Stability Assessment of an Historical Masonry Bridge through the LA Kinematic Theorem for NT Structures

I. Corbi, O. Corbi, F. Tropeano

Abstract— The paper focuses on a study case concerning an historical masonry bridge in the Campania region. The Devil's bridge on the Sele river is characterized by a single vaulted span. The arcade presents an elliptical shape with five focal points "anse de panier". It burdens on the foundation directly. The bridge is made of masonry both in its tympanum and in the fill; the fill is made of masonry bricks with mechanical properties similar to those ones of the main structure; so it is expected to cooperate to the structural function by contributing to the absorption of deformational and tensional effects. This paper is focused on the analysis of the behaviour of the bridge trough the setup of a preliminary study regarding the vault stability under its own weight; thereafter possible collapse mechanisms based on the kinematic theorem of Limit Analysis for masonry constructions under the No Tension assumption are selected, allowing to identify the most dangerous position of the variable load component for the bridge.

Keywords— *Collapse mechanisms; load multiplier; limit analysis.*

I. INTRODUCTION

The bridge can be considered, by different points of view, as an urban/landscape requalification, complex architectural element. The masonry bridge represents a very significant construction within the historical monumental heritage, both as regards the architectural and technical- executive features [1]-[7].

This is particularly true in Italy where many bridges were realized during the roman period, as main infrastructures for transportation and military scopes.

Actually, for two millenniums nearly, the masonry bridge have being built according to the roman techniques. During the roman age the construction of the bridge was playing such an important role to be committed to the "*Pontifices*", i.e. some kind of architects- ministers who supervised the technical rules for the edification of the masonry bridge and for its conservation.

In the following one develops a static analysis of a study case, concerning a bridge selected in the region Campania; the study is based on the application of the kinematic theorem of Limit Analysis for No-tension structures [8]-[32] , which is usually assumed for dealing with masonry constructions.

For the specific case at hand the objective is to give a response about the stability assessment of the bridge, that, as well known, represents a fundamental issue when addressing masonry bridges that are still currently active, like in the treated case, as transportation infrastructures on the territory.

This analysis is also aimed at selecting the worst position for accidental loads, able to identify the most dangerous condition for the bridge under the applied loads.

II. THE STUDY CASE

II.1 Introduction to the study case: the Devil's bridge on the Sele river

Starting from the pre-roman period, the use of masonry for the technical realization of constructions has been largely employed, and in particular for the construction of bridges, continuously until the beginning of the IXX century.

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The subsequent introduction and exploitation of new construction materials, like reinforced concrete and steel, although it has offered some different alternatives to masonry, has never caused the abandoning of masonry, whose adoption has never stopped. This may be probably re-conducted to the great observed stability of ancient masonry constructions, that have resisted many environmental and anthropological attacks during time, in many cases still resisting the adverse events and phenomena. Actually, during the first and second world wars, a number of masonry bridges with historical and artistic value, have being suffering some partial or total destruction.

One of the last examples of surviving masonry vaulted bridges in the Campania Region is represented by the Devil's bridge on Sele river in Barrizzo in Fig.1, on the way that from Salerno leads to Cilento, which is crossed by Sele river.

In order to overcome the natural obstacle represented by the river, after a number of attempts for building a bridge, which were unsuccessful because they were all destroyed by the overflow of the river, at the end of the IXX century, it was decided the realization of this masonry bridge.

The Devil's bridge is totally made of in masonry blocks and characterized by one single vaulted span. The big arcade is of the type usually defined as "anse de panier", with an elliptical shape and five focus points. Besides the historical and strategic relevance of the bridge, the analysis of the stability of the bridge appears interesting for its particular geometry and for the adopted technical execution.

In the following sections, the first phase of the analysis allows to verify the vault stability under its own weight, and the weight of the super-structure, including the tympanum, the fill, and the road. The stability assessment is checked through the search of an equilibrated an admissible solution under the applied fixed loads, and the relevant pressure curved surface.

