Boundary Layer Flow and Heat Transfer of Nanofluids over a Moving Plate with Partial Slip and Thermal Convective Boundary Condition: Stability Analysis

Najwa Najib, Norfifah Bachok, Norihan Md. Arifin and Norazak Senu

Abstract— The steady boundary layer flow and heat transfer of nanofluids over a moving plate with partial slip and thermal convective boundary condition is studied. The governing nonlinear partial differential equations are first transformed into a system of nonlinear ordinary equations using a similarity transformation. The system then have been solved numerically using the bvp4c solver in Matlab. The numerical results are presented in tables and graphs for the skin friction coefficient and the local Nusselt number as well as the velocity and the temperature profile for a range of various parameters such as nanoparticles, nanoparticles volume fraction, slip parameter, Biot number and velocity ratio parameter. It is observed that the skin friction coefficient and the local Nusselt number which represents the heat transfer rate at the surface are significantly influenced by these parameters. The results indicate that dual solutions (first and second solutions) exist when the plate and free stream move in the opposite direction. A stability analysis has been performed to show which solutions are stable and physically realizable. Based on the analysis, the results indicate that the first solution is linearly stable, while the second solution is linearly unstable.

Keywords— Boundary layer flow, Heat transfer, Slip, Biot number, Nanofluid, Dual solutions, Stability analysis

I. INTRODUCTION

WATER, oil and ethylene glycol are the poor heat transfer fluids because of their poor thermal conductivity. By applying such fluids as cooling tools will enhance the manufacturing and operating costs. Then the nanoparticle is suspended in liquids (called nanofluid) to enhance the thermal conductivity of the fluids (see Uddin et al. [1]). Nanofluid has become one of the most important subjects of research due to its numerous applications in engineering and biomedical area. After that, many researchers have been studied the boundary layer flow of a nanofluid in different plate and effects [2-7]. Also, the study on thermal convective at the boundary condition has become an attraction among researchers due to their significant where the higher thermal convective will enhance the heat transfer rate at the surface [8-11].

The existence of dual or multiple solutions on the flow behavior become a question which solution is stable and unstable. The procedure to validate which solutions are in stable state was developed by Merkin [12]. The implemented method was used by other researchers and they found that the first solution is stable and physically realizable while the other solutions is not [13-15].

Our main objective in this present paper is to extend the work by Bachok et al. [16] by adding thermal convective at the boundary condition. The effects of the selected parameters on energy flow characteristics will be numerically study and discussed further. The bvp4c function is implemented in order to obtain the numerical results and to validate the first solution is stable whereas the second solution is not.

II. PROBLEM FORMULATION

Consider a two-dimensional laminar boundary layer flow on a fixed or continuously moving flat surface in a water-based nanofluid containing copper (Cu), Titania (TiO_2) and Alumina (Al_2O_3) nanoparticles. It is assumed that the plate moves in the same or opposite direction to the free stream, both with constant velocities. The nanoparticles are assumed to have a uniform spherical shape and size. The boundary layer equations are given by Bachok et al. [3]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2} \qquad (3)$$

subject to the boundary conditions

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$$u = U_w + L\left(\frac{\partial u}{\partial y}\right), v = 0, -k\left(\frac{\partial T}{\partial y}\right) = h_f\left(T_f - T\right), \text{ at } y = 0$$
$$u \to U_\infty, T \to T_\infty \quad \text{as } y \to \infty, \tag{4}$$

where U_w and U_∞ are constants and correspond to the plate velocity and the free stream velocity, respectively. Further, u and v are the velocity components along the $x-$ and $y-$ directions, respectively, L denotes the slip length, T is the temperature of the nanofluid, μ_{nf} is the viscosity of the nanofluid, α_{nf} is the thermal diffusivity of the nanofluid and ρ_{nf} is the density of the nanofluid, which are given by (Oztop and Abu-Nada [2])

$$\alpha_{nf} = \frac{k_{nf}}{\left(\rho C_p\right)_{nf}}, \mu_{nf} = \frac{\mu_f}{\left(1-\varphi\right)^{2.5}},$$
$$\rho_{nf} = \left(1-\varphi\right)\rho_f + \varphi\rho_s,$$
$$\left(\rho C_p\right)_{nf} = \left(1-\varphi\right)\left(\rho C_p\right)_f + \varphi\left(\rho C_p\right)_s, \tag{5}$$
$$\frac{k_{nf}}{k_f} = \frac{\left(k_s + 2k_f\right) - 2\varphi\left(k_f - k_s\right)}{\left(k_s + 2k_f\right) + \varphi\left(k_f - k_s\right)}.$$

