Criteria and layouts for improving approaches to the improvement of the seismic resistance of masonry constructions

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Abstract— The preservation of masonry constructions poses some fundamental issues as regards the protection of the historical and monumental heritage. To this regard, a series of approaches should be suitably conceived for improving the resistance of masonry constructions to dynamic events, for example through the adoption of properly designed composite reinforcements.

Design approaches may be vary very significantly depending on the geometry and the expected failure modes of the structural system, in order to best fit the specific need of the structure under analysis.

Within this framework, the paper focuses on some design issues and criteria, presented according to some selected typologies and geometries of the masonry structures; the objective consists of strengthening the structural system and increasing the seismic resistance of the construction, tailored on its shape and overall behavior.

Keywords— Masonry, geometry, typology, composite materials, reinforcements, design

I. Introduction

The objective of preservation of the historical and monumental heritage, mostly made of masonry material, poses some important issues to be handled. A main issue consists of the modeling of the masonry material and construction in such a way to develop reliable forecasts of its behavior and response under environmental loads and anthropological actions.

Basically approaches to the study of masonry constructions require the understanding and treatment of the non-linearity related to the structure geometry and material, that is unable to resist tensile stresses and should be handled by on-linear analytical models such as the No-Tension (NT) model (for bibliography by the authors one may refer to [1]-[17] and to [18]-[25] in case of reinforcements).

Therefore, the analysis stage plays a central role for masonry structures, especially when treating structures that couple a complex geometry with the complex material behavior, such as structures with vaulted surfaces of generic shape.

Secondarily the preliminary analytical treatment is mandatory when forecasts are aimed at the protection of monumental and historical building, and at the set up of preservation strategies based on the adoption of composite reinforcements or of dynamic control systems [26]-[41].

In the following two geometries are referred to, the masonry vaults and tower chimneys, presenting some criteria and layouts for their reinforcement through composite provisions.

As regards to vaults, the paper presents some theoretical/numerical results focusing on the possibility of identifying the regions of a masonry vault to be equipped with FRP reinforcement.

The procedure starts from the premise that the solution of the problem of the NT vault under assigned loads may be searched for, after identifying the set of admissible solutions relevant to some specific load families.

The presence of the reinforcement allows some local relaxing of some of the NT admissibility constraints governing the problem.

As regards to chimneys, which are structural elements often found in parts of the world that experienced the effects of the Industrial Revolution in the nineteenth and twentieth centuries, the first approach is presented to the study of their structural strengthening in order to increase their resistance to seismic actions.

The main objective consists of finding out whether strengthening configurations based on vertical carbon fibre-

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reinforced polymer (CFRP) bands are valid for increasing the earthquake resistance and determining the height to which the chimney stack must be strengthened.

II. A Design Criterion for Composite Reinforcement in Masonry Vaults

A. Premise

When dealing with masonry constructions, the most common geometry is represented by vaulted structures.

The theoretical treatment of this typology is not trivial, and, of course, it requires the equilibrium with the applied loads and the admissibility of the material at any point of the structure.

Actually one may somehow invert the problem and perform the analysis by selecting families of load shapes equilibrated by sets of admissible solutions, outlining a strategy that allows to identify the areas of the vault to be reinforced with fiber- reinforced composite provisions.

As shown in the numerical investigation, higher intensities of the stress state are then allowed by the introduction of the reinforcement and the local relaxation of some of the constraints of the problem is possible.

