Environmental Noise Level Threshold Surpassing Analysis by Non-Homogeneous Poisson Model with Informative and Non-Informative Prior Distributions

Claudio Guarnaccia, Joseph Quartieri, Nikos E. Mastorakis and Carmine Tepedino

Abstract— The dangerous effect of acoustical noise on health, both auditory and non-auditory, is largely documented in literature. The study of noise problem and the development of several predictive models are very important issues in cities and areas in which relevant noise sources are present. A model based on threshold exceedances is suitable since a great number of national regulations define limits to acoustical levels, according to the area and to the kind of buildings and activities occurring in them.

In this paper, a non-homogeneous Poisson model will be presented and applied to a large dataset of noise measurements. The aim of this model is to predict, from a probabilistic point of view, the number of threshold exceedances in a given future period. This can be achieved estimating the parameters of the mean function, that represents the number of surpassings of the threshold as a function of the time, and of the rate function of the Poisson process. Different models of the rate function form can be chosen, according to their functional dependence from time. The choice adopted in this work will be the Goel Okumoto (GO) model.

The estimation of the parameters probability distributions will be performed using a Bayesian approach, based on Monte Carlo Markov Chains and Gibbs algorithm. Once the parameters will be tuned, the mean function can be compared with the observed exceedances plot. The two GO parameters will be estimated using non-informative uniform prior distributions and also informative gamma distributions. Different starting points sets will be implemented for the Markov chains to evaluate possible effects on the posterior distributions.

Keywords— Acoustical Noise Level, Cumulative Rate Function, Exponential Rate Function, Goel Okumoto, Non homogenous Poisson Process, Predictive Model, Threshold Surpassing.

I. INTRODUCTION

ACOUSTICAL noise is one of the major problems in urban areas [1]. Together with air pollution and other pollutant agents, in fact, it is a relevant risk for human health. In [2], [3] some of the effects related to the exposure to acoustical noise are resumed, focusing on the auditory and non-auditory effects.

The most important source of noise are transportation infrastructures and industrial areas, such as documented in [1], [4], [5]. Many models, with different approaches, have been developed to predict the noise levels produced by trains (for instance [6]-[11]), road traffic (for instance [12]-[23]), airport (for instance [24], [25]), industrial settlements (for instance [3], [26]), etc..

In this paper, the pursued approach is based on the implementation of a non-homogeneous Poisson process [27], [28] to study the probability of surpassing of a certain noise threshold. Similar models have been presented in [29], [30] for air pollution and [25], [31] for noise level. The introduction of changepoints is discussed in [25] for airport noise, while in [32] the comparison between various models, with a different number of changepoints, is discussed on four large dataset of noise levels. One of these datasets is adopted here to test the implementation of a different rate function, that is exponential instead of power law. In particular, the Goel Okumoto (GO) rate function model [33] is implemented instead of the Weibull Power Law (PL) rate function, adopted in [25] and [31].

The GO model is characterized by two parameters, \((\alpha,\beta)\) that are estimated using a Bayesian approach, based on Monte Carlo Markov Chains and Gibbs algorithm. In the first run, non-informative uniform prior distributions are adopted, while in the second run informative gamma distributions are implemented, to evaluate any improvement in the model performances. In addition, two other runs with different starting points for the Markov chains are implemented. This strategy is adopted also to evaluate possible effects on the posterior distributions caused by the different starting points.

The dataset under study is related to acoustical noise equivalent level measured in Messina, Italy, in the framework of the long term monitoring campaign promoted by the Department of Urban Mobility of the city. The levels considered here are measured during the night and are evaluated on a 8 hours range, from 10 p.m. to 6 a.m.

Since some data were missing, both single and group of data, a Time Series Analysis model, such as the ones developed in [34]-[38], has been used as imputation method, i.e. to fill the gaps [39]. This is the same procedure adopted in [32] to make the datasets continuous.
The results of the model analysis are presented in terms of variations in the parameters estimations in the different cases described above, and in terms of comparison between observed and estimated exceedances, i.e. cumulative mean function. In addition, the Deviance Information Criterion (DIC) [40] is used to measure the Bayesian adequacy of the models.

Finally, the evaluation of the probability of observing a certain number of threshold surpassings is presented in the last section, showing how the model can be helpful in decision processes by policy makers.

II. MODEL PRESENTATION

The Non-Homogeneous Poisson Process (NHPP) is a stochastic model able to count occurrences of a certain event, in our case the surpassing of a chosen noise level threshold.