The second phase of the study concerns the evaluation of a number of possible collapse mechanisms that may be activated and that may affect the arcade; consequently, the relevant kinematic displacements' sets are deducted, and the application of kinematic theorem of Limit Analysis for No-Tension structures is performed, where only unilateral constraints are admitted for modeling fractures.

The study is finally aimed to identify the most dangerous load condition within the set of considered mechanisms. The type of accidental loads is considered according to the Italian instructions.

II.2 Main technical and architectural features

The preliminary phase of the study has been based on an historical survey, in order to highlight the geometry and materials of the bridge. The vaulted span of the bridge is characterized by a diminished arch shape with the variable thickness, as shown in Fig. 2.

Figure 1: The Devil's Bridge on the Sele river.

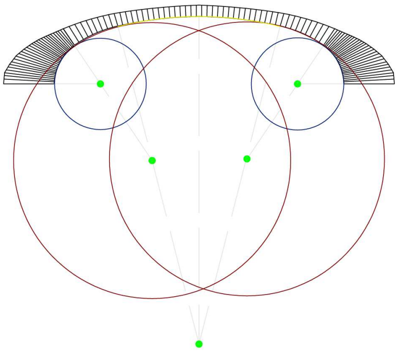

Figure 2: The geometry of the main vault of the bridge in the longitudinal plane, with the 5 focal points characterizing the elliptically shaped generator curve .

As concern the technological features, the bridge is integrally composed of masonry blocks, both in its external and internal part, in its tympani, vault, fill and other components. These components are made of different masonry materials with similar mechanical features, essentially consisting of calcareous stones and clay bricks.

The fill as well is made of blocks, and in order to be lightened in its weight, in this executive conception, some internal curved cavities have been realized. The prospective view of the bridge in its external and internal layered components is illustrated in Fig. 3. From this one may appreciate the internal positioning of the lightning holes crossing the internal fill.

Due to its particular configuration, and to the material whose mechanical characteristics are similar to those of the main structural vault, the fill is thus conceived in such a way to be able to significantly contribute to the global response of the bridge, by collaborating with the main vault to the static action against the loads which it is usually subject to.

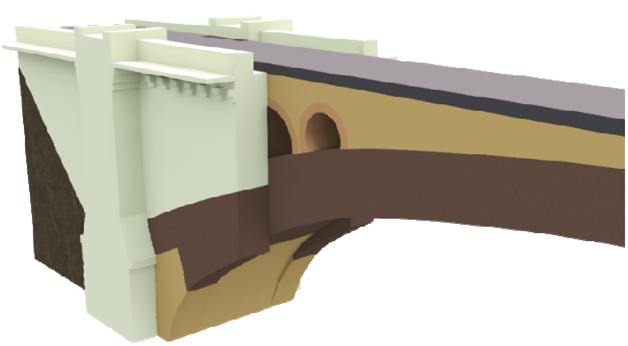

Figure 3: 3D view of the layered architectural composition of the bridge, with the internal fill lightened with inner holes.

As regards to the executive technique adopted for the realization of the vault, in the longitudinal plane it is characterized by three overlapping bricks' layers forming three superposed vaults, parallel to the intrados curved. These layers are jointed to each other in the longitudinal plane and inter- connected through special brick elements according to the technique of *integrated rolls* , shown in Fig.4. The superposed vaulted rolls, in the thickness, realize a sequence of five arches that are assembled together to compose the spatial vault.

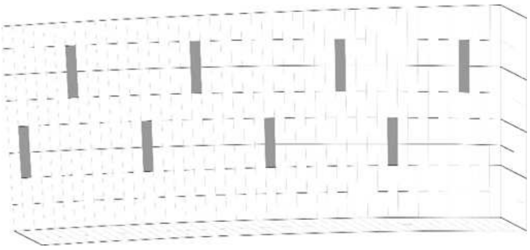

Figure 4: Detail of the vault realized by rolls jointed to each other through connection of special brick elements.