Here, φ is the nanoparticle volume fraction, $\left(\rho C_p\right)_{nf}$ is the heat capacity of the nanofluid, k_{nf} is the thermal conductivity of the nanofluid, k_f and k_s are the thermal conductivities of the fluid and of the solid fractions, respectively, and ρ_f and ρ_s are the densities of the fluid and of the solid fractions, respectively. The use of the above expression for k_{nf} / k_f is restricted to spherical nanoparticles where it does not account for other shapes of nanoparticles [2,17]. Also, the viscosity of the nanofluid μ_{nf} has been approximated by Brinkman [18] as viscosity of a base fluid μ_f containing dilute suspension of fine spherical particles.

To obtain similarity solutions for the system of Eqs. (1) – (4), we introduce the following similarity variables

$$\eta = \left(\frac{U}{\nu_f x}\right)^{1/2} y, \ \psi = \left(\nu_f x U\right)^{1/2} f(\eta), \ \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \tag{6}$$

where U is the composite velocity defined as $U = U_w + U_\infty$. This definition of U was first introduced by Afzal et al. [19]. Further, ψ is the stream function defined as $u = \partial\psi / \partial y$ and $v = -\partial\psi / \partial x$, which identically satisfies Eq. (1). Employing the similarity variables (6), Eqs. (2) and (3) reduce to the following ordinary differential equations:

$$\frac{1}{(1-\varphi)^{2.5}(1-\varphi+\varphi\rho_s/\rho_f)} f''' + \frac{1}{2} ff'' = 0 \tag{7}$$

$$\frac{1}{Pr}\frac{k_{nf}/k_f}{\left[1-\varphi+\varphi(\rho C_p)_s/(\rho C_p)_f\right]}\theta'' + \frac{1}{2} f\theta' = 0 \tag{8}$$

subjected to the boundary conditions (4) which become

$$f(0) = 0, \ f'(0) = \lambda + \sigma f''(0), \ \theta'(0) = Bi\left(\theta(0) - 1\right),$$
$$f'(\eta) \to 1 - \lambda, \quad \theta(\eta) \to 0 \quad \text{as } \eta \to \infty. \tag{9}$$

In the above equations, primes denote differentiation with respect to η. Here Pr $(= \nu_f / \alpha_f)$ is the Prandtl number, λ is the velocity ratio parameter, σ is the slip parameter and Biot number are defined as

$$\lambda = \frac{U_w}{U}, \ \sigma = LU\left(\frac{U}{\nu_f x}\right)^{1/2}, \ Bi = \frac{c}{k_f}\sqrt{\nu_f / U}, \tag{10}$$

respectively. More, for energy equation to have a similarity solution, the quantity Bi must be a constant and not a function of x. This condition can be met if the heat transfer coefficient h_f is proportional to $x^{-1/2}$. Therefore, we assume

$$h_f = cx^{-1/2} \tag{11}$$

where c is a constant (see Aziz [8]).

The case $0 < \lambda < 1$ is when the plate and the fluid move in the same direction, while they move in the opposite directions when $\lambda < 0$, and when $\lambda > 1$. If $\lambda < 0$, the free stream is directed towards the positive $x-$direction, while the plate moves towards the negative $x-$direction. If $\lambda > 1$, the free stream is directed towards the negative $x-$direction, while the plate moves towards the positive $x-$direction. However, in this paper we consider only the case $\lambda \le 1$, i.e. the direction of the free stream is fixed (towards the positive $x-$direction). It is worth mentioning that the present problem reduces to those considered by Ahmad et al. [20] when $\lambda = 0$ and $\lambda = 1$. Further, without the energy equation and when $\varphi = 0$ (regular fluid), the present problem reduces to those of Blasius [21] when $\lambda = 0$, and to those of Sakiadis [22] when $\lambda = 1$.