B. Set up of the problem

It has been proved by the authors that a class of solutions of the No-Tension vault equilibrium for a vault with plant X, and upper and lower profiles respectively $z = z_1(x,y)$ and $z = z_2(x,y)$ $[z_1(x,y) \le z_2(x,y)]$, under the assigned vertical load $\bar{p}_z(x,y) \ge 0 \;\; \forall (x,y) \in \boldsymbol{X}$ per unit area, can be obtained by solving the following Vault Inequality System

$$\begin{cases} H_z(x,y) = \\ \quad = z_{,xx}(x,y) z_{,yy}(x,y) - [z_{,xy}(x,y)]^2 = \\ \quad = \dfrac{\bar{p}_z(x,y)}{Q} \ge 0 \\ \\ z_{,xx}(x,y) \ge 0 \;\; ; \;\; z_{,yy}(x,y) \ge 0 \\ \\ z_1(x,y) \le z(x,y) \le z_2(x,y) \end{cases} \tag{1}$$

where $z = z(x,y)$ denotes any surface included in between the intrados and extrados surfaces, $H_z(x,y)$ denotes the Hessian determinant of the z-function and Q represents any positive real parameter i.e. the boundary thrust (dimensionally, a force); the variables preceded by a comma denote in Eq. (1) derivation with respect to these variables.

The first two rows in Equations (1) express the condition for the convexity of the function z(x,y), and hence of the membrane force surface (see e.g. Rockafellar, 1970 [42]; Roberts and Varberg, 1973 [43]).

After setting up the problem in Eq. (1), the vault geometry and active loads should be accounted for in order to specialize the problem to the case at hand.

Thereafter the shape of the membrane function may be selected in order to comply with the given geometry and load shape.

At this stage the membrane function should be partially defined leaving unknown its coefficients, whose identification should follow the imposition of equilibrium and admissibility conditions of the material.

As an example, for a sail vault a symmetric shape may be chosen of the type

$$z(x,y) = K + \frac{A}{2}x^2 + \frac{B}{12}x^4 + \frac{C}{2}y^2 + \frac{D}{12}y^4 \tag{2}$$

where the unknown constants K, A, B, C, D may be shown, after imposing equilibrium, to be related to the load polynomial coefficients p_o, p_x, p_y and p_{xy} by the following equations

$$\begin{cases} \dfrac{A}{B} = \dfrac{p_o}{p_x} = \dfrac{p_y}{p_{xy}} \\ \\ \dfrac{C}{D} = \dfrac{p_o}{p_y} = \dfrac{p_x}{p_{xy}} \end{cases} \tag{3}$$

which represent four equations in five unknowns.

The NT admissibility conditions should be checked by considering the values of the function z(x,y) at the origin of the axes and on the perimeter tympani.

C. Design strategy

Different criteria may be selected for designing the composite reinforcement and arranging its layout, depending on the tasks to be achieved and on the expected failure modes for the structure under dynamic action.

One possible design criterion following the overall strategy consists of considering the minimum thrust transmitted by the structure onto the abutments. In this case one is considering failures modes involving the opening of a unilateral hinge located at the keystone extrados of the arch and one aims at a further reduction of the thrust by introducing the reinforcement at the intrados.

Different levels of reduction of the thrust may be selected and achieved, allowing to finally identify the area of the vault to be reinforced according to the procedure.

According to the selected strategy (reducing the minimum thrust transmitted by the reinforced structure), and to this aim one should consider the case when the membrane surface of the reinforced structure under the applied loads is requested to be tangent to the extrados vault profile at its keystone.

Thereafter one should consequently specialize the problem to this case.

D. *Selection of composite reinforcement*

With refers to the case when the membrane surface is tangent to the extrados profile at the origin point (0,0).

The imposition of the boundary conditions relevant to the occurrence of this situation allows to identify, after some developments, the relevant (5 unknown) constants and solutions, and their range of applicability.

Actually, when introducing a reinforcement at the intrados of the vault, the (minimum) thrust may be still reduced allowing the membrane to come out from the vault thickness at the extrados.

