Dynamic modeling, simulation and control of a hybrid driven press mechanism

Mehmet Erkan Kütük, Lale Canan Dülger

***Abstract*— Hybrid driven mechanism combines the motion of a large constant velocity motor with a small servo motor, and a mechanism. The constant velocity motor provides the main torque and motion requirements, while the servomotor contributes to modulations on this motion. Hybrid driven mechanism has higher flexibility in producing motion. Dynamic modeling and simulation of the hybrid system is presented in this study. A motion scenario is designed to be used on metal forming process. The equation of motion is derived by Lagrangian technique. Actuator dynamics for both axes are included with a PID control algorithm. The system simulation is performed with an explicit method; the fourth order Runge-Kutta method as an integration technique to get an approximate solution. The simulation results are presented.

***Keywords*—** dynamic analysis, hybrid press, system simulation, PID control

I. Introduction

THE main principle of hybrid driven system is to drive a mechanism with a servo motor and a constant speed motor including a flywheel. [1]. Thus, it is possible to take advantage of both drive systems. Non-uniform motion outputs can be obtained by combining these motors with such features as flexibility and reduced power requirement. [2, 3]. The first thing to do in hybrid system analysis is to design kinematics characteristics of the output link (slider). A motion scenario was designed. It is a quick rise and slow return motion. It is different from traditional metal forming motion obtained by a slider crank mechanism. This difference is shown in fig.1.

Having designed the slider motion, inverse kinematics analysis is carried out to get kinematics properties of servo motor. The input parameters of inverse technique are kinematics of slider and the crank driven by constant velocity motor. [4,5,6].

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M.Erkan Kütük is with Department of Mechanical Engineering, Gaziantep University, Turkey (corresponding author to provide phone: +903423172534; fax: +903423601104; e-mail: mekutuk@gantep.edu.tr).

L. Canan Dülger is with Department of Mechanical Engineering, Gaziantep University, Turkey, e-mail: dulger@gantep.edu.tr

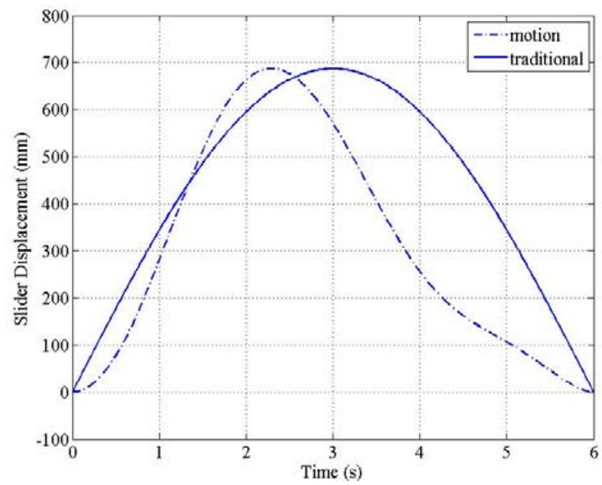
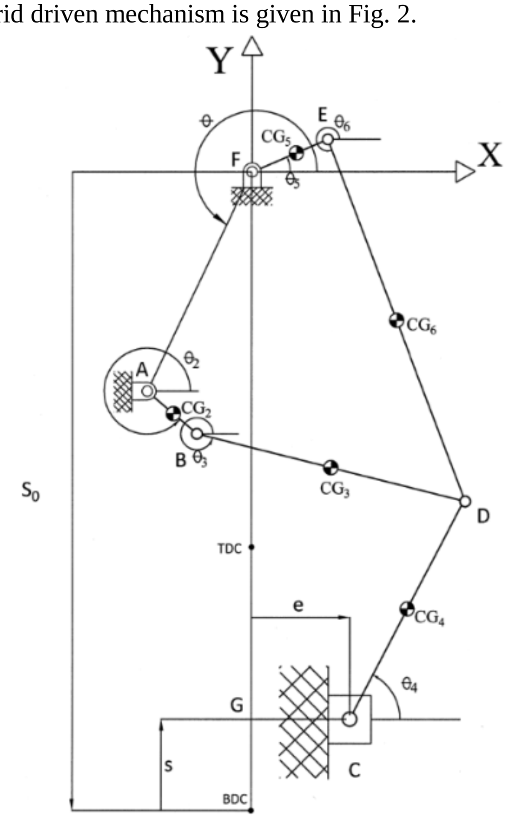


fig. 1 slider motions

A seven bar linkage system with two degrees of freedom was selected as the configuration. The schematic illustration of the hybrid driven mechanism is given in Fig. 2.



