Verification of the elasto-plastic behavior of nonlinear concrete material models

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Abstract—The aim of this paper is the verification of the behavior of nonlinear concrete material models subjected to dynamic loading. The behavior of five selected nonlinear concrete models is tested within numerical simulations of the testing of the mechanico-physical properties of concrete in uniaxial and triaxial compression, using cylinders. These numerical simulations are performed using the explicit finite element approach and their results take the form of load-displacement and load-time diagrams created for a concrete cylinder under dynamic compressive loading whose behavior was calculated by individual nonlinear models of concrete. The results gained from the numerical simulations are compared with experimental results obtained from laboratory tests performed for the purpose of verifying the nonlinear concrete models. This comparison shows the differences in the behavior of the individual nonlinear models of concrete and the approximations of the experimental data. The comparison allows conclusions to be drawn as to which nonlinear models provide the best approximation of the test data, and therefore which models best describe the behavior of the tested concrete in compression.

Keywords—Dynamic compressive loading, experiment, finite element simulation, LS-DYNA software, nonlinear concrete material model.

I. INTRODUCTION

The numerical modeling of structures or their components with the aim of investigating their behavior with the aid of computational systems based on the finite element method is a commonly used procedure during the design of structures or structural elements. Current innovative finite element procedures (implicit, explicit or combined solvers) offer an inexhaustible amount of material models. These material models are usually linear, bilinear or multilinear, but can also be strongly nonlinear. Nonlinear material models undoubtedly have their place in sophisticated analysis and for this reason it is appropriate to test and validate them with the aim of achieving their optimal function.

A number of publications have been dedicated to the nonlinear modeling of concrete, e.g. [1]–[8]. Worldwide, nonlinear material models of concrete are most often used for the numerical modeling of concrete structures subjected to specific loading during which the effect of the strain rate upon the mechanico-physical properties of concrete is apparent. Information regarding nonlinear models of concrete and results obtained from various numerical simulations in which nonlinear models of concrete were used, and in which strain rate played an important role, can be found in the literature: see the references section for examples [9]–[16].

This paper details the subjection of five different nonlinear material models of concrete to dynamic compressive loading. The behavior of the selected models was tested within the context of numerical simulations of the testing of the mechanico-physical properties of concrete in uniaxial and triaxial compression, using cylinders. The results gained from the numerical simulations take the form of load-displacement and load-time diagrams for a concrete cylinder whose behavior was calculated by individual nonlinear concrete models. These diagrams were analyzed. They show the differences in concrete behavior for the individual nonlinear models. The results gained from the numerical simulations are compared with experimental data obtained from laboratory tests.

II. EXPERIMENTS

Experiments were carried out in order to compare their results within this paper with the results obtained from the aforementioned numerical simulations. On the basis of this comparison the behavior of the nonlinear concrete models can be verified.

A. Outline of the experiments

Each experiment involved testing the mechanico-physical properties of concrete cylinders subjected to uniaxial compression.

The tests were carried out on three identical cylindrical concrete specimens with a height of approx. 305 mm and a base diameter of approx. 152 mm. The specimens were placed between steel plates and a strain gauge was installed in the middle of each of them (see Fig. 1). The mean value of the ultimate concrete strength in uniaxial compression varied around 43 MPa.
The specimens were compressed at a constant velocity of approx. 1.27 mm/s and the test duration, strain deformation, relative vertical displacement and compressive force exerted by the test apparatus were monitored during the tests.

**B. Experimental results**

The results gained from the experiments conducted on cylindrical specimens in uniaxial compression are depicted in a load-time diagram (see Fig. 2). With regard to the fact that the relationship between the displacement of the cylinder base and time is linear, this diagram describes the behaviour of the tested cylinder during compressive loading. Due to the nature of the test performed as part of each experiment, the diagram simultaneously also describes the behaviour in uniaxial compression of the concrete from which the cylindrical specimens were manufactured, and it is easy to derive the ultimate strength of the tested concrete in uniaxial compression from the diagram.