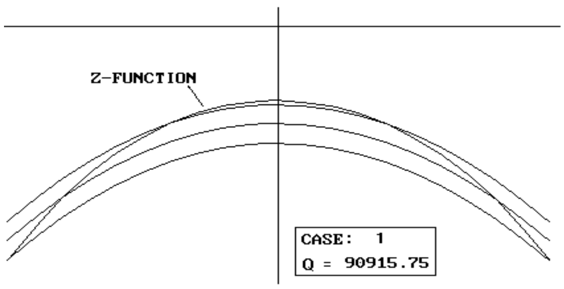

Figure 1: Vault cross section and membrane surface exceeding the extrados at the keystone.

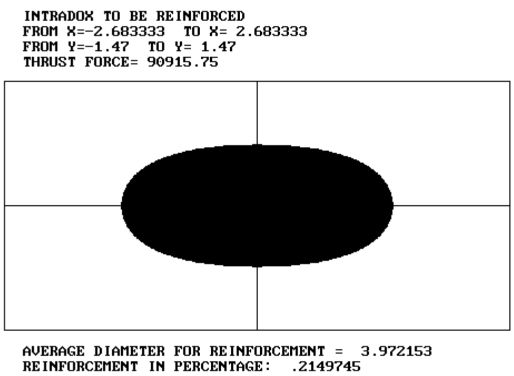

Figure 2: Region of the vault to be reinforced at the intrados.

This circumstance implies that the parameter K may be smaller and also the thrust Q may decrease as the area of the intrados reinforcement is increased.

As an example, in Fig. 1 and 2 are reported the results relevant to an application of the procedure, with the membrane surface over-passing the extrados profile of the vault and the identification of the area to be reinforced in plant.

III. THE REINFORCEMENT OF MASONRY CHIMNEYS

A. *Premise*

A specific category in the class of masonry typologies is represented by industrial chimneys, which are towered constructions often recognized a monumental and historical value.

Masonry chimney architectures are quite diffused in European countries, witnessing the XIX century industrial revolution.

Since accidental loads as earthquakes were not usually considered when designing these structures, they are often damaged and subject to restoration interventions for their preservation against future dynamic events.

The assessment of their seismic vulnerability for checking the structural integrity thus represents a fundamental issue to be addressed, as well as the design of reinforcements for their refurbishment and protection.

In the following investigations are presented relevant to a masonry chimney in Valencia, with the final layout of a fiber-reinforced composite provision.

One considers CFRP bands placed on the external surface of the chimney because this choice makes the application easier to be placed and it can be painted if desired to preserve the appearance of the chimney.

B. *A case study*

Results are reported relevant to the investigations conducted on the industrial masonry chimney represented in Fig.3, with its main dimensions obtained through a topographic survey.

The development of dynamic tests executed using four seismic accelerometers located at different heights of the chimney and with different orientations, allowed to get the structural identification of the tower, with its natural frequencies and structural damping.

Following the experimental tests, a numerical model has been calibrated adjusting he numerical frequencies in such a way to match those experimentally obtained.

The updated numerical model has then been used in a seismic analysis in order to know the seismic response of the chimney when different earthquakes act.

At the second stage the refurbishment intervention is designed based on the adoption of some FRP strips.

In this phase, the usefulness of using vertical strips of FRP in the strengthening to meet the requirements of safety, serviceability and durability for the different earthquake load is studied, so strips of FRP are introduced in the calculations to obtain the reinforced level achieved regarding seismic vulnerability.

Figure 3: The studied masonry chimney and its geometry.

C. Experimental/Numerical tests

The chimney is vertical and 36 m high made in masonry. No external cracks were visible. The masonry is made of bricks and lime mortar.

Since no experimental tests were done for the material characterization, usual values [44] for the main mechanic parameters were used to obtain a numerical model:

- uniaxial compressive strength: $f_c = 637.500$ N/m^2
- uniaxial tensile strength: $f_t = 196.200$ N/m^2
- elastic modulus: $E = 5.886e^9$ N/m^2
- Poisson coefficient $\nu = 0.2$
- density: $\gamma = 1600$ kg/m^3

The tests were conducted placing four seismic accelerometers at different heights in perpendicular directions, as shown in Fig. 4.