Let \( N(t) = \{N_t^0 : t \in [0, T]\} \) be a NHPP with mean value function \( \lambda(t|\theta) \) and \( \theta \) the parameters vector. The function \( \lambda(t|\theta) \) represents the cumulative function of the expected number of events registered by \( N(t) \) up to time \( t \). In this paper, the events are represented by the surpassing of a certain acoustical noise threshold (68 dBA). In order to fully characterize the process, the functional form of \( m(t|\theta) \), or equivalently, of its rate function \( \lambda(t|\theta) \), must be achieved. In fact, the relation between the two functions is:

\[
\lambda(t|\theta) = \frac{d}{dt} m(t|\theta) \tag{1}
\]

The probability to observe \( k \) exceedances in the time range \([t; t+s]\) is given by:

\[
P(N_t^{t+s}=k) = \frac{[m(t+s)-m(t)]^k}{k!} e^{-[m(t+s)-m(t)]} \tag{2}
\]

where \( N_t \) is the number of times a threshold is surpassed in the time range from 0 to \( t \).

The model adopted in this paper is based on the Goel-Okumoto (GO) [33] process defined by the following mean value function:

\[
m(t|\theta) = \alpha[1 - \exp(-\beta t)], \quad \alpha, \beta > 0, \tag{3}
\]

where \( \theta = (\alpha, \beta) \). The rate function associated with this process is given by

\[
\lambda(t|\theta) = \alpha \beta \exp(-\beta t). \tag{4}
\]

In the Messina dataset considered in this paper, the epochs of occurrence of noise level threshold violations up to time \( T \) are included in the dataset \( D_T = \{n; t_1, ..., t_n ; T\} \), in which \( n \) is the number of observed threshold overruns times which are such that \( 0 < t_1 < t_2 < ... < t_n < T \).

The likelihood function for \( \theta \) is:

\[
L(\theta|D_T) = \left( \prod_{i=1}^{n} \lambda(t_i|\theta) \right) \exp(-m(T|\theta)) \tag{5}
\]

A. Parameters estimation

In this work, in order to have the estimation of the model parameters, an empirical Bayesian analysis is applied. The choice was to use approximately non-informative prior distributions, in particular uniform \( U[a,b] \) distributions, with \( a \) and \( b \) chosen in an appropriate way.

The software OpenBugs has been used to obtain simulated samples from the posterior distribution of \( \theta \). In this software framework, only the distribution for the data and prior distributions for the parameters need to be specified. This software adopts standard Markov chain Monte Carlo (MCMC) methods. For further details, one may refer to [25], [29]-[32].

III. ANALYSIS AND RESULTS

The model presented above has been tested on a dataset of 1341 noise measurements, collected in Messina, Italy, in a monitoring station placed in Boccetta street, during night time (from 10 p.m. to 6 a.m.). The time range goes from the 11th of May 2007 to the 10th of January 2011. The observed exceedances of the threshold (68 dBA) are 900. The statistics of the data are resumed in Tab. 1.

<table>
<thead>
<tr>
<th>Number of data</th>
<th>Mean [dBA]</th>
<th>Std.dev [dBA]</th>
<th>Median [dBA]</th>
<th>Min [dBA]</th>
<th>Max [dBA]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1341</td>
<td>68.25</td>
<td>1.31</td>
<td>68.50</td>
<td>63.5</td>
<td>72.0</td>
</tr>
</tbody>
</table>

In this section, four runs are implemented and performed in the OpenBugs software. Each run is composed by five chains, with 100000 iterations each, and different sets of starting points. The parameters have been evaluated after a burning period of 50000 iterations and thinning every 10 steps.

The starting points are chosen in the interval of acceptance of the prior distributions that can be non-informative (uniform) or informative. The choice of the prior distributions is crucial in the Bayesian approach. An initial strongly informative distribution choice, in general, should be avoided but, once the first iteration has been run, more informative distribution can be used.

The variations in the prior distributions and in the chains starting points are one of the main aims of this work and are presented in the next subsections. The four runs that have been performed are set according to the scheme presented in Table 2.

