A Computational Mechanics using Nonlinear Analysis Method in Tensioned Fabric Structure

Yee Hooi Min, Choong Kok Keong

***Abstract*—**Nonlinear analysis method is one of the earliest methods proposed for form-finding analysis of tensioned fabric structures. However due to some inherent weaknesses, the method has not been fully developed. In this paper, computational strategies for form-finding analysis of tensioned fabric structure using the nonlinear analysis method has been proposed. For the purpose of verification, form-finding analysis using the proposed computational strategies have also been applied on cable reinforced two saddle-shaped tensioned fabric structure model, cable reinforced double Chinese hat tensioned fabric structure model and cable reinforced tent Hüfingen tensioned fabric structure model which have been studied by other researchers. The proposed computational strategies can be presented as a general tool capable of form-finding analysis of tensioned fabric structures for structural engineer.

***Keywords*—**Nonlinear analysis method, Form-Finding, Tensioned Fabric Structure, Weakness, Computational Strategies, Cable Reinforced.

I. INTRODUCTION

Tensioned Fabric Structures (TFS) include a wide variety of systems that are distinguished by their reliance upon tensile only members to support load. [1] have mentioned that structures are built to last a long time. Actually, TFS has been used over the past 50 years ago. [2] have stated there is some particular information about the development of a new sort of a temporary modular steel footbridge for pedestrians and cyclist, which was designed as a truss system with the deck below the supports and with the closed cross-section. [3] have stated researches in course of development in the field of protection of new and existing structures subject to dynamic events. Tensioned fabric structure is the best alternative to cover area with low cost. One of the greatest benefit is their translucent. Woven fabric coated with a polymeric resin allows a light transmission value of around 10%. This provides a very comfortable level of illumination compared to the full brightness of outside the structure. [4] have stated large energy consumption of energy heating for residential and commercial use in the world. [5], [6], [7], [8] and [9] have stated energy saving. The energy saving problem can be overcome by application of TFS.

Due to the flexibility of architectural fabric used, a great variety of surface form can be realized using TFS. Nevertheless, for the structure to function satisfactory, its initial equilibrium shape has to be first determined. Many technical procedures and algorithms have been developed to find the structure form and to determine the prestressed system. They can be roughly sorted into three groups: force density methods; dynamic relaxation method; non-linear displacement method. [10] has described that the non-linear displacement method is among the first computer methods applied to solve initial equilibrium problem. The principle of non-linear displacement method is based on the large displacement finite element technique used for analysis of structural behaviour under external loads. Since the method can be used for both the initial equilibrium problem and load analysis, the approach using non-linear analysis is quite common. [11] has carried out form-finding analysis by using computer program for stress analysis which is based on theory of non-linear displacement method. It concluded that a stress analysis program based on non-linear displacement method could be effectively used for form-finding analysis, provided that the material properties are controlled. [12] have presented a method of nonlinear analysis of fabric tension structures and numerical examples. An updated formulation is used to include displacements in the load. A shape-finding is performed using a technique with a small Young's modulus. A uniaxal stress-strain relationship is derived and used in a wrinkling formulation. [13] have described the initial equilibrium problem for tension structures and presented a variety of methods for solving it. [14] are among the first to use the non-linear displacement method to solve the initial equilibrium problem for cable nets. Their method was developed in order to find the form of the cable roofs at the 1972 Olympics in Munich. [15] has used non-linear displacement method. This method is an application of the dynamic relation method, where an initially out-of-balance structure is allowed to undergo damped vibrations until a steady equilibrium shape is obtained. [16] has mentioned the governing equation of shape-finding. The shape of the equilibrium of a given stress field is the theoretical basis of the form-finding analysis of membrane structures. References [17]-[20] have studied the problem of mathematical modeling in the industry.

[21] have stated that in recent years, the tensile surface

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structure business has grown considerably, and is predicted to grow further. Such structures are becoming bigger and more sophisticated. A lot of sophisticated methods for form-finding analysis have been proposed over the years. Despite all these development, nonlinear analysis method is worth relooking based on the reason that it is easily available (e.g. any conventional FE program with capability for initial stress input can be used) and until now there are lack of used on this method. From past research works, have no detailed studies on the nonlinear analysis method have been carried out. This study is aimed to propose a strategy of form-finding analysis using the nonlinear analysis method to overcome the inherent weakness so that it can be presented as a general tool capable of form-finding analysis. Furthermore this method has been demonstrated to be effective by typical examples. The aspects of modeling of surface in the form of cable reinforced tensioned fabric structure models and the form as well as pre-stress pattern of the resulting cable reinforced tensioned fabric structure models through form-finding analysis are also studied.

