Geodetic support in the context of GIS for monitoring mechanics of movement of the earth's surface in mining subsidence

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***Abstract*—**The mining activity influence on the environment belongs to the most negative industrial influences. As a result of underground mining of the mineral deposits in the surface creates the subsidence trough, i.e. caving zone which could be dangerous for any movement of people in this zone. Character and size of the mining subsidence on the surface depends mainly on the geotectonic ratios of rock massif above the mined out area. Knowing the extent of the subsidence trough in mining areas is determining to prevent the entry of people into these dangerous zones. Conditioning factors to establish the extent of the movement of the earth's surface above the mined out area are a geodetic way surveyed deformation vectors which can be derived from the processing of measurements at monitoring stations based on these mining tangent territories. The limits of undermined regions in many cases equal to isolines connected so called break points occurred in the front of the subsidence borders. The theory for the estimation of polynomial break points in the case of subsidence analysis is presented. The theory was developed as a part of the kinematics analysis procedures for the evaluation of the magnesite deposit in the suburb of Kosice-Bankov on the northern outskirts of the city of Kosice in the eastern Slovakia. The subsurface abandoned mine Kosice-Bankov is located in the immediate vicinity of the recreational and tourist zone in the northern suburb of the city of Kosice. Some numerical and graphical results from the break points estimation in the magnesite deposit Kosice-Bankov are presented. The obtained results from the abandoned mining area Kosice-Bankov were transferred into GIS for the needs of the local governments in order to conduct the reclamation of this mining landscape.

***Keywords*—**Break points, GIS, Mining subsidence, Rock movement, Test statistics.

I. INTRODUCTION

ON the present in accretive exigencies to people and its property protection, there is security one from priority needs and tasks of all countries or their groupings around the world. In the environment protection, which an unspoiled ecosystem is a condition of human living, it is needed to protect people and its property against the negative industrial influences. The mining activity influence on the environment belongs to the most negative industrial influences. As a result of underground mining of the mineral deposits in the surface creates the subsidence trough, i.e. caving zone (area) dangerous for the movement of people in this zone [1], [3], [15], [18].

The gradual subsidence development at the mine region Kosice-Bankov in the eastern Slovakia was monitored by geodetic measurements from the beginning of mine underground activities in the magnesite deposit. The analysis of time factor of the gradual subsidence development continuing with underground exploitation allows production of more exact model situations in each separate subsidence processes and especially, it provides an upper degree in a prevention of deformations in the surface, Possibility in improving polynomial modelling the subsidence is conditioned by the knowledge to detect position of so-called "break points", i.e. the points in the Earth's surface in which the subsidence borders with a zone of breaches and bursts start to develop over the mineral deposit exploitation. It means that the break points determine a place of the subsidence, where it occurs to the expressive fracture of the continuous surface consistence [5], [7], [12], [13], [16]. Currently in mapping of the settlement trough it is used a lot of advanced surveying and recording (mapping) fully automated techniques and technologies [2], [4], [6], [9], [14], [24].

II. RESEARCH OVERVIEW – THE STUDY CASE KOSICE-BANKOV

Problems of mine damages on the surface, dependent on the underground mine activities at the magnesite deposit, did not receive a systematic research attention in Slovakia till 1976. After that, the requirements for a scientific motivation in the subsidence development following out from rising exploitations and from introducing progressive mine technologies were taken in consideration. The monitoring deformation station Kosice-Bankov covers an area around the mine field of the magnesite mine in Kosice-Bankov. Kosice-Bankov is in the northern part of the city of Kosice, where the popular city recreational and tourist centre of the city of Kosice is situated. This popular urban recreational area is located in close proximity to the mining field of the magnesite mine Kosice-Bankov (Fig. 1).

The paper followed out from the project VEGA No.: 1/0473/14 researched at the Institute of Geography, Faculty of Science, Pavol Jozef Safarik University in Kosice, Slovak Republic. The research was supported in part by the Scientific Grant Agency (VEGA) of the Ministry of Education, Science, Research and Sport of the Slovak Republic.