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x, which are defined as

$$C_f = \frac{\tau_w}{\rho_f U^2}, \quad Nu_x = \frac{xq_w}{k_f(T_w - T_\infty)}, \tag{12}$$

where the surface shear stress τ_w and the surface heat flux q_w are given by

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}, q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0}, \tag{13}$$

with μ_{nf} and k_{nf} being the dynamic viscosity and thermal conductivity of the nanofluids, respectively. Using the similarity variables (6), we obtain

$$C_f Re_x^{1/2} = \frac{1}{(1-\varphi)^{2.5}} f''(0), \tag{14}$$

$$Nu_x / Re_x^{1/2} = -\frac{k_{nf}}{k_f}\theta'(0), \tag{15}$$

where $Re_x = Ux/\nu_f$ is the local Reynolds number.

III. Stability flow

In order to perform a stability analysis, we consider the unsteady problem. Eq. (1) holds, while Eqs. (2) and (3) are replaced by

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} \tag{16}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} \tag{17}$$

where t denotes the time. Based on the variables (6), we introduce the following new dimensionless variables:

$$\eta = \left(\frac{U}{\nu_f x}\right)^{1/2} y, \quad \psi = \left(\nu_f x U\right)^{1/2} f(\eta,\tau), \quad \theta(\eta,\tau) = \frac{T-T_\infty}{T_w - T_\infty}, \quad \tau = \frac{U}{x}\mathrm{t}, \tag{18}$$

so that Eqs. (2) and (3) can be written as

$$\frac{1}{(1-\varphi)^{2.5}(1-\varphi+\varphi\rho_s/\rho_f)}\frac{\partial^3 f}{\partial\eta^3} + \frac{1}{2} f\frac{\partial^2 f}{\partial\eta^2} - \frac{\partial^2 f}{\partial\eta\partial\tau} = 0 \tag{19}$$

$$\frac{1}{Pr}\frac{k_{nf}/k_f}{\left[1-\varphi+\varphi(\rho C_p)_s/(\rho C_p)_f\right]}\frac{\partial^2\theta}{\partial\eta^2} + \frac{1}{2} f\frac{\partial\theta}{\partial\eta} - \frac{\partial\theta}{\partial\tau} = 0 \tag{20}$$

and are subjected to the boundary conditions

$$f(0,\tau)=0,\ \frac{\partial f}{\partial\eta}(0,\tau) = \lambda + \sigma\frac{\partial^2 f}{\partial\eta^2}(0,\tau), \quad \left(\frac{\partial\theta}{\partial\eta}\right)(0,\tau) = Bi\left(\theta(0,\tau)-1\right), \quad \frac{\partial f}{\partial\eta}(\eta,\tau)\to 1-\lambda,\ \theta(\eta,\tau)\to 0 \quad \text{as } \eta\to\infty \tag{21}$$

To test the stability of the steady flow solution $f(\eta) = f_0(\eta)$ and $\theta(\eta) = \theta_0(\eta)$ satisfying the boundary value problem (1) – (4), we write

$$f(\eta,\tau) = f_0(\eta) + e^{-\gamma\tau}F(\eta), \quad \theta(\eta,\tau) = \theta_0(\eta) + e^{-\gamma\tau}G(\eta), \tag{22}$$

where γ is an unknown eigenvalue, and $F(\eta)$ and $G(\eta)$ are small relative to $f_0(\eta)$ and $\theta_0(\eta)$. Solutions of the eigenvalue problem (19) – (21) give an infinite set of eigenvalues $\gamma_1 < \gamma_2 < ...$; if the smallest eigenvalue is negative, there is an initial growth of disturbances and the flow is unstable but when γ_1 is positive, there is an initial decay and the flow is stable. Introducing (22) into (19) and (20), we get the following linearized problem

$$\frac{1}{(1-\varphi)^{2.5}(1-\varphi+\varphi\rho_s/\rho_f)}F_0''' + \frac{1}{2}\left(f_0 F_0'' + f_0'' F_0\right) + \gamma F_0' = 0 \tag{23}$$

$$\frac{1}{Pr}\frac{k_{nf}/k_f}{\left[1-\varphi+\varphi(\rho C_p)_s/(\rho C_p)_f\right]}G_0'' + \frac{1}{2}\left(f_0 G_0' + F_0\theta_0'\right) - G_0' + \gamma G_0 = 0 \tag{24}$$

along with the new boundary conditions

$$F_0(0)=0,\ F_0'(0) = \sigma F_0''(0),\ G_0'(0) = BiG_0(0), \quad F_0'(\eta)\to 0,\quad G_0(\eta)\to 0 \quad \text{as } \eta\to\infty \tag{25}$$

It should be stated that for particular values of Pr and γ, the stability of the corresponding steady flow solutions $f_0(\eta)$ and $\theta_0(\eta)$ are determined by the smallest eigenvalue γ. As it has been suggested by Harris et al. [23], the range of possible eigenvalues can be determined by relaxing a boundary condition on $F_0(\eta)$ or $G_0(\eta)$. For the present problem, we relax the condition that $F_0'(\eta)\to 0$ as $\eta\to\infty$ and for a fixed value of γ we solve the system (26, 27, 28) along with the new boundary condition $F_0''(0)=1$.