II. COMPUTATIONAL STRATEGY FOR FORM-FINDING ANALYSIS USING NONLINEAR ANALYSIS METHOD

The principle of nonlinear analysis method is based on the large displacement finite element formulation used for analysis of structural behaviour under external loads. Since the method can be used for both the initial equilibrium problem and load analysis, the approach using nonlinear analysis is quite common. The basic equation used is expressed as follows:

$$({}_{0}^{t}\boldsymbol{K}_{L}+{}_{0}^{t}\boldsymbol{K}_{G})\boldsymbol{u}={}^{t+\Delta t}\boldsymbol{F}-{}_{0}^{t}\boldsymbol{f} \tag{1}$$

Where ${}_{0}^{t}\boldsymbol{K}_{L}$ is linear strain incremental stiffness matrix, ${}_{0}^{t}\boldsymbol{K}_{G}$ is nonlinear strain incremental stiffness matrix, ${}_{0}^{t}\boldsymbol{f}$ is vector internal forces, ${}^{t+\Delta t}\boldsymbol{F}$ is load vector and $\boldsymbol{u}$ is vector of increment in displacement.

Nonlinear finite element analysis procedures for stress analysis of TFS have been used as basis for form-finding analysis in this study. As a first shape for the start of form-finding analysis procedure adopted in this study, initial guess shape is needed. The software [22] has been used for the purpose of model generation. For the generation of such initial guess shape, anticlastic feature is incorporated into the model in order to produce a better initial guess shape. Such anticlastic feature has been incorporated by means of specification of selected sag, Δ, relative to two suitably chosen points on the fixed boundary with span, L. Using this Δ/L sag ratio, geometry for approximates the surface of TFS is then generated. Such geometry is then meshed to produce initial guess shape with anticlastic feature.

The proposed computational strategy involves two stages of analysis in one cycle. The first stage (denoted as SF1) is an analysis which starts with an initial assumed shape in order to obtain an updated shape for initial equilibrium surface. This is then followed by the second stage of analysis (denoted as SS1) aimed at checking the convergence of updated shape obtained at the end of stage SF1. During stage SF1, elastic modulus E with very small values, are used. Both warp and fill stresses, σ_W and σ_F are kept constant. In the second stage SS1, the actual tensioned fabric properties values are used. Resulting warp and fill stresses are checked at the end of the analysis against prescribed stresses. Iterative calculation has to be carried out in order to achieve convergence. The resultant shape at the end of iterative step n (SSn) is considered to be in the state of initial equilibrium under the prescribed warp and fill stresses and boundary condition if difference between the obtained and the prescribed tensioned fabric stresses relative to the prescribed stress satisfied the specified criteria. Such checking of difference in the obtained and prescribed stresses have been presented in the form of total stress deviation in warp and fill direction versus stress analysis stage.

III. COMPUTATIONAL EXAMPLES

Form-finding analysis in TFS using the proposed computational strategies have been applied with surface shapes in the forms of cable reinforced two saddle-shaped tensioned fabric structure model, cable reinforced double Chinese hat tensioned fabric structure model and cable reinforced tent Hüfingen tensioned fabric structure model which have been studied by other researchers such as [23], [24] and [16], respectively.

A. Cable Reinforced Two Saddle-Shaped Tensioned Fabric Structure Model

Table 1 Material properties, cable pretension and area cables of cable reinforced two saddle-shaped tensioned fabric structure model

Tensile modulus in warp direction , $E_W t$	900000N/m
Tensile modulus in fill direction, $E_F t$	648000N/m
Poisson's ratio corresponds to warp direction, v_W	0.018
Poisson's ratio corresponds to fill direction, v_F	0.025
Shear modulus, Gt	31400N/m
Modulus of elasticity of cable, E_c	2×10^{11} N/m^2
Area cables, A_c	0.00021m^2
Cable pretension, σ_c for edge Cable 1	40000N
Cable pretension, σ_c for edge Cable 2	50000N
Cable pretension, σ_c for edge Cable 3	75000N
Thickness, t	0.001m

A cable-membrane structure composed of two saddle-shaped surfaces from [23] in Fig. 1 has been studied. Fabric prestress in warp and fill of the first part cable reinforced two saddle-shaped tensioned fabric structure model are 2000N/m and for the second part cable reinforced two saddle-shaped TFS model are 2500N/m. The material properties, cable pretension and area cables of cable reinforced two saddle-shaped TFS model are shown in Table 1.

Fig. 2 shows the converged shape of the cable reinforced two saddle-shaped tensioned fabric structure model after form-finding. The converged value of cable pretension 1, 2 and 3 for the cable reinforced two saddle-shaped TFS model is 43032.14N, 52234.20N and 77549.56N, respectively. The least square error (LSE) of total warp and fill stress deviation for the first and second part of cable reinforced two saddle-shaped TFS model are < 0.01.