![Load-time diagram](image)

During the compressive loading, each tested concrete cylinder first exhibited linearly elastic behavior, and then elasto-plastic behavior. From the moment when the maximum compressive force (i.e. the maximum compression load capacity of the concrete cylinder) was exceeded, the concrete began to show signs of compressive strain softening. In other words, deformation of the concrete cylinder began to increase at the same time as compressive force started decreasing. The compressive strain softening of the concrete began to assert itself as a result of the damage to the concrete cylinder as the ultimate strength of the concrete in uniaxial compression was exceeded. The drop in compressive strength to zero as deformation increased was very rapid, which suggests that the concrete fractured in a brittle manner (see Fig. 2).

The maximum measured compressive force was 736.028 kN. The measured ultimate strength of the concrete in uniaxial compression was therefore approximately 42 MPa.

### III. NONLINEAR MATERIAL MODELS OF CONCRETE

Five types of nonlinear material model, which are implemented in LS-DYNA explicit finite element software ([17], [18]), were used for the purposes of this paper. Their brief description is given in the following subsections.

**A. The Continuous Surface Cap Model**

The Continuous Surface Cap material model ([19], [20]) is based on a yield surface which is described as a function of three stress invariants [21], [22]:

$$ Y(I_1, J_2, J_3) = J_2 - 9R(J_3)^2 F_c^2(I_1) F_c(I_1, \kappa) $$

where $I_1$ is the first invariant of the stress tensor, $J_2$ and $J_3$ are the invariants (second and third) of the deviatoric stress tensor, $9R(J_3)$ is the Rubin strength reduction factor and $\kappa$ is the cap hardening parameter. The yield surface is composed of two parts, these being the shear failure surface $F_c(I_1)$ and the hardening compaction surface $F_c(I_1, \kappa)$. The shear failure surface and hardening compaction surface are combined using a multiplicative formulation which allows their combination to be continuous and smooth at their intersection. The shear failure surface is defined as:

$$ F_c(I_1) = \alpha - \lambda \exp^\frac{-\beta}{I_1} + \theta I_1 $$

where $\alpha$, $\beta$, $\lambda$, and $\theta$ are the material constants determined from the triaxial compression test data. The expression of the hardening compaction surface is given by:

$$ F_c(I_1, \kappa) = 1 - \frac{(I_1 - L(\kappa))^2}{(X(\kappa) - L(\kappa))^2} \quad \text{for } I_1 > L(\kappa) $$

$$ F_c(I_1, \kappa) = 1 \quad \text{for } I_1 \leq L(\kappa) $$

with

$$ L(\kappa) = \kappa \quad \text{for } \kappa > \kappa_0 $$

$$ L(\kappa) = \kappa_0 \quad \text{for } \kappa \leq \kappa_0 $$

$$ X(\kappa) = L(\kappa) + RF_c(I_1) $$

where $R$ is the cap aspect ratio.

The model includes the ability to generate parameters based on the uniaxial compressive strength, and allows the effect of strain rate, failure and different mechanico-physical properties in compression and tension to be taken into account.

**B. The Winfrith Concrete Model**

The Winfrith Concrete material model ([3], [23], [24]) is based on Ottosen's shear failure surface, and is defined as a
smeared crack and smeared rebar model. The Ottosen's shear failure surface is defined as [25]:

$$F(I_1, J_2, \cos 3\theta) = a \frac{J_2}{(f_c)^2} + \lambda \sqrt{J_2} \frac{f_c}{f_c + b} I_1 - 1$$

with

$$\cos 3\theta = \frac{3\sqrt{3}}{2} J_2$$

$$\lambda = k_1 \cos \left[ \frac{1}{3} \cos^{-1}(k_2 \cos(3\theta)) \right]$$

$$\lambda = k_1 \cos \left[ \frac{\pi}{3} - \frac{1}{3} \cos^{-1}(-k_2 \cos(3\theta)) \right]$$

where $I_1$ is the first invariant of the stress tensor, $J_2$ and $J_3$ are the invariants (second and third) of the deviatoric stress tensor, $f_c$ is the unconfined (uniaxial) compressive strength, $\theta$ is the lode angle and parameters $a$, $b$, $k_1$, and $k_2$ are functions of the ratio of tensile strength to compressive strength (f/f_c). Parameters $a$, $b$, $k_1$, and $k_2$ are determined from the uniaxial, biaxial and triaxial compression tests.