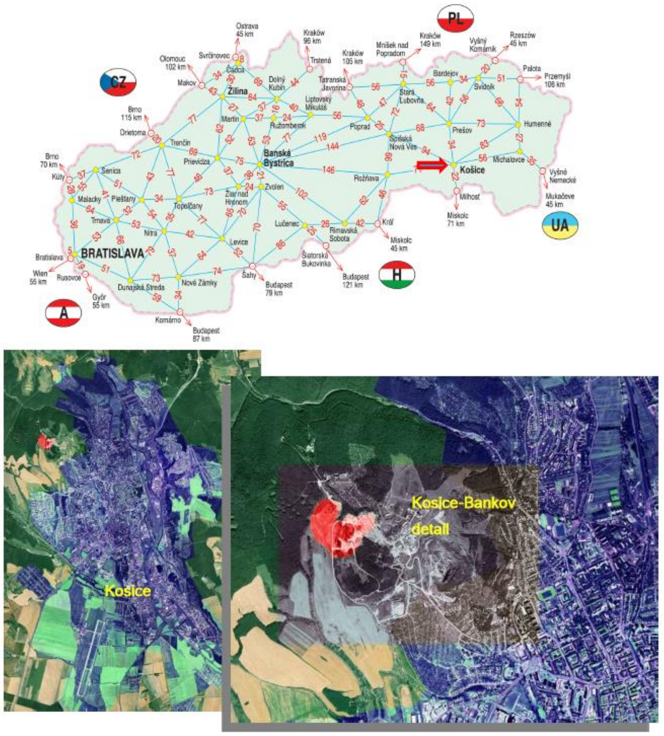

Fig. 1 Ortho-photo map of the city of Kosice with the detail view on the mine field mine Kosice-Bankov; red area is the mining subsidence over the magnesite mine

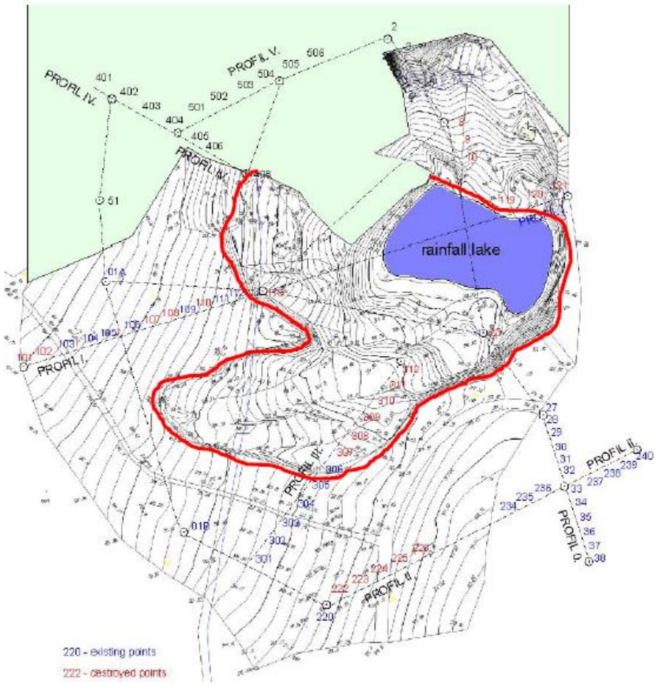

Fig. 2 Monitoring station Kosice-Bankov (1:3,000);red curve is the outline of the subsidence, green area – the forest park Kosice-Bankov

All surveying profiles of the monitoring station Kosice-Bankov are deployed across and along the expected movements in the subsidence (Fig. 2). 3D data were firstly observed by 3D (positional and levelling measurements) terrestrial geodetic technology (since 1976) using total electronic surveying equipment and later also by GPS technology (since 1997). Periodic monitoring measurements are performed at the monitoring station Kosice-Bankov twice a year (usually in the spring and autumn) [19]–[22].

II.1 One Dimensional Deformation Analysis from Levelling Networks

In accordance with the general phases of the geodetic deformation analysis the project at hand was defined to contain the following phases [20], [22]:

1. Single epoch evaluation of the levelling data available.
2. Stability evaluation of reference benchmarks (points of the monitoring station).
3. Estimation of the most likely deformation model.

The single epoch evaluation concentrates on the evaluation of the functional model, the observational data and the stochastic model. By means of the integration of the hypothesis testing, including outliner detection and variance component estimation the consistent mathematical model is obtained. In the second phase of the project the assumption in the functional model of stable reference benchmarks is tested. Unstable benchmarks are removed from the set and will further be treated as objective points. After establishing the correct functional model, the stochastic model may be improved as well. Again, we obtain a consistent mathematical model results.