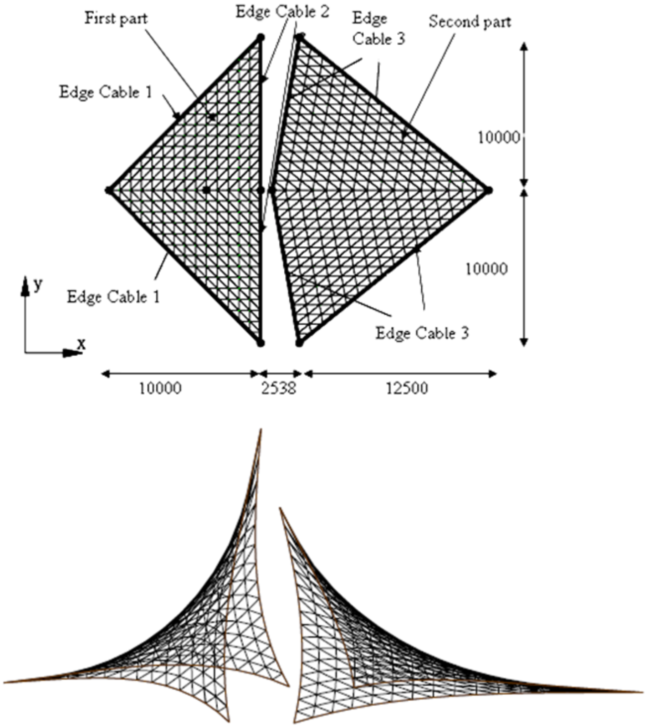

Fig. 1 A cable reinforced two saddle-shaped TFS model. (all units in mm). [23]

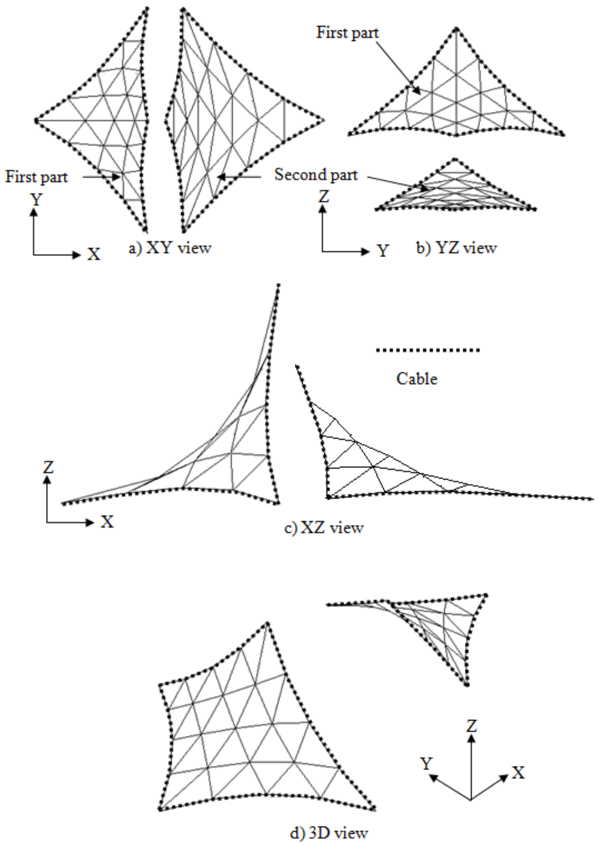

Fig. 2 Different views of the cable reinforced two saddle-shaped tensioned fabric structure model after form-finding

IV. CABLE REINFORCED DOUBLE CHINESE HAT TENSIONED FABRIC STRUCTURE MODEL

Form-finding in cable reinforced double Chinese hat tensioned fabric structure model with two hoops as shown in Fig. 3 from [24] have been carried out. The dimension of cable reinforced double Chinese hat TFS model is 10×6×2m. The fabric prestress in warp and fill direction is 3000N/m and cable pretension is 10000N.

Fig. 4 shows the converged shape of cable reinforced double Chinese hat tensioned fabric structure model after form-finding. The converged value of cable pretension for the cable reinforced double Chinese hat tensioned fabric structure model is 10862.43N. The LSE of total warp and fill stress deviation is 0.151083 and 0.072694, respectively. LSE of total warp and fill stress is same for several last few stress analysis stage. Hence, the value of LSE has been used as criteria of convergence instead of 0.01 that has been specified as convergence criteria.