<table>
<thead>
<tr>
<th>Run</th>
<th>Prior distributions</th>
<th>Chains starting points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Uniform</td>
<td>First set</td>
</tr>
<tr>
<td>2</td>
<td>Gamma</td>
<td>First set</td>
</tr>
<tr>
<td>3</td>
<td>Uniform</td>
<td>Second set</td>
</tr>
<tr>
<td>4</td>
<td>Uniform</td>
<td>Third set</td>
</tr>
</tbody>
</table>
A. Run 1: non-informative prior distributions and first set of chains starting points

In run 1, a uniform distribution $U[a,b]$ has been chosen both for $\alpha$ and $\beta$, with suitable boundaries $a$ and $b$. The uniform prior distributions, with $a$ and $b$ parameters (boundaries), are:

$$\alpha \rightarrow U[1000, 3000]$$
$$\beta \rightarrow U[0.0001, 0.00055]$$.

The first set of starting points of the five chains has been chosen in all the interval $[a, b]$ and is:

$$\alpha = 1100; \beta = 0.0005;$$
$$\alpha = 1800; \beta = 0.00045;$$
$$\alpha = 2500; \beta = 0.00012;$$
$$\alpha = 2100; \beta = 0.00035;$$
$$\alpha = 2900; \beta = 0.0003.$$  

The convergence of the five chains is achieved quite soon, independently from the starting points resumed above, as shown in Fig. 1 and Fig. 2.

The resulting posterior probability density functions of the parameters are reported in Figg. 3 and 4, while the statistics are resumed in Tab. 3.

With the mean values of the parameters, the mean function can be plotted versus time and compared with the observed surpassings of the threshold. This comparison is reported in Fig. 5.

As it can be noticed, the agreement between the two curves is very good, except for a region that goes approximately from 800 to 1150. In this time interval, a lower number of exceedances is observed but the model is not able to follow this pattern. A possible solution is represented by the introduction of a suitable number of change-points, such as in [25], [30], in order to evaluate the parameters in different ranges and with a better agreement. This improvement of the model is part of an ongoing work [32].

**Table 3**: Summary of statistics of the $\alpha$ and $\beta$ parameters; run 1.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std.dev</th>
<th>Median</th>
<th>val 2.5%</th>
<th>val 97.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>2462.0</td>
<td>310.0</td>
<td>2460.0</td>
<td>1884.0</td>
<td>2989.0</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$3.5 \times 10^{-4}$</td>
<td>$5.9 \times 10^{-4}$</td>
<td>$3.4 \times 10^{-4}$</td>
<td>$2.6 \times 10^{-4}$</td>
<td>$4.8 \times 10^{-4}$</td>
</tr>
</tbody>
</table>
B. Run 2: informative prior distributions and first set of chains starting points

In run 2, a gamma distribution $dgamma[c, d]$ has been chosen both for $\alpha$ and $\beta$, with opportune $c$ and $d$ parameters. The chosen gamma prior distributions, with $c$ and $d$ parameters, are:

\[
\begin{align*}
\alpha & \rightarrow dgamma[12123, 4.924] \\
\beta & \rightarrow dgamma[0.00204, 5.88].
\end{align*}
\]

Let us remind that $c$ and $d$ are related to the mean of the gamma distribution, $m$, and to the variance, $s^2$:

\[
\begin{align*}
d &= \frac{m}{s^2}, \quad (6) \\
c &= m \cdot d. \quad (7)
\end{align*}
\]

The values of the mean and of the variance have been chosen considering the results of run 1.

The starting points of the chains are the same as in run 1 (first set of starting points).

Again, the convergence of the chains is quite fast, as reported in Fig. 6 and Fig. 7.

Figs. 8 and 9 report the resulting posterior probability density functions of the parameters. The estimated statistics are resumed in Tab. 4.

The plot of simulated exceedances (mean function) and observed surpassings is presented in Fig. 10, showing very small differences with respect to run 1 (Fig. 5).
C. Run 3-4: non-informative prior distributions, second and third set of chains starting points

In runs 3 and 4 the same uniform distributions $U[a,b]$ used in run 1 have been chosen, both for $\alpha$ and $\beta$, with opportune boundaries $a$ and $b$. Thus, the uniform prior distributions, with $a$ and $b$ parameters (boundaries), are:

\[
\begin{align*}
\alpha & \rightarrow U[1000, 3000] \\
\beta & \rightarrow U[0.0001, 0.00055].
\end{align*}
\]

In run 3, the starting points of the five chains have been modified (second set of starting points) as follows:

\[
\begin{align*}
\alpha &= 1150; \beta = 0.00017; \\
\alpha &= 1750; \beta = 0.00054; \\
\alpha &= 2550; \beta = 0.00020; \\
\alpha &= 2050; \beta = 0.0004; \\
\alpha &= 2950; \beta = 0.00025.
\end{align*}
\]