To arrive at the most likely mathematical model describing the deformation pattern underlying the data is the aim of the third phase. The functional model part is restricted to 1D, 2D, 3D and 4D polynomials. The mathematical model is again balanced by modifications of the stochastic model [19], [21].

II.2 Polynomial Break Points

In the project described the third step consists again of three different steps, i.e. [21], [22]: the following phases [20], [22]:

1. Estimation of 1D-polynomial model per benchmark.
2. Estimation of 3D-polynomial model per selection benchmarks.
3. Evaluation of possible external height-information available.

When evaluating the estimated time-dependent polynomials per benchmark it become more and more apparent, that such a polynomial could not accurately describe the behaviour of these benchmarks which came under the influence of the mineral deposit extraction sometime after the start of the exploration. Such behaviour was described by higher order polynomials, whereas it was actually due to a break in the trend of the subsidence.

Allowing the polynomial function to have a so-called “break point”, which is defined as, may solve this problem, which is defined: A point in time at which a benchmark, due to the mineral deposit extraction, enters the subsidence area (Fig. 3). The estimation of the polynomial break points is a part of the procedure developed to establish the most likely mathematical model, describing the subsidence behaviour of a specific benchmark in time. The procedure is based on the concept of least-square estimation and multiple hypotheses testing [11], [17].

Fig. 3 Break points zone on the subsidence edge of the undermined territory Kosice-Bankov;
red arrow – the zones of break points

II.3 Hypothesis Testing

In general, the mathematical model under null-hypothesis may be modelled in terms of observation equations [11], [17], [21], [22]

$$H_o : E\{\underline{y}\} = \boldsymbol{A}x; \qquad D\{\underline{y}\} = \boldsymbol{Q}_y , \tag{1}$$

where $E\{.\}$ is the mathematical expectation; $\boldsymbol{y}$ is m-by-1 vector of observations; $\boldsymbol{A}$ is m-by-n design matrix; $\boldsymbol{x}$ is n-by 1 vector of unknowns; $D\{.\}$ is the mathematical dispersion; $\boldsymbol{Q_y}$ is m-by-m variance covariance matrix of the observations; and underlined values stand for stochastic. Moreover, m equals the number of observations and n is the number of unknowns.

The validity of the null-hypothesis may be tested against the widest possible alternative hypothesis, by means of the test statistics

$$T = \hat{\boldsymbol{e}}^{\boldsymbol{T}} \boldsymbol{Q_y}^{-1} \hat{\boldsymbol{e}} , \tag{2}$$

where $\hat{\boldsymbol{e}}$ is m-by-1 vector of the least-squares corrections of the observations.

In the case of rejection of the null-hypothesis, one will try to detect the cause of a rejection by formulating a (number of) possible alternative hypothesis. In general, the model under the alternative hypothesis may be written as a linear extension of the model under the null-hypothesis

$$H_o : \boldsymbol{E}\{\underline{y}\} = \boldsymbol{A}x + \boldsymbol{CL}; \qquad \boldsymbol{D}\{\underline{y}\} = \boldsymbol{Q}_y , \tag{3}$$

where $\boldsymbol{C}$ is m-by-q matrix; $\boldsymbol{L}$ is q-by-1 vector; and $\boldsymbol{CL}$ describes the assumed model error. The dimension of the linear extension of the functional model q may vary from $g = 1$ to $q = m - n$.

The validity of the alternative hypothesis may be tested by the test statistics

$$T_q = \hat{\boldsymbol{e}}^T \boldsymbol{Q}_y{}^{-1} \boldsymbol{C} \left[\boldsymbol{C}^T \boldsymbol{Q}_y{}^{-1} \boldsymbol{Q}_{\hat{e}} \boldsymbol{Q}_y{}^{-1} \boldsymbol{C} \right]^{-1} \boldsymbol{C}^T \boldsymbol{Q}_y{}^{-1} \hat{\boldsymbol{e}} , \tag{4}$$

in which $\boldsymbol{Q}_{\hat{\boldsymbol{e}}}$ is the covariance matrix of the least-squares residuals. Under the null-hypothesis the test statistics $T_{\boldsymbol{q}}$ has a central distribution χ^2 with q degrees of freedom, i.e. $\chi^2(q,0)$.