$$\left|\overrightarrow{AF}\right| = r_1,\ \left|\overrightarrow{AB}\right| = r_2,\ \left|\overrightarrow{BD}\right| = r_3,\ \left|\overrightarrow{CD}\right| = r_4,\ \left|\overrightarrow{FE}\right| = r_5,\ \left|\overrightarrow{ED}\right| = r_6$$

fig.2 a 2-DOF planar mechanism

The crank r_5 is driven by a constant velocity motor. The crank r_2 is driven by a servo motor. All inverse and forward kinematics analysis and motion design criteria are given with all details in [4]. Also, command graphs used in Simulation Results (ıv. section) are directly taken from [4]. It is also possible to simulate the hybrid mechanism with SimMechanics without deriving kinematics expressions as given in fig. 3 [6].

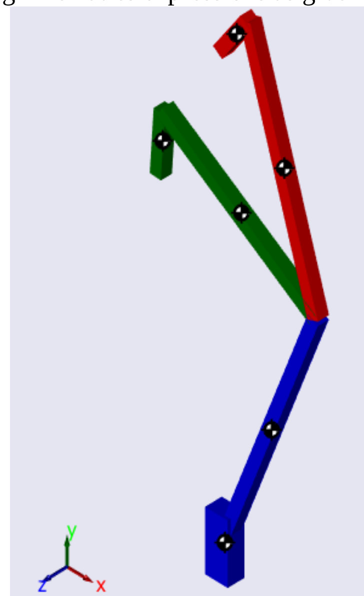


fig. 3 3D SimMechanics simulation [6]

A dynamic model plays a very important role in the terms of system validation, analysis, and control system. It is very essential to study the dynamic modeling of the hybrid mechanism in order to provide the desirable performance [7, 8].

In this study, dynamic model of hybrid system is derived by Lagrangian approach and solved. DC motor is used in simulation of Motor-load model and PID technique is used in control.

II. Dynamic Modeling

Lagrangian mechanics is formed by differentiation of the energy expressions with respect to the variables of system and time. Lagrangian expression is obtained by subtracting the potential energy of the system from the kinetic energy of the system.

$$L=T-V \tag{1}$$

The equations of motion are then derived by taking energy expressions of the system.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i}=Q_i \tag{2}$$

Q_i is the generalized torque or force and qi is the generalized coordinate. Q_i and q_i in linear motions are the force (F) and the linear displacement (x), respectively. Q_i and qi rotational motions are torque (τ) and the angular displacement (θ), respectively. [8]. The rotation angles of the cranks are taken as the generalized coordinates; $q_1=\theta_2$, $q_2=\theta_5$. Equation of motion for two axes are given in (3-4).

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right)-\frac{\partial L}{\partial \theta_2}=\tau_2 \tag{3}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_5}\right)-\frac{\partial L}{\partial \theta_5}=\tau_5 \tag{4}$$

Fifth link is driven by a constant velocity motor whereas the second link is driven by a servo motor as seen in fig.2.

A. Kinetic Energy of the System

The velocities of the mass center of every links should be obtained to calculate the kinetic energy of the mechanism.

X_{cg_i} and Y_{cg_i} are respectively the horizontal and vertical mass center positions of the i^{th} link of the hybrid mechanism. r_{cg_i} is the location of the center of mass of the i^{th} link.

Link lengths of the mechanism is given in Table 1 in mm.

Table 1. Link Lengths

r_1	r_2	r_3	r_4	r_5	r_6	e	θ
530	200	650	900	170	800	6	245^0

Inertial values are given in Table 2.

