A Computational Mechanics using Nonlinear Analysis Method in Tensioned Fabric Structure

Yee Hooi Min, Choong Kok Keong

Abstract—Nonlinear analysis method is one of the earliest methods proposed for form-finding analysis of tensioned fabric structures. However due to some inherent weaknesses, the method has not been fully developed. In this paper, computational strategies for form-finding analysis of tensioned fabric structure using the nonlinear analysis method has been proposed. For the purpose of verification, form-finding analysis using the proposed computational strategies have also been applied on cable reinforced two saddle-shaped tensioned fabric structure model, cable reinforced double Chinese hat tensioned fabric structure model and cable reinforced tent Hüfingen tensioned fabric structure model which have been studied by other researchers. The proposed computational strategies can be presented as a general tool capable of form-finding analysis of tensioned fabric structures for structural engineer.

Keywords—Nonlinear analysis method, Form-Finding, Tensioned Fabric Structure, Weakness, Computational Strategies, Cable Reinforced.

I. INTRODUCTION

Tensioned Fabric Structures (TFS) include a wide variety of systems that are distinguished by their reliance upon tensile only members to support load. [1] have mentioned that structures are built to last a long time. Actually, TFS has been used over the past 50 years ago. [2] have stated there is some particular information about the development of a new sort of a temporary modular steel footbridge for pedestrians and cyclist, which was designed as a truss system with the deck below the supports and with the closed cross-section. [3] have stated researches in course of development in the field of protection of new and existing structures subject to dynamic events. Tensioned fabric structure is the best alternative to cover area with low cost. One of the greatest benefit is their translucent. Woven fabric coated with a polymeric resin allows a light transmission value of around 10%. This provides a very comfortable level of illumination compared to the full brightness of outside the structure. [4] have stated large energy consumption of energy heating for residential and commercial use in the world. [5], [6], [7], [8] and [9] have stated energy saving. The energy saving problem can be overcome by application of TFS.

Due to the flexibility of architectural fabric used, a great variety of surface form can be realized using TFS. Nevertheless, for the structure to function satisfactory, its initial equilibrium shape has to be first determined. Many technical procedures and algorithms have been developed to find the structure form and to determine the prestressed system. They can be roughly sorted into three groups: force density methods; dynamic relaxation method; non-linear displacement method. [10] has described that the non-linear displacement method is among the first computer methods applied to solve initial equilibrium problem. The principle of non-linear displacement method is based on the large displacement finite element technique used for analysis of structural behaviour under external loads. Since the method can be used for both the initial equilibrium problem and load analysis, the approach using non-linear analysis is quite common. [11] has carried out form-finding analysis by using computer program for stress analysis which is based on theory of non-linear displacement method. It concluded that a stress analysis program based on non-linear displacement method could be effectively used for form-finding analysis, provided that the material properties are controlled. [12] have presented a method of nonlinear analysis of fabric tension structures and numerical examples. An updated formulation is used to include displacements in the load. A shape-finding is performed using a technique with a small Young's modulus. A uniaxial stress-strain relationship is derived and used in a wrinkling formulation. [13] have described the initial equilibrium problem for tension structures and presented a variety of methods for solving it. [14] are among the first to use the non-linear displacement method to solve the initial equilibrium problem for cable nets. Their method was developed in order to find the form of the cable roofs at the 1972 Olympics in Munich. [15] has used non-linear displacement method. This method is an application of the dynamic relation method, where an initially out-of-balance structure is allowed to undergo damped vibrations until a steady equilibrium shape is obtained. [16] has mentioned the governing equation of shape-finding. The shape of the equilibrium of a given stress field is the theoretical basis of the form-finding analysis of membrane structures. References [17]-[20] have studied the problem of mathematical modeling in the industry.

[21] have stated that in recent years, the tensile surface...
structure business has grown considerably, and is predicted to
grow further. Such structures are becoming bigger and more
sophisticated. A lot of sophisticated methods for form-finding
analysis have been proposed over the years. Despite all these
development, nonlinear analysis method is worth relooking
based on the reason that it is easily available (e.g. any
conventional FE program with capability for initial stress input
can be used) and until now there are lack of used on this
method. From past research works, have no detailed studies on
the nonlinear analysis method have been carried out. This
study is aimed to propose a strategy of form-finding analysis
using the nonlinear analysis method to overcome the inherent
weakness so that it can be presented as a general tool capable
of form-finding analysis. Furthermore this method has been
demonstrated to be effective by typical examples. The aspects
of modeling of surface in the form of cable reinforced
tensioned fabric structure models and the form as well as pre-
stress pattern of the resulting cable reinforced tensioned fabric
structure models through form-finding analysis are also studied.