IV. Results and discussion

Numerical solutions to the governing ordinary differential equations (7) and (8) with the boundary conditions (9) are obtained using bvp4c solver in Matlab. We consider three different types of nanoparticles namely copper (Cu), Titania (TiO_2) and Alumina (Al_2O_3) with water as the base working fluid. Following to Oztop and Abu-Nada [2], we have considered the range for nanoparticle volume fraction φ as $0 \le \varphi \le 0.2$, where $\varphi = 0$ is regular fluid with Prandtl number $Pr = 6.2$. Further, the thermophysical properties of nanoparticles and fluid are shown in Table 1. In order to validate the numerical results obtained, we have compare the present results for λ_c with those reported by Bachok et al. [16] as illustrated in Table 2. The comparison shows very good agreement. The dual solutions are obtained by setting different initial guesses for the missing values of the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$, where all profiles satisfy the far field boundary conditions (9) asymptotically but with different shapes of profiles and then being illustrated in graphs.

Figures 1 and 7 illustrate the variation of skin friction $f''(0)$ and local Nusselt number $-\theta'(0)$ for different nanoparticles, Biot number, nanoparticle volume fraction and also slip parameter. It is seen that the solution is unique when $\lambda \geq 0$, while multiple (dual) solution exist up to $\lambda_c < \lambda < 0$, where the plate and free stream moves in the opposite direction. However, no solutions are found to exist when $\lambda > \lambda_c$. The variations of the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ for different nanoparticles were shown in Figs. 1 and 2, respectively. From the figures, Cu have higher surface shear stress and heat transfer rate compared to TiO_2 and Al_2O_3.

Figure 3 presents the local Nusselt number $-\theta'(0)$ for Cu nanoparticle for various values of Biot number *Bi*. The higher value of *Bi* will cause the stronger convection occur and hence will lead to increase the heat transfer rate at the surface.

The variations of the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ for TiO_2 and some values of nanoparticle volume fraction φ were shown in Figs. 4 and 5, respectively. As the nanoparticles volume fraction φ is increasing, then the skin friction coefficient and heat transfer rate at the surface will increased.

Figure 6 and 7 illustrate the variations of skin friction coefficient $f''(0)$ and local Nusselt number $-\theta'(0)$ of slip parameter σ for Al_2O_3 nanoparticle. When the slip occur in between the boundary layer, the boundary layer separation is drag to happen and hence the λ_c will become larger (if $\sigma > 0$). The skin friction coefficient and the heat loss from the surface will increases as the slip parameter σ is increasing. Therefore, the presence of slip will accelarates the separation of the boundary layer.

The validity of these numerical solutions and dual nature solutions is supported by the velocity and temperature profile presented in Figs. 8 – 10. These profiles are satisfied the boundary conditions (9) assymptotically with different shapes of graphs. The solid line represents the first solution while the dash line represents second solution. As we can see, the boundary layer thickness for second solution will always greater then the first solution.

A stability analysis is performed using bvp4c function in Matlab software to determine which solution is stable (first or second solution). The linear eigenvalue problem (26) and (27) is used to find the unknown eigenvalue γ subjected to the new boundary condition (28). If the smallest eigenvalue is negative, there is an initial growth of disturbance and the flow is unstable while for the positive smallest eigenvalue, there is an initial decay and the flow is stable. The smallest eigenvalue γ for some values of σ at selected values of λ are stated in Table 3 which shows that γ is positive for the first solution and negative for second solution. The eigenvalue γ is approaching 0 as λ is approaching λ_c ($\gamma \to 0$ as $\lambda \to \lambda_c$) either from positive or negative sign. Thus, the first solution is stable, while the second solution is unstable.

Table 1. Thermophysical properties of fluid and nanoparticles (Oztop and Abu-Nada, [2]).