Table 2. Inertial Parameters

Parameters	*Link 2*	*Link 3*	*Link 4*	*Link 5*	*Link6*	*Slider*
mass (kg)	3	9.95	13.78	2.6	12.25	20
Mass Moment of Inertia (kg m^2)	0.0105	0.3519	0.9323	0.0067	0.6552	-

$$X_{cg_2}=X_A+r_{cg_2}\cos\theta_2 \tag{5}$$

$$Y_{cg_2}=Y_A+r_{cg_2}\sin\theta_2 \tag{6}$$

$$X_{cg_3}=X_A+r_2\cos\theta_2+r_{cg_3}\cos\theta_3 \tag{7}$$

$$Y_{cg_3}=Y_A+r_2\sin\theta_2+r_{cg_3}\sin\theta_3 \tag{8}$$

$$X_{cg_4}=r_5\cos\theta_5+r_6\cos\theta_6-r_{cg_4}\cos\theta_4 \tag{9}$$

$$Y_{cg_4}=r_5\sin\theta_5+r_6\sin\theta_6-r_{cg_4}\sin\theta_4 \tag{10}$$

$$X_{cg_5}=r_{cg_5}\cos\theta_5 \tag{11}$$

$$Y_{cg_5}=r_{cg_5}\sin\theta_5 \tag{12}$$

$$X_{cg_6}=r_5\cos\theta_5+r_{cg_6}\cos\theta_6 \tag{13}$$

$$Y_{cg_6}=r_5\sin\theta_5+r_{cg_6}\sin\theta_6 \tag{14}$$

$$X_{slider}=e \tag{15}$$

$$Y_{slider}=S_0+s \tag{16}$$

Velocity and acceleration of center of masses of the links are obtained by taking first and second order derivatives of (5-16) with respect to time, respectively [4].
Velocities and acceleration of mass centers of the links are;

$$V_{cg_i} = \sqrt{\dot{X}_{cg_i}^{\ 2} + \dot{Y}_{cg_i}^{\ 2}} \tag{17}$$

$$a_{cg_i} = \sqrt{\ddot{X}_{cg_i}^{\ 2} + \ddot{Y}_{cg_i}^{\ 2}} \tag{18}$$

Kinetic energy of the system;

$$T = \sum_{i=2}^{6} \frac{1}{2}(m_i V_{cg_i}^2 + I_i \dot{\theta}_i^2) + \frac{1}{2} m_{slider} V_{slider}^2 \tag{19}$$

where I_i is mass moment of inertia of the i^{th} link of the mechanism. The links used in the mechanism are taken as solid cylinders [9].

$$I_i = \frac{1}{12} m_i (r_i^2 + 3d_i^{\ 2}) \tag{20}$$

where m_i, r_i and d_i are mass, length and radius of the i^{th} link, respectively.

Total kinetic energy of the mechanical system is given in (21).

$$\begin{aligned} T = {} & (m_3((r_2 w_2 \cos\theta_2 + r_{cg_3} w_3 \cos\theta_3)^2 + (r_2 w_2 \sin\theta_2 + r_{cg_3} w_3 \sin\theta_3)^2)/2 + (m_6((r_5 w_5 \cos\theta_5 + \\ & r_{cg_6} w_6 \cos\theta_6)^2 + (r_5 w_5 \sin\theta_5 + r_{cg_6} w_6 \sin\theta_6)^2))/2 + (m_4((r_5 w_5 \cos\theta_5 - r_{cg_4} w_4 \cos\theta_4 + r_6 w_6 \\ & \cos\theta_6)^2 + (r_5 w_5 \sin\theta_5 - r_{cg_4} w_4 \sin\theta_4 + r_6 w_6 \sin\theta_6)^2))/2 + (m_2(r_{cg_2}^{\ 2} w_2^{\ 2}))/2 + (m_5(r_{cg_5}^{\ 2} w_5^{\ 2}))/ \\ & 2 + (m_{slider}(r_5 w_5 \cos\theta_5 - r_4 w_4 \cos\theta_4 + r_6 w_6 \cos\theta_6)^2)/2 + (m_2 w_2^{\ 2}(3d_2^{\ 2} + r_2^{\ 2}))/24 + (m_3 w_3^{\ 2} \\ & (3d_3^{\ 2} + r_3^{\ 2}))/24 + (m_4 w_4^{\ 2}(3d_4^{\ 2} + r_4^{\ 2}))/24 + (m_5 w_5^{\ 2}(3d_5^{\ 2} + r_5^{\ 2}))/24 + (m_6 w_6^{\ 2}(3d_6^{\ 2} + r_6^{\ 2}))/ \\ & 24 \end{aligned} \tag{21}$$