If $q = 1$ then $\boldsymbol{C}$ matrix reduces to m-by-1 vector $\boldsymbol{c}$, and the vector $\boldsymbol{L}$ reduces to a scalar, causing (4) to reduce to

$$T_1 = \left(\boldsymbol{c}^T \boldsymbol{Q}_y{}^{-1} \hat{e} \right)^2 . \left(\boldsymbol{c}^T \boldsymbol{Q}_y{}^{-1} \boldsymbol{Q}_{\hat{e}} \boldsymbol{Q}_y{}^{-1} \boldsymbol{c} \right)^{-1} , \tag{5}$$

which is described as $\chi^2(1,0)$ under the null-hypothesis. The well-known application (5) is found in the method of data-snooping, where the data are checked for possible measurement errors by computing the so-called conventional alternative hypotheses. These hypotheses are of the form: $\boldsymbol{c_i}^T = [0 \ldots 010 \ldots 0]$, in which 1 is found at the position j.

In the study case Kosice-Bankov in estimation and testing it is custom to compute, next to the overall model test all test statistics under indication w-test statistics for the conventional alternative hypotheses. In the present paper are used all three types of tests: (2), (4) and (5).

II.4 Mathematical Model under H_0

Given benchmark, its height at the various epochs as computed after the stability analysis of the reference benchmarks from, together with their covariance matrix, the starting point for the evaluation of the benchmarks subsidence behaviour. The general form of 1D time-dependent polynomials of order n for the benchmarks heights is given as [17], [19]–[23]

$$H_k = a_0 t_k^0 + a_1 t_k^1 + a_2 t_k^2 + \ldots + a_n t_k^n , \tag{6}$$

where H_k is a height of the benchmark as determined at epoch k; a_i is an unknown coefficient, $i = 0, \ldots, n$; t_k^i is measurement time of the epoch k to the power i.

II.5 Alternative Hypotheses Considered

The assumptions are as follows. The polynomial order before the break-point is restricted to a maximum of one ($n_1 \leq 1$), which is also the case under the null-hypothesis. This assumption is based on the fact that a possibly natural subsidence in the study case Kosice-Bankov shows at the most a linear behaviour.

The polynomial order before the break point does not exceed the polynomial order after the break point, i.e. $n_2 \geq n_1$. The function is required to be continuous in its break point, meaning that the function values of both polynomials before and after the break point should be the same.

III. RESULTS OF TESTING FOR POLYNOMIAL BREAK POINTS IN THE STUDY CASE KOSICE-BANKOV

III.1 Identification of Polynomial Break Points

The aim of the procedure is to arrive at a consistent mathematical model, i.e. both the functional and the statistical model. In short the procedure is as follows. First a least-squares adjustment of the mathematical model under the null-hypothesis is performed. The validity of this model is tested by the application of the Overall Model test (*OM*-test), given in (2).

Depending on the test result, the next steps are following [20]–[23]:

1. Acceptance H_0: The estimated slope-coefficient (a_1) is tested for its significance. If the parameter is significant, the functional model is replaced by a constant polynomial with implies stability of the benchmark considered.
2. Rejection H_0: Test all alternative hypotheses as described above for their validity and determine the most likely alternative hypothesis. Depending on the most likely hypothesis selected, the following actions are taken:
    a) *w*-test: Remove the observation concerned, i.e. the benchmark height at the epoch which was identified by the largest *w*-test value.
    b) *01*- or *02*-test: Adapt the mathematical model under the selected alternative hypothesis to be the new mathematical model under the null-hypothesis. Possibly more parameters are needed to describe the benchmarks behaviour accurately. Hence, the null-hypothesis is again tested for its validity. In case of a rejection of the alternative hypotheses mentioned before, the benchmarks are once more tested.
    c) *B*-test: Adapt a break point at the epoch which was identified by the largest *B*-test value. The order of the polynomial before and after the break point is now determined for each part separately.