II. COMPUTATIONAL STRATEGY FOR FORM-
FINDING ANALYSIS USING NONLINEAR ANALYSIS
METHOD

The principle of nonlinear analysis method is based on the
large displacement finite element formulation used for analysis
of structural behaviour under external loads. Since the method
can be used for both the initial equilibrium problem and load
analysis, the approach using nonlinear analysis is quite
common. The basic equation used is expressed as follows:

\[ (\varepsilon^0 K_L + \varepsilon^0 K_G) u - \varepsilon^+ F = \varepsilon^+ f \]  \hspace{1cm} (1)

Where \( \varepsilon^0 K_L \) is linear strain incremental stiffness matrix,
\( \varepsilon^0 K_G \) is nonlinear strain incremental stiffness matrix, \( \varepsilon^+ f \) is
vector internal forces, \( \varepsilon^+ F \) is load vector and \( u \) is vector of
increment in displacement.

Nonlinear finite element analysis procedures for stress
analysis of TFS have been used as basis for form-finding
analysis in this study. As a first shape for the start of form-
finding analysis procedure adopted in this study, initial guess
shape is needed. The software [22] has been used for the
purpose of model generation. For the generation of such initial
guess shape, anticlastic feature is incorporated into the model
in order to produce a better initial guess shape. Such anticlastic
feature has been incorporated by means of specification of
selected sag, \( \Delta \), relative to two suitably chosen points on the
fixed boundary with span, \( L \). Using this \( \Delta/L \) sag ratio,
geometry for approximates the surface of TFS is then
generated. Such geometry is then meshed to produce initial
guess shape with anticlastic feature.

The proposed computational strategy involves two stages
of analysis in one cycle. The first stage (denoted as SF1) is an
analysis which starts with an initial assumed shape in order to
obtain an updated shape for initial equilibrium surface. This is
then followed by the second stage of analysis (denoted as SS1)
aimed at checking the convergence of updated shape obtained
at the end of stage SF1. During stage SF1, elastic modulus \( E \)
with very small values, are used. Both warp and fill stresses,
\( \sigma_W \) and \( \sigma_F \) are kept constant. In the second stage SS1, the
actual tensioned fabric properties values are used. Resulting
warp and fill stresses are checked at the end of the analysis
against prescribed stresses. Iterative calculation has to be
carried out in order to achieve convergence. The resultant
shape at the end of iterative step \( n \) (SSn) is considered to be in
the state of initial equilibrium under the prescribed warp and
fill stresses and boundary condition if difference between the
obtained and the prescribed tensioned fabric stresses relative
to the prescribed stress satisfied the specified criteria. Such
checking of difference in the obtained and prescribed stresses
have been presented in the form of total stress deviation in
warp and fill direction versus stress analysis stage.

III. COMPUTATIONAL EXAMPLES

Form-finding analysis in TFS using the proposed
computational strategies have been applied with surface shapes
in the forms of cable reinforced two saddle-shaped tensioned
fabric structure model, cable reinforced double Chinese hat
tensioned fabric structure model and cable reinforced tent
Hüfingen tensioned fabric structure model which have been
studied by other researchers such as [23], [24] and [16],
respectively.

A. Cable Reinforced Two Saddle-Shaped Tensioned Fabric
Structure Model

Table 1 Material properties, cable pretension and area cables
of cable reinforced two saddle-shaped tensioned fabric
structure model

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile modulus in warp direction, ( E_W )</td>
<td>900000N/m</td>
</tr>
<tr>
<td>Tensile modulus in fill direction, ( E_F )</td>
<td>648000N/m</td>
</tr>
<tr>
<td>Poisson’s ratio corresponds to warp direction, ( \nu_W )</td>
<td>0.018</td>
</tr>
<tr>
<td>Poisson’s ratio corresponds to fill direction, ( \nu_F )</td>
<td>0.025</td>
</tr>
<tr>
<td>Shear modulus, ( G )</td>
<td>31400N/m</td>
</tr>
<tr>
<td>Modulus of elasticity of cable, ( E_c )</td>
<td>2 × 10^{11} N/m²</td>
</tr>
<tr>
<td>Area cables, ( A_c )</td>
<td>0.00021m²</td>
</tr>
<tr>
<td>Cable pretension, ( \sigma_c ) for edge Cable 1</td>
<td>40000N</td>
</tr>
<tr>
<td>Cable pretension, ( \sigma_c ) for edge Cable 2</td>
<td>50000N</td>
</tr>
<tr>
<td>Cable pretension, ( \sigma_c ) for edge Cable 3</td>
<td>75000N</td>
</tr>
<tr>
<td>Thickness, ( t )</td>
<td>0.001m</td>
</tr>
</tbody>
</table>

A cable-membrane structure composed of two saddle-
shaped surfaces from [23] in Fig. 1 has been studied. Fabric
prestress in warp and fill of the first part cable reinforced two saddle-shaped tensioned fabric structure model are 2000N/m and for the second part cable reinforced two saddle-shaped TFS model are 2500N/m. The material properties, cable pretension and area cables of cable reinforced two saddle-shaped TFS model are shown in Table 1.