A different set of starting points (third set) has been used in run 4; the values are:

\[
\begin{align*}
\alpha &= 1185; \beta = 0.00032; \\
\alpha &= 1715; \beta = 0.00022; \\
\alpha &= 2585; \beta = 0.00042; \\
\alpha &= 2015; \beta = 0.00015; \\
\alpha &= 2985; \beta = 0.00032.
\end{align*}
\]

The results of run 3 and run 4 are reported in the following figures. In particular, the convergence of the chains are reported from Fig. 11 to Fig. 14, while the posterior probability density functions are shown from Fig. 15 to Fig. 18. Tab. 5 reports the resume of the statistics of the parameters. Finally, the mean functions and the observed surpassings are plotted in Fig. 19.

From these analyses, it is clear that, in this application, the starting points have not any significant influence on the model results, since the differences of the estimations are very small.
D. Model performances evaluation

The results of the parameters statistics obtained in the different runs are reported in Table 6 for a comparison.

Table 6: Summary of statistics of the $\alpha$ and $\beta$ parameters; all runs.

<table>
<thead>
<tr>
<th>Run</th>
<th>Parameter</th>
<th>Mean</th>
<th>Std.dev</th>
<th>Median</th>
<th>2.5%</th>
<th>97.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\alpha$</td>
<td>2462.0</td>
<td>310.0</td>
<td>2460.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td>$3.5 \times 10^{-4}$</td>
<td>$5.9 \times 10^{-5}$</td>
<td>$3.4 \times 10^{-4}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$\alpha$</td>
<td>2463.0</td>
<td>22.33</td>
<td>2463.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td>$3.4 \times 10^{-4}$</td>
<td>$1.5 \times 10^{-5}$</td>
<td>$3.4 \times 10^{-4}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$\alpha$</td>
<td>2444.0</td>
<td>293.8</td>
<td>2466.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td>$3.5 \times 10^{-4}$</td>
<td>$5.7 \times 10^{-5}$</td>
<td>$3.4 \times 10^{-4}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$\alpha$</td>
<td>2430.0</td>
<td>290.0</td>
<td>2423.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td>$3.5 \times 10^{-4}$</td>
<td>$5.5 \times 10^{-5}$</td>
<td>$3.4 \times 10^{-4}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Looking at the results of run 1 and run 2, it can be noticed that the introduction of informative distributions does not significantly change the mean and the median, but strongly reduces the standard deviation. In the case of $\alpha$ parameter, the decrease is of one order of magnitude.

On the contrary, comparing results of run 1 and runs 3 and 4, it can be deduced that the starting points do not significantly influence the model outcomes.

These results leads us to affirm that the mean functions related to the four runs, plotted using the mean values of the parameters, are very similar and that all of them track the general slope of the observed surpassings (see Fig. 20), but are not able to follow the variations that occur in the second part of the curve (see zoom in Fig. 21). It can be noticed that the curve related to run 2, slightly better approaches the last part of the observed curve, showing that informative distributions may lead to better results, even if in this case the differences are very small.

As already stated in subsection III.A, the introduction of a suitable number of change-points, with a subsequent increase of the number of parameters, can lead to a better agreement between the observed and predicted curves.
E. Deviance Information Criterion (DIC) application results

A possible tool to measure the model performances is the Deviance Information Criterion (DIC) [38], and related statistics, which is an approximation for the Bayes factor. Smaller values of DIC indicate better models. The DIC can be used also to assess model complexity and compare different models.

DIC is given by:

\[
DIC = D(\bar{\theta}) + 2p_D
\]

where \(D(\bar{\theta})\) is the deviance evaluated at the posterior mean and \(p_D = D - D(\bar{\theta})\) is the effective number of parameters in the model, with \(D\) the deviance averaged over the posterior distribution.

DIC results for the various runs are reported in Tab. 7.

<table>
<thead>
<tr>
<th>Run</th>
<th>DIC</th>
<th>(p_D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2508.0</td>
<td>1.316</td>
</tr>
<tr>
<td>2</td>
<td>2507.0</td>
<td>1.159</td>
</tr>
<tr>
<td>3</td>
<td>2508.0</td>
<td>1.401</td>
</tr>
<tr>
<td>4</td>
<td>2507.0</td>
<td>1.205</td>
</tr>
</tbody>
</table>

The main conclusion of this analysis is that there are not any significant differences in the fitting and predicting abilities of the model in the four runs.