III. STABILITY ANALYSIS

III.1 Vault stability assessment

A preliminary study of the stability of the main vault has been developed under the hypothesis of No-Tension model as regards the masonry material of the bridge.

The study is based on the data that have been obtained after the initial survey study, about the geometry, materials, technological details, and mechanical properties.

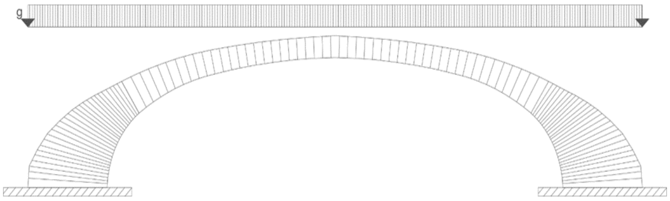

Figure 5: The model of the main vaulted span subject to permanent loads of the vault and of the superstructure.

The first step of the analysis has concerned the individuation of the static scheme of the main span and of the vault in Fig.5, under the action of the permanent loads, identified in the vault self-weight, and the weight of the structural and non-structural components, i.e. the tympanum, the fill, and so on.

Subsequently the stability assessment has been performed by searching for an admissible and equilibrated solution through the allocation in the vault profile of the funicular curve, which has been determined.

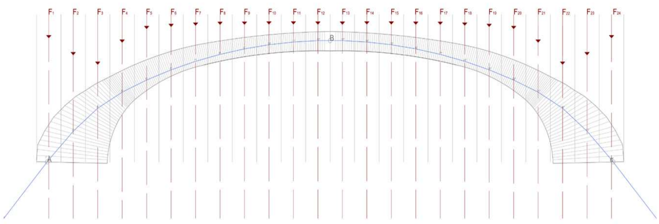

Figure 6: Funicular curve internal to the vault profile under the selected permanent loads.

The permanent loads are selected and placed in the downward vertical direction with regards to the relevant weights of the individual elements according to the varying sections of the vault, and to the vertical bands in which the structure is subdivided, as shown in Fig.6.

For the vault stability, the curve of subsequent stress resultants has been identified, completely bounded by the extrados and intrados vault profiles, as illustrated in Fig.6.

III.2 Limit Analysis

As well know, masonry vaults are generally characterized by static redundancies. Usually cracks appear at sections where the funicular line under a given load condition touches the vault contour. The activation of cracks may be then considered a physiological behavior of masonry.

The number and localization of cracks, and consequently of unilateral hinges activated allowing relative rotations, may turn the structure in a possible collapse mechanism depending on the number of redundancies. Therefore a preliminary analysis is necessary aimed at investigating the level of stability of the vault. Tools form Limit Analysis may be employed to this purpose according to the Static or the Kinematic Theorem, suitably extended to masonry constructions, under the NT hypothesis.

The Static Theorem searches for the largest load multiplier associated with the statically admissible stress distribution. The Kinematic Theorem searches for the kinematic multiplier, identified as the smallest multiplier associated with sufficient kinematic mechanisms.

III.3 Selection of kinematic mechanisms and related load multipliers

The approach adopted in this analysis refers to extension of the Kinematic Theorem to the case under exam.

Therefore one searches for the smallest kinematically sufficient multiplier. The first phase concerns the identification of probable collapse mechanisms.

A number of mechanisms are identified, by supposing the development of cracks on the arch and consequently the formation of unilateral hinges with relative rotations on the intrados or extrados of the arch.

Some examples of considered mechanisms are depicted in Fig.7, 8 and 9, where the allowed kinematic motion and the compatible deformed configuration of the vault are reported for any considered mechanism.

The identified mechanisms have been selected in a limited number compared to the infinity of possible collapse mechanisms. Therefore the calculated load multiplier does not coincide with the real multiplier but an approximation within the set of selected kinematic modes. Of course, within this set, the final load coefficient identifies the more dangerous load condition in the considered class that may produce the worst response of the vault of the bridge.