The model allows the effect of strain rate, reinforcement and different mechanico-physical properties in compression and tension to be taken into account.

**C. The Damage Plastic Concrete Model**

The Damage Plastic Concrete material model [26] is based on a yield surface described in Haigh-Westergaard coordinates, the flow rule, hardening law and evolution law, and is defined as a constitutive model based on the combination of damage mechanics and plasticity [27].

The model allows the effect of strain rate, failure and different mechanico-physical properties in compression and tension to be taken into account.

**D. The Karagozian & Case Concrete Model - Release 3**

The Karagozian & Case Concrete material model - Release 3 ([28], [29]) is defined as a three-invariant model using three shear failure surfaces (initial yield surface, maximum shear failure surface and residual failure surface). These strength surfaces are independent and their formulation in a generalized form is given by [30]:

$$F_i(p) = a_{i0} + \frac{p}{a_{i1} + a_{i2}p}$$

where $i$ stands for $y$ (initial yield surface), $m$ (maximum shear failure surface) or $r$ (residual failure surface) and are parameters calibrated from the test data and $p$ is the pressure, which is dependent on the first invariant of the stress tensor ($p = J_1/f_c$). The failure surface is interpolated between the maximum shear failure surface and either the initial yield surface or the residual failure surface:

$$F(I_1, J_2, J_3) = r(J_3)[\eta(\lambda)(F_m(p) - F_y(p)) + F_y(p)]$$

for $\lambda > \lambda_m$ (14)

where $I_1$ is the first invariant of the stress tensor, $J_2$ and $J_3$ are the invariants (second and third) of the deviatoric stress tensor, $\lambda$ is the modified effective plastic strain or the internal damage parameter, $\eta(\lambda)$ is the function of the internal damage parameter $\lambda$ and $r(J_3)$ is the scale factor in the form of the William-Warnke equation [31].

The model includes the ability to generate parameters based exclusively on the uniaxial compressive strength, and allows the effect of strain rate, failure and different mechanico-physical properties in compression and tension to be taken into account.

**E. The Karagozian & Case Concrete Model**

The Karagozian & Case Concrete material model is the original release of the Karagozian & Case Concrete Model - Release 3. The model allows the effect of strain rate, failure, reinforcement and different mechanico-physical properties in compression and tension to be taken into account.

**IV. NUMERICAL SIMULATIONS**

**A. Description of the simulated uniaxial compression tests**

In a real uniaxial compression test, a concrete cylinder is placed between the pressure plates of a test press. For the purposes of the numerical simulations in this paper, it was assumed that the friction between the surface of the cylinder base and the surface of the pressure plates is so great that it prevents any kind of horizontal displacement. This assumption enabled cylindrical specimen with crosswise-supported basal nodes to be modeled alone without the pressure plates. Schematically, this assumption can be represented in the form of a cylinder with fixed bases that allow vertical freedom of movement (see Fig. 3).

**Fig. 3 Idealization of boundary conditions for the uniaxial compression test simulations**

Within the numerical uniaxial compression test simulations, the cylinder was considered to be loaded via compression at a constant velocity from both sides. The vertical displacement of both bases of the cylinder over time thus grew linearly and evenly, during which the cylinder was loaded beyond its limits, and failed (see Fig. 4).
B. Results of the numerical uniaxial compression test simulations, and their comparison with experimental results

The results of the numerical simulations performed in LS-DYNA software are depicted in a load-time diagram (see Fig. 5). It is clear from the diagram that during the compressive loading, the concrete cylinder first exhibited linearly elastic behavior and then elasto-plastic behavior for all of the nonlinear models of concrete used. From the moment when the maximum compressive force (i.e. the maximum compression load capacity of the concrete cylinder) was exceeded, the concrete in most of the nonlinear models began to show signs of compressive strain softening. In other words, deformation of the concrete cylinder began to increase at the same time as compressive force started decreasing. The compressive strain softening of the concrete began to assert itself as a result of the damage to the concrete cylinder as the ultimate strength of the concrete in uniaxial compression was exceeded.