Physical properties	Fluid phase (water)	*Cu*	TiO_2	Al_2O_3
C_p (J/kg K)	4179	385	686.2	765
ρ (kg/m^3)	997.1	8933	4250	3970
k (W/mK)	0.613	400	8.9538	40
$\alpha \times 10^7$ (m^2/s)	1.47	1163.1	30.7	131.7
$\beta \times 10^{-5}$ (1/K)	21	1.67	0.9	0.85

Table 2. Variations of λ_c with φ = 0.1 and σ = 0.2 for different nanoparticles.

σ	Bachok et al. [16]	Present Result
Cu	-0.6821	-0.6821
TiO_2		-0.6595
Al_2O_3		-0.6581

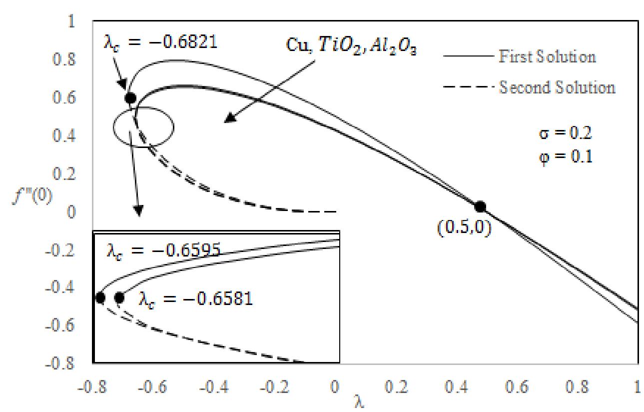

Fig. 1. Skin friction coefficient $f''(0)$ as a function of λ for various nanoparticles.

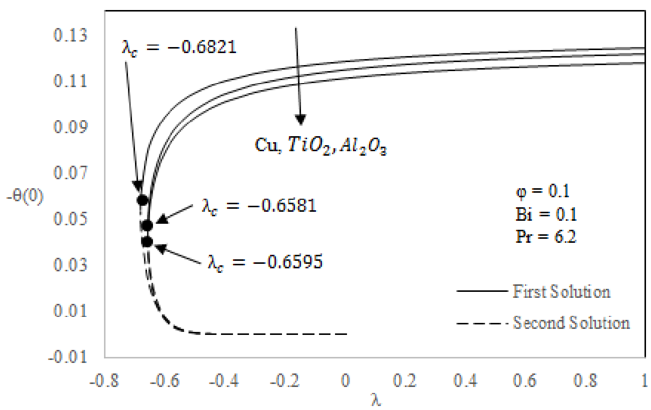

Fig. 2. Local Nusselt number coefficient $-\theta'(0)$ as a function of λ for various various nanoparticles.

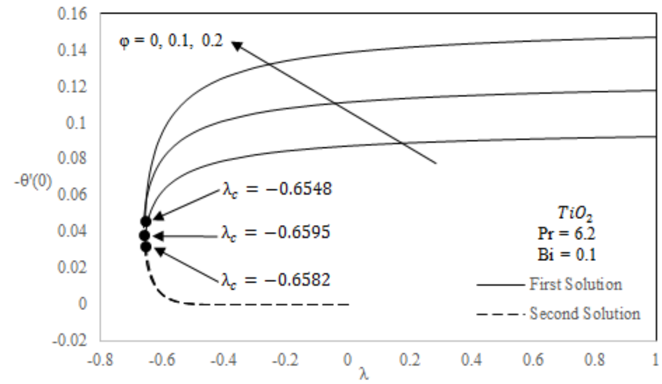

Fig. 5. Local Nusselt number coefficient $-\theta'(0)$ as a function of λ for various values of φ.

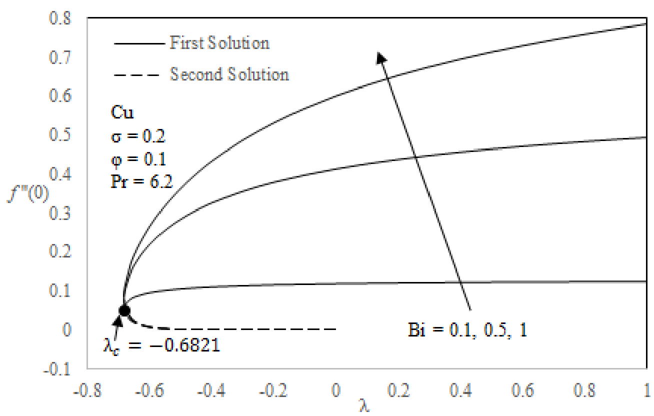

Fig. 3. Local Nusselt number coefficient $-\theta'(0)$ as a function of λ for various values of Biot number.