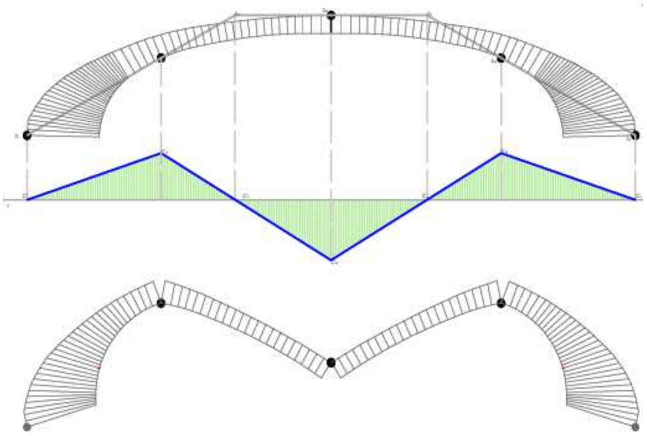

Figure 7: A symmetric mechanism with activation of five hinges and its compatible deformed configuration.

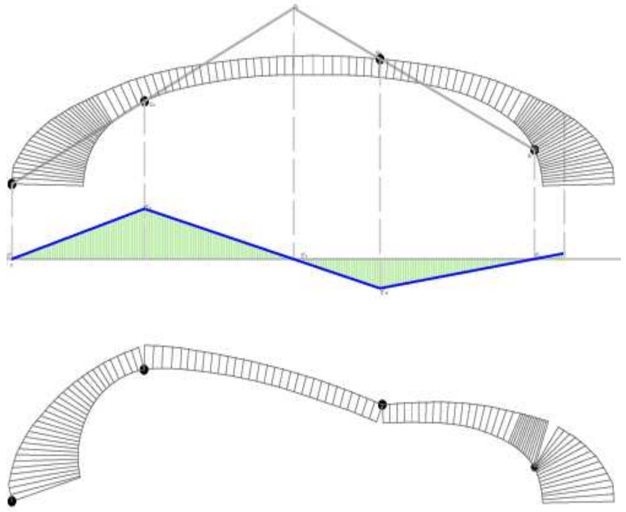

Figure 8: An asymmetric mechanism with activation of four hinges with its compatible deformed configuration.

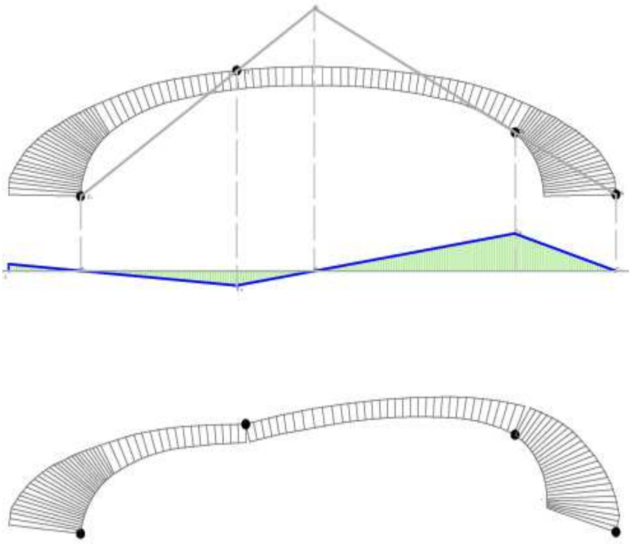

Figure 9: An asymmetric mechanism with activation of four hinges and its compatible deformed configuration.

For any of the selected mechanisms, the accidental loads are chosen in way to produce the worst response for the given allowed motion. The variable loads are deducted, according to the Italian instructions NTC2008, and then suitably placed on the first and second lanes of the roadways following the same instructions.

In this way, for each mechanism the most dangerous is identified by considering the maximization of the positive detrimental work developed by the variable loads for the allowed motion, as shown in Fig.10 and 11. The accidental loads are then placed each time in the worst position for the given mechanism, knowing that for each of them the most dangerous load condition is unique and uniquely determined.