First consider the case where the dimensions of the hypotheses considered are equal. In our procedure this occurs when all *w*-tests or when all *B*-tests are compared. Since those test statistics T^{i} are all of the form (5) and thus all have the same central distribution with one degree of freedom, i.e.

$$T^{i} \approx \chi^{2}(1,0)\ \forall\ i \tag{7}$$

and the largest value implies the most likely alternative hypothesis. Hence, in this case the most likely alternative hypothesis is the one for which

$$T^{i} > T^{j}\ \forall\ j \neq i\ , \tag{8}$$

where the indices i and j refer to the hypothesis i and j, respectively.

However, at a certain point in the procedure the most likely alternative hypothesis should be selected from a number of hypotheses with different dimensions. This is the case when it is necessary to discriminate between, for instance, the *01*- and *02*-tests. Although the related test statistics χ^{2} are again all χ^{2} distributed, the number of degrees of freedom differs, i.e. we compare the test statistics of the form (5) with the test statistics of the form (4). Therefore the largest value does not automatically refer to the most likely alternative hypothesis.

In order to deal with this problem in the present case, a practical solution may be found, comparing the test quotients, which are defined as $T_q^i / \chi_a^2(q_i,0)$, where T_q^i is the test statistics of the form (4), referring to the i-the alternative hypothesis; $\chi_\alpha^2(q_i,0)$ is a critical value $(l = 5\%)$ of the central χ^2 distribution with q_i degrees of freedom for a certain choice of a_i.

Here it should be noted that the test quotients might only be used if the significance levels a_i of the tests involved are matched through an equal power. Those test quotients that are less than 1 are not taken into account, since the hypothesis in question is certainly not more likely than the null-hypothesis. For the order test quotients it is assumed that the most likely alternative hypothesis is the one, which is rejected strongest, i.e. differs most from 1. Hence, the most likely hypothesis is the one for which the test statistics T_q^i and T_q^j are in the relation: $T_q^i / \chi_\alpha^2(q_i,0) > T_q^j / \chi_\alpha^2(q_j,0) \forall\ j \neq i$.

III.2 Global Test of the Congruence

Significant stability, respectively instability of the network points is rejected or not rejected by verifying the null-hypothesis H_0 respectively, also other alternative hypothesis [23]

$$H_0 : d\hat{\mathbf{C}} = 0; \quad H_\alpha : d\hat{\mathbf{C}} \neq 0\ , \tag{9}$$

where H_0 expresses insignificance of the coordinate differences (deformation vector) between epochs $t_{(0)}$ and $t_{(i)}$. To testing can be use e.g. test-statistics T_G for the global test

$$T_G = \frac{d\hat{\bar{\boldsymbol{C}}}\boldsymbol{Q}^{-1}_{d\hat{\bar{C}}}\, d\hat{\bar{\boldsymbol{C}}}^T}{k\bar{s}_0^2} \approx F\left(f_1, f_2\right), \tag{10}$$

where $\boldsymbol{Q}_{d\hat{\bar{C}}}$ is cofactor matrix of the final deformation vector $d\hat{\bar{\boldsymbol{C}}}$, k is coordinate numbers entering into the network adjustment (k = 3 for 3D coordinates) and $\bar{s}_0^2$ is posteriori variation factor (square) common for both epochs $t_{(0)}$ and $t_{(i)}$.

The critical value T_{KRIT} is searched in the tables of F distribution (Fisher–Snedecor distribution) according to the degrees of freedom $f_1 = f_2 = n - k$ or $f_1 = f_2 = n - k + d$, where n is number of the measured values entering into the network adjustment and d is the network defect at the network free adjustment. Through the use of methods MINQUE is $s_0^{2t(0)} = s_0^{2t(i)} = \bar{s}_0^2 = 1$ [23]. The test-statistics T should be subjugated to a comparison with the critical test-statistics T_{KRIT}. T_{KRIT} is found in the tables of F distribution according the network stages of freedom. Two occurrences can be appeared:

i) $T_G \le T_{KRIT}$: The null-hypothesis H_0 is accepted, i.e. the coordinate values differences (deformation vectors) are not significant;

ii) $T_G > T_{KRIT}$: The null-hypothesis H_0 is refused: i.e. the coordinate values differences (deformation vectors) are statistically significant. In this case the deformation with the confidence level α is occurred. Table 1 presents the results of the global testing of the geodetic network congruence for the selected points.