B. Potential Energy of the System

By looking at fig. 2, point F is the reference point in the calculation of the potential energy of the system. Then the potential energy of the system is;

$$V = \sum_{i=2}^{6} m_i g Y_{cg_i} + m_{slider} g Y_{slider} \tag{22}$$

The total potential energy of the hybrid driven mechanism is given in (23).

$$\begin{aligned} V = {} & m_6 g(r_5 \sin\theta_5 + r_{cg_6} \sin\theta_6) + m_3 g(Y_A + r_2 \sin\theta_2 + r_{cg_3} \sin\theta_3) + \\ & m_3 g(Y_A + r_2 \sin\theta_2 + r_{cg_3} \sin\theta_3) + m_4 g(r_5 \sin\theta_5 - r_{cg_4} \sin\theta_4 + r_6 \sin\theta_6) + \\ & m_{slider} g(r_5 \sin\theta_5 - r_4 \sin\theta_4 + r_6 \sin\theta_6) + m_2 g(Y_A + r_{cg_2} \sin\theta_2) + m_5 g(Y_A + \\ & r_{cg_5} \sin\theta_5) \end{aligned} \tag{23}$$

III. MOTOR-LOAD MODELING

The electric equivalent circuit of the armature and the free-body diagram of the rotor are shown in fig. 4. The rotor and shaft are assumed to be rigid [10,11].

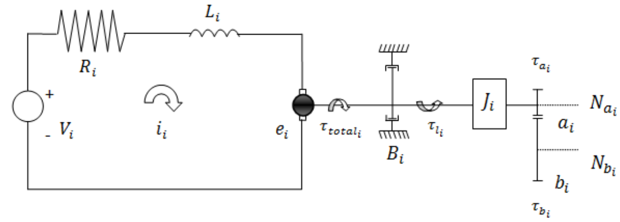


fig. 4 schematic representation of a DC motor

Gear ratio of the speed reducer,

$$n_i = \frac{\tau_{b_i}}{\tau_{a_i}} = \frac{w_{a_i}}{w_{b_i}} = \frac{N_{b_i}}{N_{a_i}} \qquad i = (2,\ 5) \tag{24}$$

where i=2 and 5 for speed reducer of servo and constant velocity motor, respectively.

w_{a_i} and w_{b_i} are the angular velocities of the input shaft a and the output shaft b, respectively. N_{a_i} and N_{b_i} are the numbers of teeth of the gears a and b. It is assumed that the input of the system is the voltage source (V_i) applied to the motor's armature, the output is torque (τ_{b_i}). i_i, R_i and L_i represent the armature current, resistance and inductance, respectively. Shaft b is connected to the crank of the mechanism in order to drive the mechanism. B_i is the viscous friction coefficient. τ_{l_i} is constant mechanical torque caused of friction of gear and brush. J_i is the total mass moment of inertia including flywheel, reducer and rotor of motor.

$$J_i = J_{f_i} + J_{r_i} + J_{m_i} \tag{25}$$

where J_{f_i} is the moment of inertia of the flywheel, J_{r_i} is the moment of inertia of the speed reducer and J_{m_i} is the moment of inertia of the motor rotor.
Kirchhoff voltage law is applied around windings of the armature [11];

$$V_i = i_i R_i + L_i \frac{di_i}{dt} + e_i \tag{26}$$

where e_i is back electromotive force of the motor.
Newtonian equation is used to get the torque equation below [10,12];

$$\tau_{b_i} = n_i(\tau_{total_i} - \tau_{l_i} - B_i w_{a_i} - J_i \frac{dw_{a_i}}{dt}) \tag{27}$$

where τ_{total_i} is the magnetic motor torque. τ_{total_i} and e_i are defined as.

$$\tau_{total_i} = K_{t_i} i_i \tag{28}$$

$$e_i = K_{e_i} w_{a_i} \tag{29}$$

where K_t is the motor torque constant and K_e is the motor voltage constant.
In general,

$$w_{a_i} = n_i w_{b_i} = n_i \dot{q}_i \tag{30}$$

$$\frac{di_i}{dt} = \frac{V_i - R_i - n_i K_{e_i} \dot{q}_i}{L_i} \tag{31}$$

$$\tau_{b_i} = n_i K_{t_i} i_i - n_i \tau_{l_i} - n_i^2 B_i \dot{q}_i - n_i^2 J_i \ddot{q}_i \quad (32)$$

τ_{b_i} for servo motor is τ_2 which is given in (3) and τ_{b_i} for constant velocity motor is τ_5 which is given in (4).