The load multiplier with regards to the single load condition, is calculated through the ratio

$$\gamma_m = \frac{L_G^- - L_G^+}{L_a^+ - L_a^-}$$

where:
L_G *denotes the work produced by the permanent loads*
L_a *denotes the work produced by the accidental loads*
The index $(\cdot)^+$ *and* $(\cdot)^-$ *denote the positive and negative contributions respectively.*

In *Table 1* one reports the load multipliers identified for the considered mechanisms, and the individuation of the smallest multiplier which allows to identify the most dangerous load condition in the considered set.

Table 1: Load conditions and related multipliers.

Load condition	*Multiplier*	*Load Multiplier*
CdC1	γ_1	*13,2*
CdC2	γ_2	*12,01*
CdC3	γ_3	*16,9*
CdC4	γ_4	*18,3*
CdC5	$\boldsymbol{\gamma_5}$	***9,1***
CdC6	γ_6	*19,5*
CdC7	γ_7	*24,9*
CdC8	γ_8	*19,3*
CdC9	γ_9	*13,2*
CdC10	γ_{10}	*25,8*

IV. CONCLUSIONS

In this paper the stability assessment of an ancient masonry vaulted bridge is performed, and the most dangerous load condition is identified for the structure. A selection of more probable collapse mechanisms is evaluated in order to figure out the worst situation as regards the response of the main structural vault.

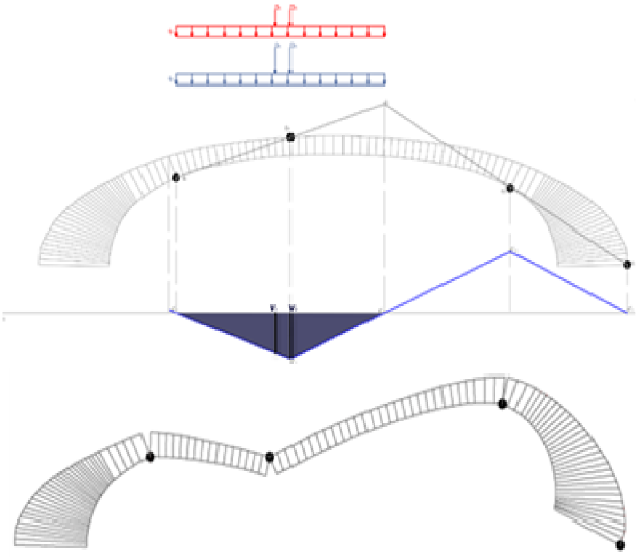

Figure 10: The most dangerous load condition identified within the set of selected mechanisms and the relevant compatible deformed configuration.

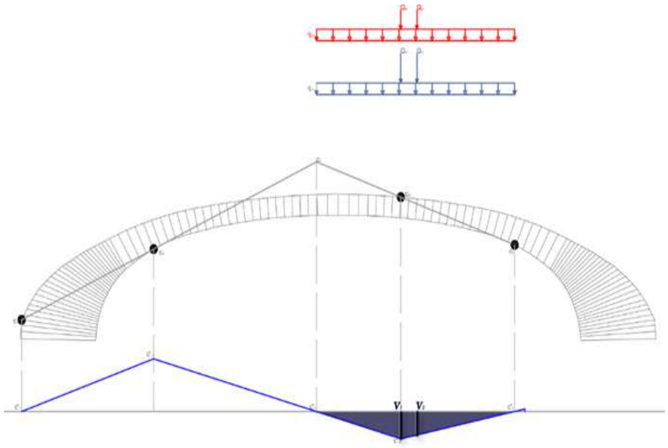

Figure 11: The less dangerous load condition identified within the set of selected mechanisms.

They may be considered some general schemes since the event that would affect the structure are several and they may be the consequence of different causes. The adopted approach, based on the Kinematic Theorem of Limit analysis extended to structures under the hypothesis of No Tension material, is aimed to identify the most dangerous load condition in the selected class, by considering the smaller load multiplier associated to the single mechanism.

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