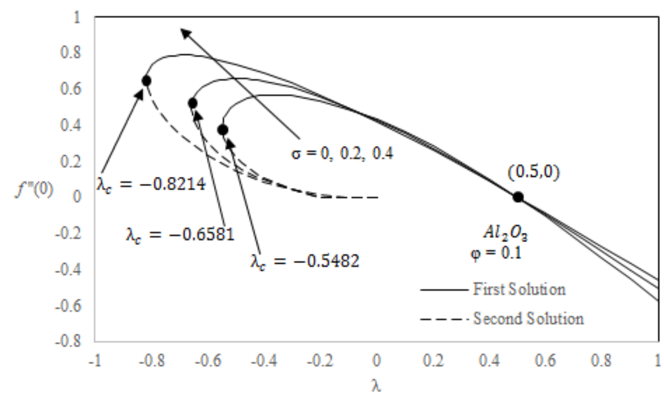

Fig. 6. Skin friction coefficient $f''(0)$ as a function of λ for various values of σ.

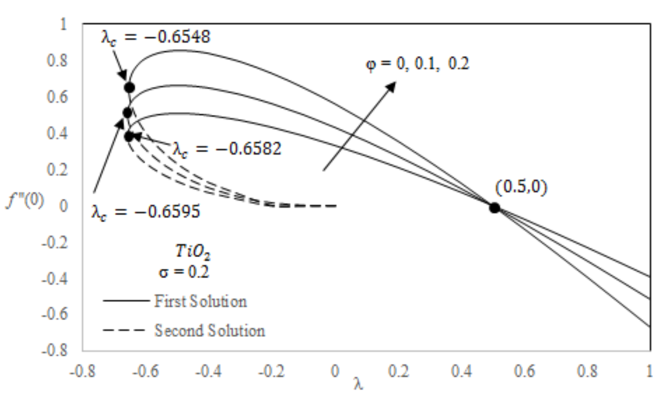

Fig. 4. Skin friction coefficient $f''(0)$ as a function of λ for various values of φ.

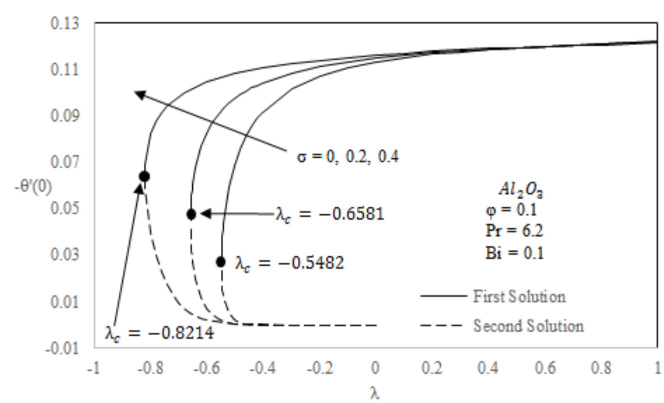

Fig. 7. Local Nusselt number coefficient $-\theta'(0)$ as a function of λ for various values of σ.

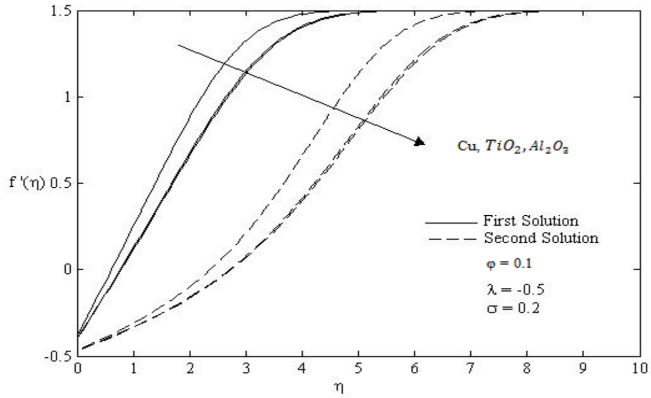

Fig. 8. Velocity profiles $f'(\eta)$ for various nanoparticles.