Hybrid driven press mechanism has got two degrees of system. Equation (32) is written for two axes (2nd and 5th link). The expressions obtained are highly nonlinear and coupled. All details are given in [4].

A. Control of Hybrid Driven System

PID controllers use a control signal including position, velocity and integral of position error functions. The mathematical representation of standard PID control is described as follows; [10,12,13,14];

$$V_i = k_{p_i}(q_{i_i} - q_{i_a}) + k_{d_i}(\dot{q}_{i_i} - \dot{q}_{i_a}) + k_{I_i}\int_0^t (q_{i_i} - q_{i_a})dt \quad (33)$$

where k_{p_i} is the proportional gain constant, k_{d_i} is the derivative gain constant and k_{I_i} is the integral gain constant. The required angular displacement and angular velocity of the crank i are q_{i_i} and $\dot{q}_{i_i}$. The actual angular displacement and angular velocity of the crank i are q_{i_a} and $\dot{q}_{i_a}$, respectively. So the motor current equation is modified as follows;

$$\frac{d_{i_i}}{dt} = \frac{k_{p_i}(q_{i_i} - q_{i_a}) + k_{d_i}(\dot{q}_{i_i} - \dot{q}_{i_a}) + k_{I_i}\int_0^t (q_{i_i} - q_{i_a})dt - R_i i_i - n_i K_{e_i}\dot{q}_{i_a}}{L_i} \quad (34)$$

B. State Space Representation

The dynamic behaviour of the hybrid driven system is studied by using a numerical method to get an approximate solution. The fourth order Runge-Kutta is used as the integration technique. It is an explicit method used in integration of nonlinear systems. It is necessary to form state-space representation. The aim in state-space representation is to introduce a suitable set of state variables. Then the equations of motion of the system are formed as a system of first order differential equations [15]. The angular displacements, the angular velocities and the currents are treated as the state variables.

$$X_1 = \theta_2,\ X_2 = w_2,\ X_3 = \theta_5,\ X_4 = w_5,\ X_5 = i_2,\ X_6 = i_5$$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \\ X_5 \\ X_6 \end{bmatrix} = \begin{bmatrix} X_2 \\ \alpha_2 \\ X_4 \\ \alpha_5 \\ \left(k_{p_2}(\theta_{2_i} - X_1) + k_{d_2}(w_{2_i} - X_2) + k_{I_2}\int_0^t (\theta_{2_i} - X_1)dt - R_2 X_5 - n_2 K_{e_2} X_2\right)/L_2 \\ \left(k_{p_5}(\theta_{5_i} - X_3) + k_{d_5}(w_{5_i} - X_4) + k_{I_5}\int_0^t (\theta_{5_i} - X_3)dt - R_5 X_6 - n_5 K_{e_5} X_4\right)/L_5 \end{bmatrix} \quad (35)$$

α_2 and α_5 expressions are taken from (3) and (4), respectively. Their explicit forms with addition of the motor dynamics are given in [4].

IV. Simulation Results

Modeling is carried out to understand the dynamic behaviour of the complex systems. It is used to investigate the relationship between the system variables. The mathematical model of the system is described by a set of mathematical expressions. Then simulation is performed on a computer [7].

The properties of the motors used are given in Table III and Table IV. The gear ratio for DC motor is 60. Servo motor is directly driven to the crank r_2. A flywheel is coupled to DC motor. Total mass moment of inertia of the flywheel and the speed reducer is 50 kgm^2 $(J_{r_5} + J_{f_5})$. The time required for one cycle is 6 seconds. Incremental time of simulation is taken as 0.166 seconds. Viscous friction coefficient, B_i and the constant mechanical torque, τ_{l_i} are assumed as zero in calculations. Simulation results are presented as;