IV. Probability of Threshold Surpassings

In this section, the authors present how the Poisson distribution can be used to evaluate the probability of observing a certain number \(k\) of events of noise level threshold surpassings, in an interval of \(s\) periods, starting from the period \(t\). Let us remind that the time base for the equivalent level is 8 hours, from 10 p.m. to 6 p.m. (night level), thus each period corresponds to a night measurement.

In order to define a suitable Poisson distribution, an estimation of the parameters of the GO rate function model is needed. In this section, the mean values of these parameters, evaluated in run 1 and presented in section III.A (Table 3), are used.

The simulation reported here is based on an interval of 30 periods (\(s\)), i.e. 30 nights, starting from period 1200 (\(t\)), i.e. the 22\textsuperscript{nd} of August 2010, in the case of 68 dBA noise level threshold. The actual observed exceedances are 23.

Fig. 22 and 23 report respectively the probability to observe \(k\) threshold exceedances and the cumulative probability of observing up to \(k\) threshold exceedances in the 30 periods time interval.

Table 8, instead, presents the following data:

1) probability of observing exactly the actual number of threshold surpassings;
2) cumulative probability of observing up to 10 threshold surpassings;
3) cumulative probability of observing up to 20 threshold surpassings.

It can be noticed that the probability of observing the real number of exceedances is quite low (about 3%), that means that looking for the exact value of threshold surpassings is not the most useful way to use this tool.

On the contrary, the other two results are more relevant. The cumulative probability (\(p\)) of observing up to 10 surpassings is very low, i.e. there is a high probability (1-\(p\) is about 95%) of having more than 10 exceedances in the 30 periods (days) that follow the 22\textsuperscript{nd} of August 2010 (\(t = 1200\)).

Finally, the cumulative probability of observing up to 20 surpassings is about 81%, stating that there is an almost low probability (1-\(p\) is about 19%) of observing more than 20 surpassings of the threshold. These results show that the model is able to give strong indications about the range of the threshold exceedances. In the case presented here, the indication is that the site is affected by many threshold surpassings and it is critical from the noise pollution point of view.

Fig. 22: Probability to observe \(k\) exceedances in an interval of 30 days from the period 1200, as a function of \(k\).
Fig. 23: Cumulative distribution function of observing up to \( k \) exceedances in an interval of 30 days from the period 1200.

### Table 8: Probability of observing the actual number of exceedances (\( k=23 \)) and cumulative probabilities of observing up to 10 and up to 20 threshold surpassings.

<table>
<thead>
<tr>
<th>Probability</th>
<th>Probability</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(k = 23) )</td>
<td>( p(k \leq 10) )</td>
<td>( p(k \leq 20) )</td>
</tr>
<tr>
<td>0.030815</td>
<td>0.051557</td>
<td>0.812575</td>
</tr>
</tbody>
</table>

### V. CONCLUSIONS

In this paper, a model that predicts the number of threshold exceedances of a fixed noise level threshold, in a given period of time, has been presented. This model is based on the assumption that the surpassing events are distributed over the time according to a non-homogeneous Poisson process. The adopted rate function is taken from the Goel Okumoto (GO) model.

The two GO parameters have been estimated following the Bayesian approach adopting a Monte Carlo Markov Chain technique, in the OpenBugs software framework.

The two parameters were estimated using non-informative uniform prior distributions for parameters probability density and also informative gamma distributions. Both the choices for the prior distributions resulted in an almost quick convergence of the chains.

Moreover, different starting points sets were implemented for the Markov chains, to evaluate possible effects on the posterior distributions of the parameters. The different starting points sets don't affect the estimation results: this can be a proof of the robustness of the model and of the reliability of the found solutions.

In all the runs, the posterior distributions have been presented and the mean values used to calculate and plot the mean function, i.e. the cumulative function of the surpassing predicted versus time. The comparison of the model results with the real observed exceedances is good, except for a region of the plot in which a lower number of actual surpassings is detected, with respect to the prediction of the model.

The evaluation of the probability of observing a certain number, or a given interval, of threshold surpassings has been presented in the last section, showing that the model is able to give strong indications about the range of threshold exceedances.

The implementation of different rate functions in the Poisson process represents a possible future step of this work, in order to achieve an always better prediction of the probability of noise level threshold surpassings.

### REFERENCES