Table 1 Test-statistics results of the geodetic network points at the monitoring station Kosice-Bankov

Benchmark No.	$T_{G(i)}$	<, ≤, >	***F***	**Notice**
2	1.297	<	3,724	deformation vectors are not significant
3	3.724	≤		
30	3.501	<		
38	3.724	≤		
104	2.871	<		
105	1.403	<		
227	2.884	<		

IV. RESULTS IN THE CASE KOSICE-BANKOV

It will be clear that both polynomials with and without a break point may result from the procedure described in the previous paragraph. In this section examples of estimated polynomials in the study case Kosice-Bankov are presented and discussed.

In the following the test quotient belonging to the overall model test is denoted by *OM*-test (refer to Table 2) [22], [23]:

- ***Benchmark No. 8:*** The behaviour of this benchmark caused the original null-hypothesis to be rejected. The validation of the alternative hypotheses, as specified before identified an extra parameter for the polynomial to be the most likely alternative hypothesis. After the adaptation of this alternative hypothesis as the new null-hypothesis, the overall model test value became 0.9733, which is clearly smaller than its critical value of 1.548 (the significance level of α = 5% to derive deviation mean height values). Hence a quadratic polynomial model was accepted.
- ***Benchmark No. 109:*** This benchmark is a clear (typical) example of the break point estimation at the point in time of 1986 (autumn). After adapting the model including a polynomial break point as the null-hypothesis, the order of the polynomial after the break point was determined to be of the order two.
- ***Benchmark No. 112:*** This benchmark is also a clear (typical) example of the break point estimation with two breaks: at the point in time of 1986 (autumn) and 1995 (spring). The null-hypothesis with the polynomial determined to be of order two can be again considered of the null-hypothesis in time of 1986–1997. And the polynomial is determined to be of the order three after time of 1988.
- ***Benchmark No. 306:*** For this benchmark the original null-hypothesis, assuming a linear subsidence, was accepted. The overall model test statistics was determined to be of 0.468 which is clearly smaller than the critical value of 0.85. However, the first epoch (spring 1990) was considered as a break point possibility. And the alternative hypothesis after the break point was accepted as the polynomial of order two.

Table 2 Test-quotients overview

Benchmark No.	**Quotients**					**Break point [%]**
	Test					
	w	***OM***	***B***	***01***	***02***	
8	1.826	0.779	1.995	2.189	1.521	**0**
109	7.691	2.238	7.796	4.381	5.146	**100**
112	6.175	2.002	7.013	4.199	4.903	**100**
306	6.070	1.908	6.510	4.056	4.216	**70**

The graphical representations of the tested benchmarks are shown in Fig. 4. Fig. 5 shows the panoramic view to the subsidence Kosice-Bankov with the eastern edge of this subsidence (years: 1983 and 2000). Fig. 6 presents the same panoramic view like Fig. 5 but after the reclamation of the subsidence and surrounding mining landscape (year: 2015).

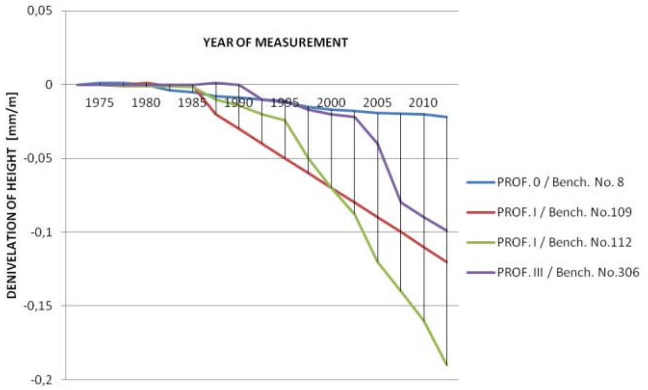

Fig. 4 Polynomial model: Profiles 0, I and III, Benchmarks No. 8, 109, 112 and 306

Fig. 5 Subsidence Kosice-Bankov before the reclamation; panoramic views – years: 1983(A), 2000(B)

Fig. 6 Subsidence Kosice-Bankov after the reclamation; panoramic view – year: 2015

V. GIS APPLICATIONS

GIS (Geographical Information Systems) of the interested area is based on the next decision points [22], [23]:

i) basic and easy data presentation,
ii) basic database administration,
iii) wide information availability.