Table III. DC Motor Parameters

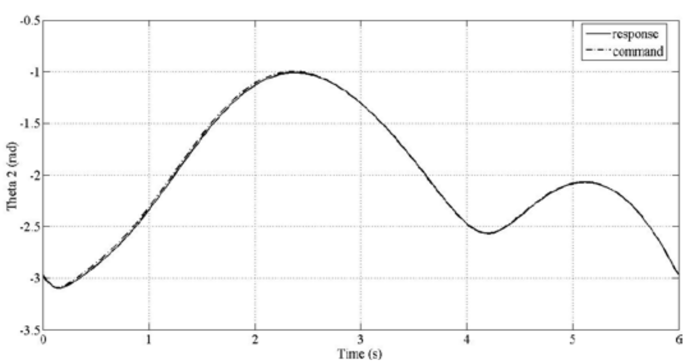
Rated Power (P_5)	18.5 KW
Rated Voltage (V_5)	440 V
Rated Current (i_5)	52 A
Rated Speed/Maximum Speed (W_{max})	600 rpm
Moment of Inertia (J_{m_5})	1.72 kg m^2
Moment of Inertia $(J_{r_5} + J_{f_5})$	50 kg m^2
Gear Ratio (n_5)	60
Winding Resistance (R_5)	0.973 Ω
Winding Inductance (L_5)	19.9*10^{-3} H
Motor Voltage Constant (K_{e_5})	6.2 V/rad/s
Motor Torque Constant (K_{t_5})	5.6627 Nm/A

Table IV. DC Servo Motor Parameters

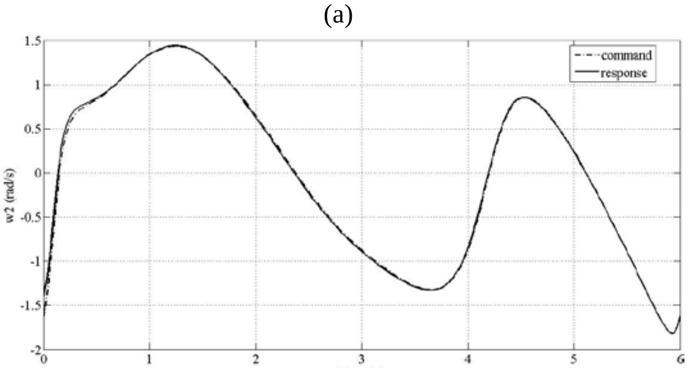
Rated Power (P_2)	3 KW
Maximum Speed (W_{max})	3000 rpm
Moment of Inertia (J_{m_2})	6.8*10^{-4} kg m^2
Winding Resistance (R_2)	0.8 Ω
Winding Inductance (L_2)	5.8*10^{-3} H
Motor Voltage Constant (K_{e_2})	0.8598 V/rad/s
Motor Torque Constant (K_{t_2})	0.76 Nm/A

The initial conditions of the angular displacement and velocity of the servo motor, the angular displacement and velocity of the constant velocity motor and the currents of the servo motor and constant velocity motor are given as X_{ic_1} = [-2.96(rad) -1.64(rad/sec) 4.51(rad) -1.0472(rad/sec) 0(A) 0(A)], respectively. The gain constants to control the system are found by using Ziegler Nichols method [11, 14]. Then they are modified by trial and error. The proportional (k_{p_2}), derivative (k_{d_2}) and integral constant (k_{i_2}) for the servo motor are taken as 1000, 100 and 50, respectively. The proportional gain constant (k_{p_5}) for the constant velocity motor is 4000. Since it is constant velocity motor, no derivative and integral gain constants are used.

The simulation results of Motion are shown in fig. 4. The visible lines in fig. 4 represent the response results of the simulation. They are the actual results. The hidden lines in fig. 4 are for command values. They are obtained from the kinematics analysis of the hybrid driven mechanism [4]. Fig.4 is composed of four parts. Fig.4a and fig.4b are for angular displacement and velocity for servomotor, respectively. Fig.4c and fig. 4d are for angular displacement and velocity for constant velocity motor, respectively. Response displacement values taken from fig. 4a and 4c are given to forward kinematics equation as input data to get response for slider displacement. Command displacement values taken from fig. 4a and 4c are given to forward kinematics equations as input data to get command for slider displacement. It is given in fig.5. Displacement error for slider is shown in fig.6. Command line is the ideal motion designed.