Accelerations were registered due to ambient vibrations, as shown in Fig. 5. It can be observed the impact excitation introduced during the test by an impact hammer.

A Fast Fourier Analysis was applied to the registered accelerations to obtain the natural frequencies.

The two first values are: 1.07 Hz for the first mode and 3.32 Hz for the second mode.

Artificial accelerograms compatible with the Spanish Seismic Standard (NCSE 2002) were then generated and introduced in the numerical model using the Gasparini and Vanmarcke technique [45].

Figure 4: Positions of the accelerometers

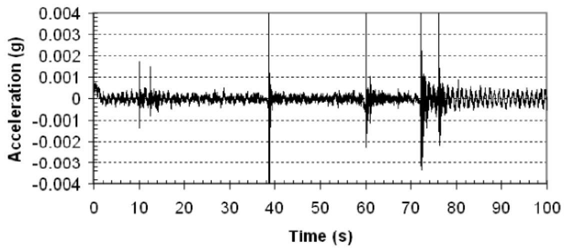

Figure 5: Instrumentally recorded accelerations

Fig. 6 shows the 3D numerical model used in the calculations to reproduce the actual geometry of the chimney, and cracks obtained for one of the accelerograms scaled up to a peak ground acceleration of 0.06g.

A macro-model with a homogeneous material was used with the mechanical characteristics shown previously, and the failure surface adopted was that proposed by Willam&Warnke [46], based on a smeared crack approach.

D. FRP composite strengthening

With reference to the above reported results from experimental/numerical investigation, some fiber reinforced provisions were considered for improving the strength of the chimney.

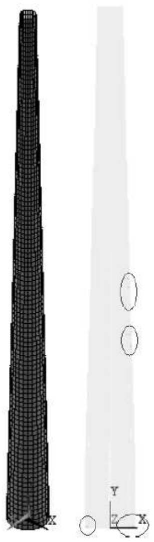

Figure 6: Numerical model and cracks obtained for a_b=0.06g

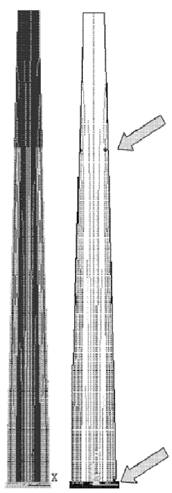

Figure 7: Longitudinal section of the reinforced chimney, and rotational symmetry angle of the reinforcement (45º).

Fig. 7 shows the layout of the strips up to a height of 20 m and 45º rotational symmetry angle, with the cracks displayed for the same seismic load previously considered on the unreinforced structure, that appear translated to the point where the strengthening ends.

IV. Conclusion

In the paper one has focused on the problem of dealing with masonry constructions with historical or monumental value, and setting up a strategy of protection against seismic loading.

Generally, modeling masonry structures represents a difficult task that requires a deep understanding of the behavior involved, which also majorly depend on the structural geometry.

In the paper two typologies are addressed, one referred to vaulted structures and the other one relevant to chimneys, selection of the refurbishment provision to be realized with fiber reinforced composites.

The approach proposed for vaulted structures shows that some easy application of the theoretical set up developed by the authors may be performed, also with the specific objective of identifying the regions of the vault to be reinforced with the introduction of FRP provisions.

By this approach, as shown in the investigation, one may evaluate the extension and the position of the areas where the reinforcement should be applied according to the expected collapse mechanism of the vault and to the desired reduction of the thrust at given locations.

As regards chimneys, it has been shown a technique to calibrate a numerical model using experimental results, for a dynamic analysis.

Investigations are developed based on seismic analyses developed on an industrial masonry chimney before and after the strengthening by using CFRP.

Different configurations and FRP materials are possible to improve the seismic capacity of this type of structures. Here it is shown how using CFRP and vertical strips, the seismic capacity of a masonry chimney is enhanced.

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