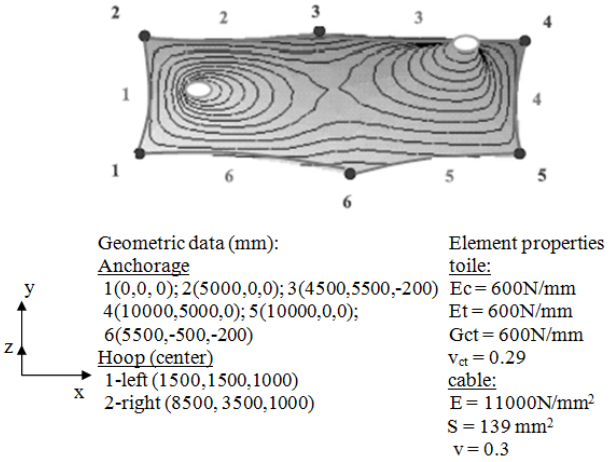

Fig. 3 Cable reinforced double Chinese hat TFS model. [20]

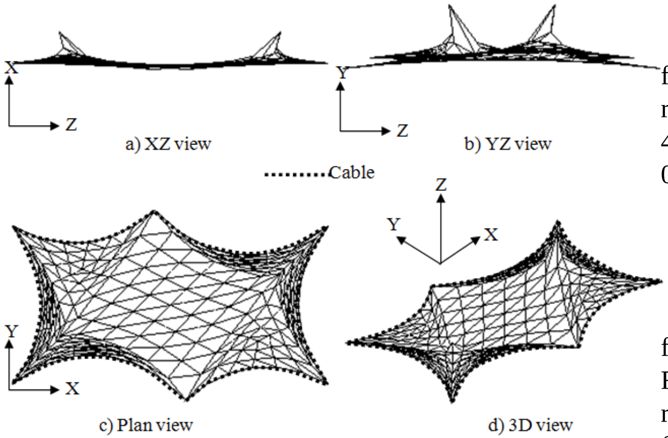

Fig. 4 Different views of cable reinforced double Chinese hat tensioned fabric structure model after form-finding

V. Cable Reinforced Tent Hüfingen Tensioned Fabric Structure Model

Form-finding in cable reinforced tent Hüfingen tensioned fabric structure model as shown in Fig. 5 from [16] have been studied. The dimension of the tent is 22.007×13.979×10.706m. Fabric prestress in warp σ_W and fill σ_F direction is 2500N/m. Material properties, cable pretension and tensioned fabric structure model as shown in Table 2.

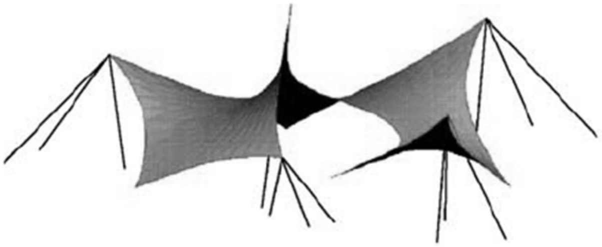

Fig. 5 Cable reinforced tent Hüfingen tensioned fabric structure model. [16].

Table 2 Material properties, cable pretension and area cables of cable reinforced tent Hüfingen tensioned fabric structure model

Tensile modulus in warp direction , E_Wt	900000N/m
Tensile modulus in fill direction, E_Ft	648000N/m
Poisson's ratio corresponds to warp direction, v_W	0.018
Poisson's ratio corresponds to fill direction, v_F	0.025
Shear modulus, Gt	31400N/m
Modulus of elasticity of cable, E_c	2×10^{10} N/m^2
Area cables, A_c	$0.00021m^2$
Cable pretension, σ_c	50000N

Fig. 5 shows the converged shape of the model after form-finding. The converged value of cable pretension for cable reinforced tent Hüfingen tensioned fabric structure model is 45480.80N. The LSE of total warp and fill stress deviation is 0.009022 and 0.007027, respectively.

I. Results and Discussion

Table 3 shows the cable pretension in previous research is found to be closed with cable pretension in current research. Form-finding analysis on all the above mentioned cable reinforcement tensioned fabric structure models have been found to converge. LSE of deviation of cable pre-tension for all models less than 0.01 except cable reinforced double Chinese hat tensioned fabric structure model has been found to satisfy the tolerance of 0.072694 to 0.842229. The results show that nonlinear analysis method is proven to be a simple method of form-finding analysis which can yield sufficiently accurate results.

Table 3 Cable pretension in previous research and cable pretension in current research

Types of cable reinforced tensioned fabric structure models	Cable pretension in previous research (N)	Cable pretension in current research (N)
Cable reinforced two saddle-shaped tensioned fabric structure model [16]		
Cable 1	40000.00	43032.14
Cable 2	50000.00	52234.20
Cable 3	75000.00	77549.56
Cable reinforced double Chinese hat tensioned fabric structure model [17]	10000.00	10862.43

Cable reinforced tent Hüfingen tensioned fabric structure model [18]	50000.00	45480.80

I. CONCLUSION

The computational procedures based on nonlinear analysis method in combination with properly formulated computational strategies can be presented as a general tool capable of form-finding of TFS. It has the added advantage that the program used to solve the initial equilibrium problem can also be used for further stress analysis.

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