(a)



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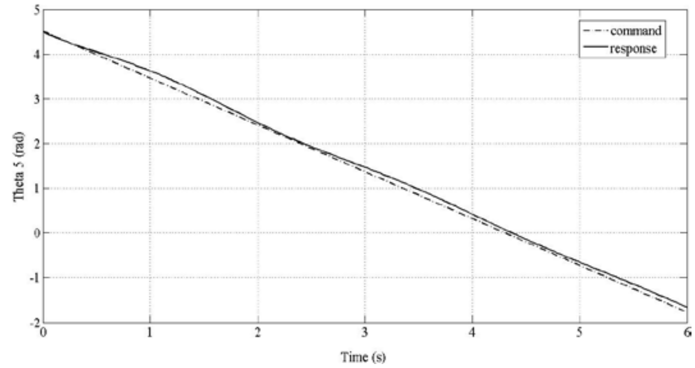

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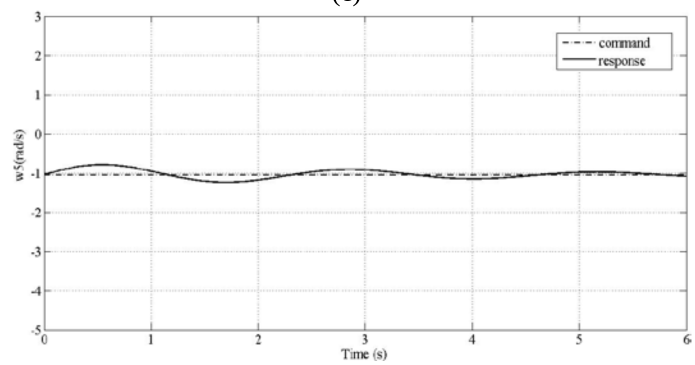

(d)

fig. 4 simulation results

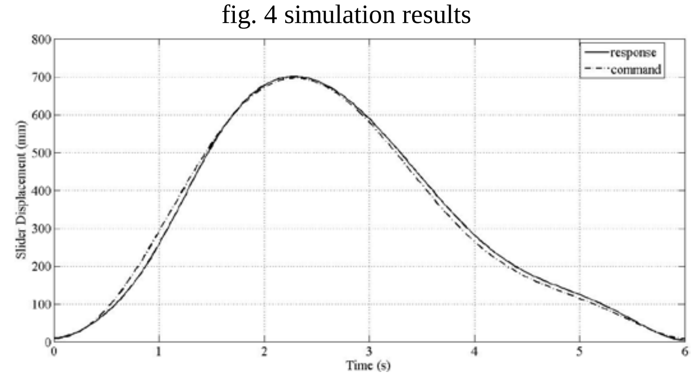

fig. 5 slider simulation results

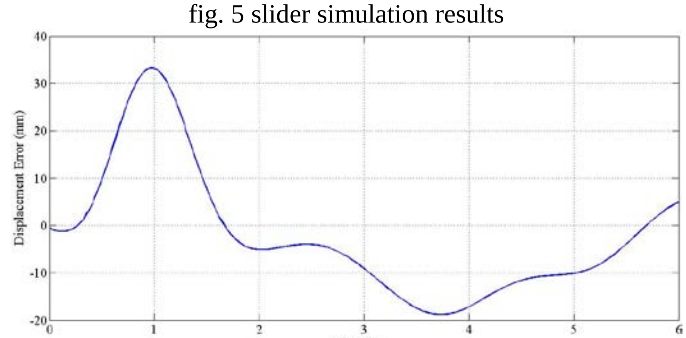

fig. 6 displacement error

V. CONCLUSION

Dynamic model of the hybrid system is derived with Lagrangian approach in this study. Kinematics relationship in the dynamic analysis is based on forward kinematics which two cranks are used as independent input parameters. Generalized angular acceleration and torque expressions are obtained as a result of Lagrangian expressions. Because the system has such a feature of highly nonlinear, there are a lot of coupled equations in the analysis.

The fourth order Runge-Kutta which is an explicit method used in integration of nonlinear systems is used as the integration technique to get an approximate solution. State space representation of the system is carried out to express the system in first order differential equations. A control system is studied to succeed design objectives. A PID control technique is applied on the system. Controllability of the system is demonstrated by throughout the simulation results. Matlab ® is used to get complete analysis and design stages.

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