Fig. 2 shows the converged shape of the cable reinforced two saddle-shaped tensioned fabric structure model after form-finding. The converged value of cable pretension 1, 2 and 3 for the cable reinforced two saddle-shaped TFS model is 43032.14N, 52234.20N and 77549.56N, respectively. The least square error (LSE) of total warp and fill stress deviation for the first and second part of cable reinforced two saddle-shaped TFS model are < 0.01.

![Fig. 2](image)

**Fig. 2** Different views of the cable reinforced two saddle-shaped tensioned fabric structure model after form-finding

IV. **CABLE REINFORCED DOUBLE CHINESE HAT TENSIONED FABRIC STRUCTURE MODEL**

Form-finding in cable reinforced double Chinese hat tensioned fabric structure model with two hoops as shown in Fig. 3 from [24] have been carried out. The dimension of cable reinforced double Chinese hat TFS model is 10×6×2m. The fabric prestress in warp and fill direction is 3000N/m and cable pretension is 10000N.

Fig. 4 shows the converged shape of cable reinforced double Chinese hat tensioned fabric structure model after form-finding. The converged value of cable pretension for the cable reinforced double Chinese hat tensioned fabric structure model is 10862.43N. The LSE of total warp and fill stress deviation is 0.151083 and 0.072694, respectively. LSE of total warp and fill stress is same for several last few stress analysis stage. Hence, the value of LSE has been used as criteria of convergence instead of 0.01 that has been specified as convergence criteria.
CABLE REINFORCED TENT HÜFINGEN TENSIONED FABRIC STRUCTURE MODEL

Form-finding in cable reinforced tent Hüfingen tensioned fabric structure model as shown in Fig. 5 from [16] have been studied. The dimension of the tent is 22.007×13.979×10.706m. Fabric prestress in warp $\sigma_W$ and fill $\sigma_F$ direction is 2500N/m. Material properties, cable pretension and tensioned fabric structure model as shown in Table 2.

Table 2 Material properties, cable pretension and area cables of cable reinforced tent Hüfingen tensioned fabric structure model

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile modulus in warp direction, $E_{Wt}$</td>
<td>900000N/m</td>
</tr>
<tr>
<td>Tensile modulus in fill direction, $E_{Ft}$</td>
<td>648000N/m</td>
</tr>
<tr>
<td>Poisson’s ratio corresponds to warp direction, $\nu_W$</td>
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</tr>
<tr>
<td>Shear modulus, $G_t$</td>
<td>31400N/m</td>
</tr>
<tr>
<td>Modulus of elasticity of cable, $E_c$</td>
<td>$2 \times 10^{10}$ N/m²</td>
</tr>
<tr>
<td>Area cables, $A_c$</td>
<td>0.00021m²</td>
</tr>
<tr>
<td>Cable pretension, $\sigma_c$</td>
<td>50000N</td>
</tr>
</tbody>
</table>

Fig. 5 shows the converged shape of the model after form-finding. The converged value of cable pretension for cable reinforced tent Hüfingen tensioned fabric structure model is 45480.80N. The LSE of total warp and fill stress deviation is 0.009022 and 0.007027, respectively.

I. RESULTS AND DISCUSSION

Table 3 shows the cable pretension in previous research is found to be closed with cable pretension in current research. Form-finding analysis on all the above mentioned cable reinforcement tensioned fabric structure models have been found to converge. LSE of deviation of cable pre-tension for all models less than 0.01 except cable reinforced double Chinese hat tensioned fabric structure model has been found to satisfy the tolerance of 0.072694 to 0.842229. The results show that nonlinear analysis method is proven to be a simple method of form-finding analysis which can yield sufficiently accurate results.

Table 3 Cable pretension in previous research and cable pretension in current research

<table>
<thead>
<tr>
<th>Types of cable reinforced tensioned fabric structure models</th>
<th>Cable pretension in previous research (N)</th>
<th>Cable pretension in current research (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cable reinforced two saddle-shaped tensioned fabric structure model [16]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cable 1</td>
<td>40000.00</td>
<td>43032.14</td>
</tr>
<tr>
<td>Cable 2</td>
<td>50000.00</td>
<td>52234.20</td>
</tr>
<tr>
<td>Cable 3</td>
<td>75000.00</td>
<td>77549.56</td>
</tr>
<tr>
<td>Cable reinforced double Chinese hat tensioned fabric structure model [17]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10000.00</td>
<td>10862.43</td>
</tr>
</tbody>
</table>
I. CONCLUSION

The computational procedures based on nonlinear analysis method in combination with properly formulated computational strategies can be presented as a general tool capable of form-finding of TFS. It has the added advantage that the program used to solve the initial equilibrium problem can also be used for further stress analysis.

REFERENCES


[21] Dr. Yee is a member of Board of Engineers Malaysia, Institution of Engineers Malaysia. After graduating, she worked as assistant professor at Faculty of Civil Engineering, Universiti Teknologi MARA, Pulau Pinang, Malaysia. Dr. Yee has obtained 9 awards in Invention, Innovation and Research Design Platform and has published 50 up-to-date publications.

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