The best viable solution is to execute GIS project as the Free Open Source application available on Internet. The general facility feature is free code and data source viability through the HTTP and FTP protocol located on the project web pages. Inter among others features range simple control, data and information accessibility, centralized system configuration, modular stuff and any OS platform (depends on PHP, MySQL and ArcIMS port) [8], [10], [25], [26].

Network based application MySQL is in a present time the most preferred database system on Internet. This database is relational database with relational structure and supports SQL language. At the present time MySQL 4.0 is released and supports transaction data processing, full text searching and procedure executing. PHP, which stands for “PHP: Hypertext Pre-processor” is a widely used Open Source general purpose scripting language that is especially suited for Web development and can be embedded into HTML. Its syntax draws upon C, Java, and Perl, and is easy to learn.

The main goal of the language is to allow web developers to write dynamically generated web pages quickly, but you can do much more with PHP. The database part of GIS for the subsidence Kosice-Bankov at any applications is running into MySQL database (Fig. 7).

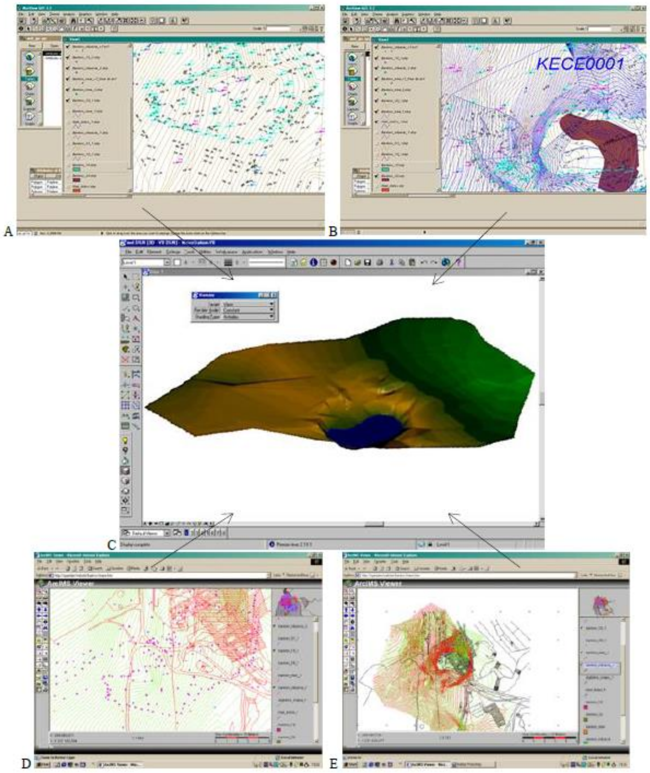

Fig. 7 ArcView user interface Entity visualization (A, B); MicroStation V8 with Terramodeler MDL application (C); Screenshot of ARC IMS - Application internet interface (D, E)

PHP supports native connections to many databases, for example MySQL, MSSQL, Oracle, Sybase, AdabasD, PostgreSql, mSQL, Solid, Informix. PHP supports also older database systems: DBM, dBase, FilePro, PHP etc. can communicate with databases with ODBC interface and this feature represents PHP to work with desktop applications supporting ODBC interface. PHP cans attend to another Internet services, because includes dynamics libraries of some Internet protocols (i.e. HTTP, FTP, POP3, SMTP, LDAP, SNMP, NNTP, etc.) [22], [23].

VI. CONCLUSION

The examples of the chosen benchmarks taken from the monitoring station Kosice-Bankov can give an overview of some resulting polynomial models, representing trends in the deformation developments over an extracted mine space theory of the estimated subsidence polynomial break points follows out from a consideration of 1D deformation model of monitoring points. Similar 3D deformation model analysis at the polynomial break points can be taken into consideration. It will be the subject of a future research of the estimated differential polynomial points in the subsidence. Knowledge about the edges of the undermine areas on the ground surface (edges of the mining subsidence) certainly can be helpful to the environment protection as well as to human live and property protection. Given the fact that extraction of magnesite has been completed at the mine Kosice-Bankov and these mine workings are abandoned, the local governments of the city of Kosice adopted a plan for a renovation of the mine area. The mine subsidence began to gradually backfill by imported natural material. On the territory of the former extensive mine subsidence area the forest park Kosice-Bankov is built as the environmental green-forest part of the urban recreation area of the city of Kosice [23].

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