, thus looking for the worst situation that can occur for a structure when it is acted on by badly defined forcing time-histories.

To this aim, one may set up a constrained optimization process where the response maximization procedure consists of an iterative process, based on the generation at each step of a new accelerogram compatible with the assigned properties of the disturbance and on the evaluation of the response of the structural model.

IV. Conclusions

The paper focuses on approaches to forecasts about the dynamic response of structural systems in the cases when uncertainty deeply affects the response itself of the structure and make ordinary analyses not effective.

In particular one refers to a large class of structures whose behaviour under seismic motion may be simulated by means of the rigid assumption.

The pure rocking mode, which is the more dangerous as regards to possible structural failure, is focused on.

Some considerations are outlined about the low robustness of the rocking mode response, its randomness and sensitivity to imperfections and errors.

Therefore one finally concludes that the seismic assessment of structural response of structures moving according to the unilateral hinges rigid scheme represents a big deal, especially because of the high degree of uncertainty of the seismic action, but even with reference to any forcing action of any shape.

On this basis, one figures out the need of making recourse to different strategies for vulnerability assessment, mostly based on the forecast of worst-case situations.

Acknowledgement

The authors acknowledge the financial contribution of the Dept. Civil Protection through the ReLuis pool (convention signed 27/12/2013).

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