C. Effect of boundary conditions on the results of the numerical simulations of uniaxial compression tests

The load-displacement diagram in Fig. 7 shows the results of the numerical uniaxial compression test simulations for different boundary conditions. Two types of curves are shown in this diagram. The dashed curves represent the calculations for the finite element model of the cylinder with crosswise-supported nodes used for one of its bases. The solid curves represent the calculations for the model of the cylinder with no supports. Nonlinear concrete models A - E are indicated in a manner corresponding to the note below Fig. 5. It is evident from the comparison of the calculations in Fig. 6 (calculations for the model of the cylinder with crosswise-supported bases) with the calculations in Fig. 7 that before the ultimate concrete strength in uniaxial compression is reached, the evolution of compressive force versus displacement is practically identical for all given boundary conditions. The calculations for the given boundary conditions differ only in terms of the post-peak compressive behavior of the concrete; in some cases they do not differ at all.
Fig. 7 Results of the numerical uniaxial compression test simulations for different boundary conditions

Fig. 8 shows a comparison between the results of the numerical simulations for different boundary conditions and the experimental results. In this figure, the solid curves represent the calculations for the model of the cylinder with crosswise-supported bases. The dashed curves represent the calculations for the finite element model of the cylinder between crosswise-supported plates whose material was modeled as steel. Nonlinear concrete models A - E are again indicated in a manner corresponding to the note below Fig. 5. It is clear from the load-time diagram in Fig. 8 that in the case of the cylinder held between plates, the maximum compressive force was reached later in time than in the case of the cylinder without plates for practically all of the nonlinear models used. The results for the cylinder held between plates therefore provide a better approximation of the experimental results, while the Winfrith Concrete material model provides the best approximation of the experimental data in terms of the evolution of compressive force over time before the ultimate concrete strength in uniaxial compression is reached.

D. Description of the simulated triaxial compression tests

In a real triaxial compression test, a concrete cylinder is placed between the pressure plates in the triaxial test chamber of a triaxial apparatus. For the numerical triaxial compression test simulations, the same assumptions as in the case of the uniaxial compression test simulations (Fig. 3) were established in terms of boundary conditions (see Fig. 9).

Within the triaxial compression test simulations, the cylinder was considered to be loaded via compression at a constant velocity from both (top and bottom) sides and via constant pressure (confinement) over time from the side. The vertical displacement of both bases of the cylinder over time thus grew linearly and evenly, during which the cylinder was loaded beyond its limits, and failed (see Fig. 10).

E. Results of the numerical triaxial compression test simulations, and their comparison with experimental data

The results of the numerical triaxial compression test simulations for two levels of confinement (7 MPa and 20 MPa) are depicted in the load-displacement diagrams in Fig. 11 and Fig. 12. It is clear from the diagrams that during the compressive loading at both levels of confinement, the concrete cylinder first exhibited linearly elastic behavior and then elasto-plastic behavior for all of the nonlinear models of concrete used. It is also evident from the diagrams that the maximum compressive force (i.e. the maximum compression load capacity of the concrete cylinder) increased along with the increasing level of confinement for all of the nonlinear models used. Furthermore it is seen that in the case of both levels of confinement, the cylinder exhibited much more ductile post-peak compressive behavior than in the case of the cylinder without confinement (see Fig. 6) for most of the nonlinear material models used. Nonlinear concrete models A - E in Fig. 11 and Fig. 12 are indicated in a manner corresponding to the note below Fig. 5.

Fig. 11 and Fig. 12 also compare the calculations for the levels of confinement of 7 MPa and 20 MPa with the relevant test data taken from [32]. Since the boundary conditions...
established for the numerical simulations in this paper correspond to the boundary conditions used within the experiments from which the test data were obtained, the results of the numerical simulations demonstrate a good degree of agreement with the test data in terms of the evolution of compressive force versus displacement before the ultimate concrete strength in triaxial compression is reached. This is also true with regard to the behavior of concrete in triaxial compression. It is evident from the figures that the Karagözian & Case Concrete material models provide the best approximation of the test data. Therefore, these nonlinear material models with the given parameter values best describe the behavior of the given concrete in triaxial compression.

In addition, the effect of boundary conditions on the results of the numerical simulations of uniaxial compression tests was studied. This study showed how the boundary conditions affected the behavior of a concrete cylinder in uniaxial compression as calculated within the numerical simulations, especially in terms of the evolution of compressive force over time.

**REFERENCES**