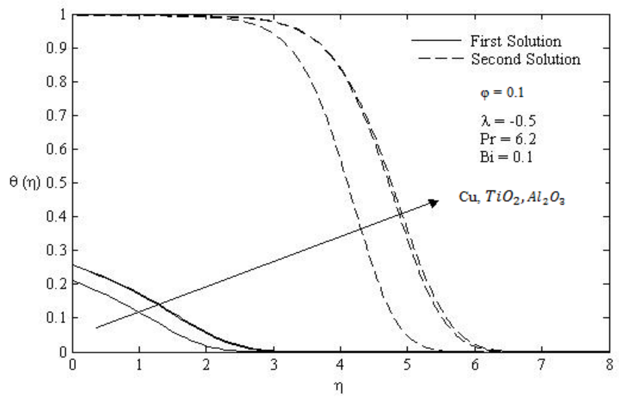

Fig. 9. Temperature profiles $\theta(\eta)$ for various nanoparticles.

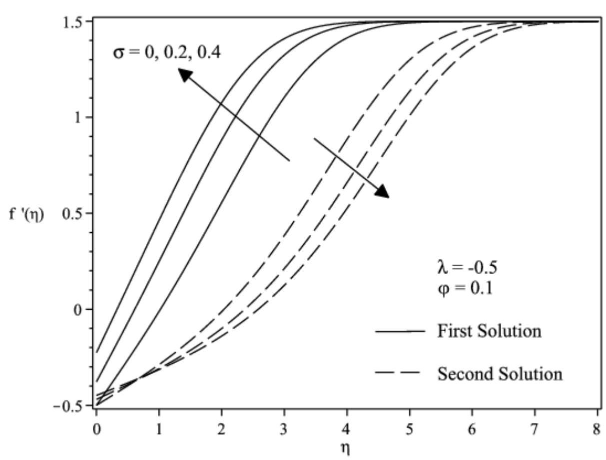

Fig. 10. Velocity profiles $f'(\eta)$ for various of σ.

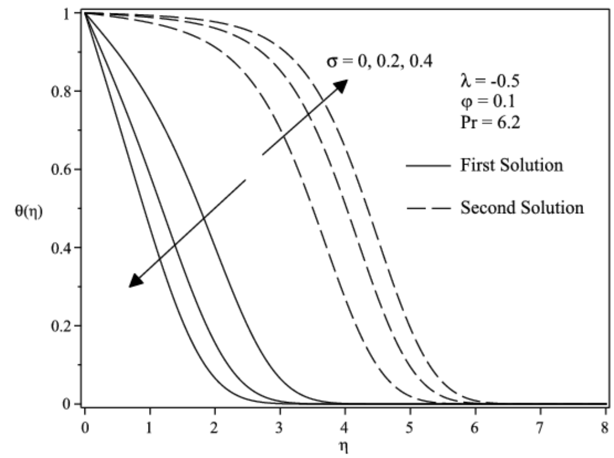

Fig. 11. Temperature profiles $\theta(\eta)$ for various of σ.

Table 3. Smallest eigenvalues γ at selected values of λ for different nanoparticle when φ = 0.1 and σ = 0.2.

Nanoparticle	λ	Present result	
		First Solution	Second Solution
Cu	-0.6821	0.0009	-0.0009
	-0.682	0.0041	-0.0040
	-0.68	0.0186	-0.0175
	-0.6	0.1299	-0.0884
Al_2O_3	-0.6581	0.0022	-0.0022
	-0.658	0.0046	-0.0045
	-0.65	0.0379	-0.0336
	-0.6	0.1090	-0.0783
TiO_3	-0.6595	0.0027	-0.0027
	-0.659	0.0094	-0.0092
	-0.65	0.0412	-0.0362
	-0.6	0.1104	-0.0790

V. CONCLUSION

This paper considers the steady boundary layer flow and heat transfer on a moving plate in nanofluid with presence of slip effect and thermal convective. The stability analysis is also implemented to determine which solution is stable an unstable. The effects of nanoparticles, nanoparticle volume fraction φ, slip parameter σ as well as Biot number on skin friction coefficient and heat transfer rate at the surface were investigated and discussed. The results indicate that

- dual solution existed in a certain range when the free stream and plate move in opposite direction.
- Cu has higher surface shear stress and heat transfer rate

compared with TiO_2 and Al_2O_3.

- as Biot number increase, heat transfer rate also increases.
- skin friction and heat transfer rate increase as nanoparticle volume fraction increase.
- the presence of slip parameter σ will widen the range of velocity ratio parameter λ.
- the first solution is linearly stable and can be realize physically while the second solution is linearly unstable and would not be realize physically.

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